

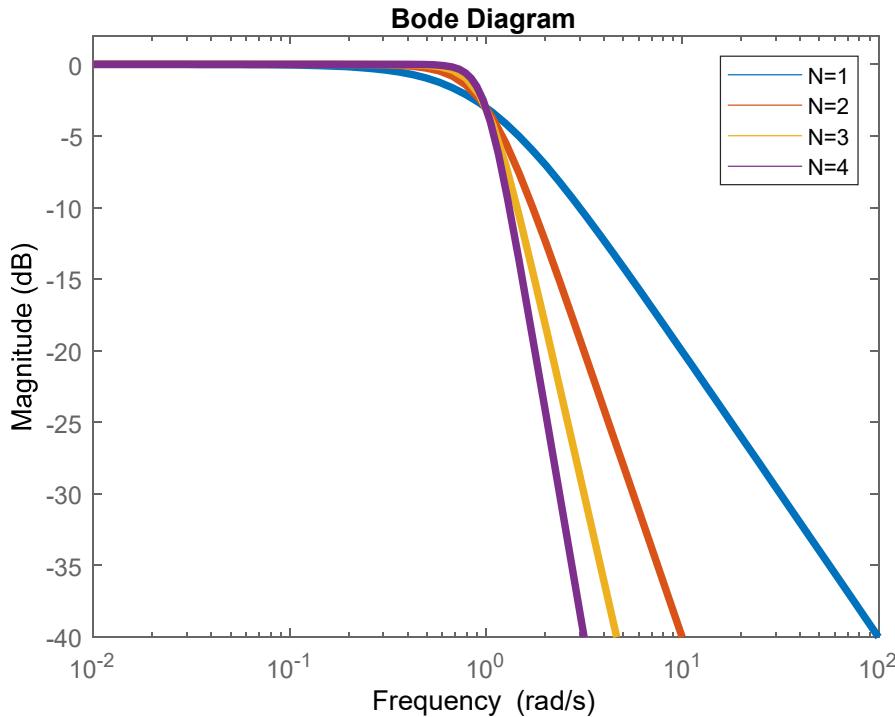
Announcements

- Return Analog Discovery Studio & Exp 1 parts in class
- Final Exam
- TNvoice Open
 - <https://utk.campuslabs.com/eval-home/>
 - Please fill out – Closes midnight May 11
 - Currently **79%** response rate
 - +5 pts EC on final for 100% response rate

Canonical N^{th} Order Filter Designs

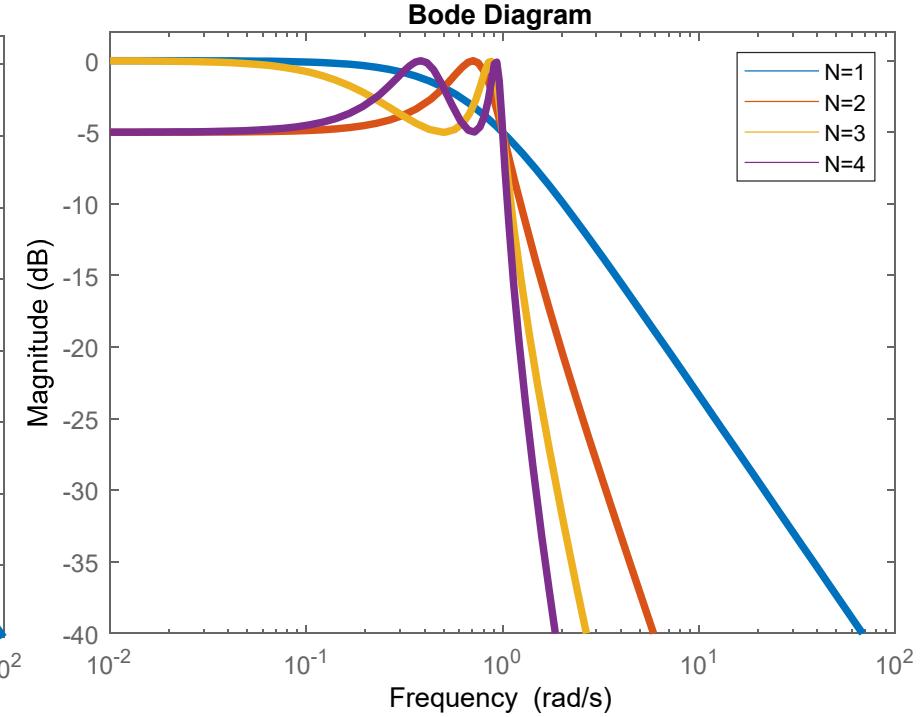
Butterworth

$$|\mathbf{H}(j\omega)| = \frac{K}{\sqrt{1 + (\omega/\omega_c)^{2n}}} \quad n = 1, 2, 3, \dots$$



Chebyshev

$$|\mathbf{H}(j\omega)| = \frac{K}{\sqrt{1 + \beta^2 C_n^2(\omega/\omega_c)}} \quad n = 1, 2, 3, \dots$$



Example 15.15

EXAMPLE 15.15

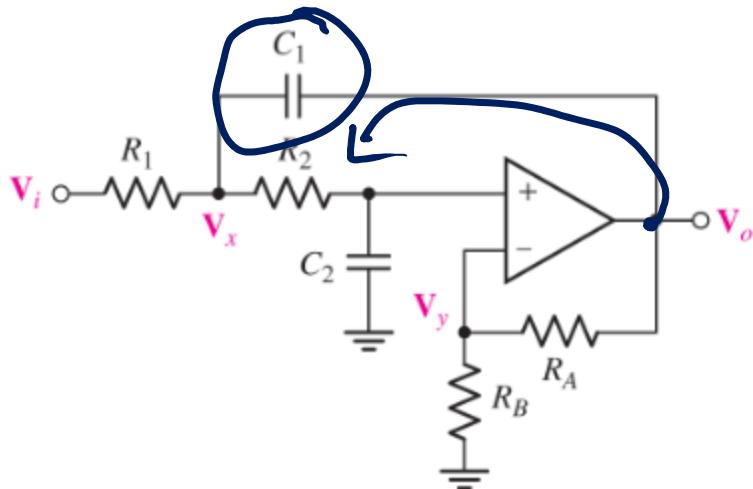
Design a second-order low-pass Butterworth filter having a gain of 4 and a corner frequency at 1400 rad/s.

TABLE 15.2 Coefficients for Low-Pass Butterworth and Chebyshev ($\beta = 0.9976$, or 3 dB) Filter Functions,
Normalized to $\omega_c = 1$

Butterworth					
n	a_0	a_1	a_2	a_3	a_4
1	1.0000				
2	1.0000	1.4142			
3	1.0000	2.0000	2.0000		
4	1.0000	2.6131	3.4142	2.6131	
5	1.0000	3.2361	5.2361	5.2361	3.2361

Chebyshev ($\beta = 0.9976$)					
n	a_0	a_1	a_2	a_3	a_4
1	1.0024				
2	0.7080	0.6449			
3	0.2506	0.9284	0.5972		
4	0.1770	0.4048	1.1691	0.5816	
5	0.0626	0.4080	0.5489	1.4150	0.5744

The Sallen-Key Amplifier



$$G \equiv \frac{R_A + R_B}{R_B}$$

$$\frac{V_o}{V_i} = \frac{\frac{G}{R_1 R_2 C_1 C_2}}{s^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1 - G}{R_2 C_2} \right] s + \frac{1}{R_1 R_2 C_1 C_2}}$$

Final Exam

- Friday May 13th, 3:30-6:00pm
- Roughly 2x midterm in length, w/ 3x time
- Covers all course material
 - Chapters 10-11, 13-15 & 17(partial)
 - All homeworks, quizzes, exams, and experiments
1&2
 - All lectures

Final Exam Problems (Tentative)

- Power in the sinusoidal steady-state
- Bode plot (plot $\rightarrow H(s)$ and/or $H(j\omega)$ \rightarrow plot)
- Find $H(s)$ given a circuit schematic
- Identify bounded/stable systems
- Solve Laplace with complex and repeated poles and zeroes
- Evaluate Fourier Series and frequency response
- Additional Notes:
 - Circuits will contain coupled inductors and/or transformers

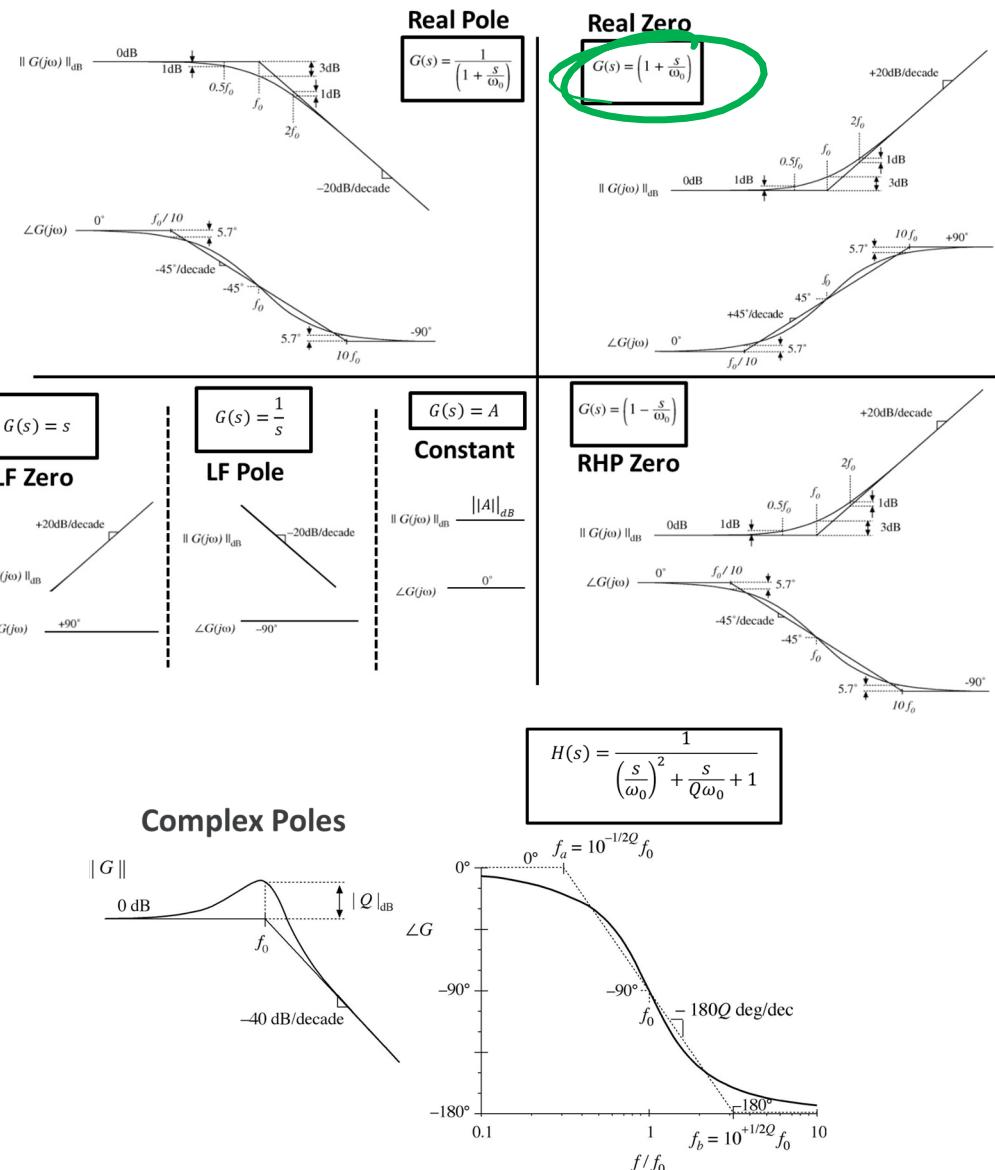
Exam Tables

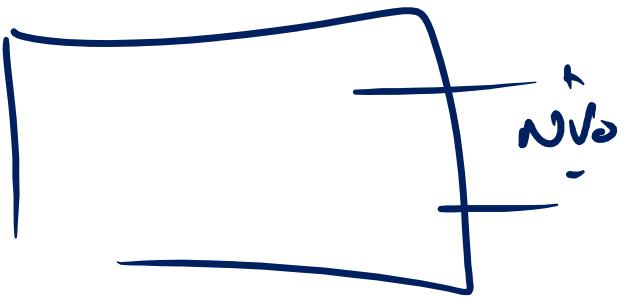
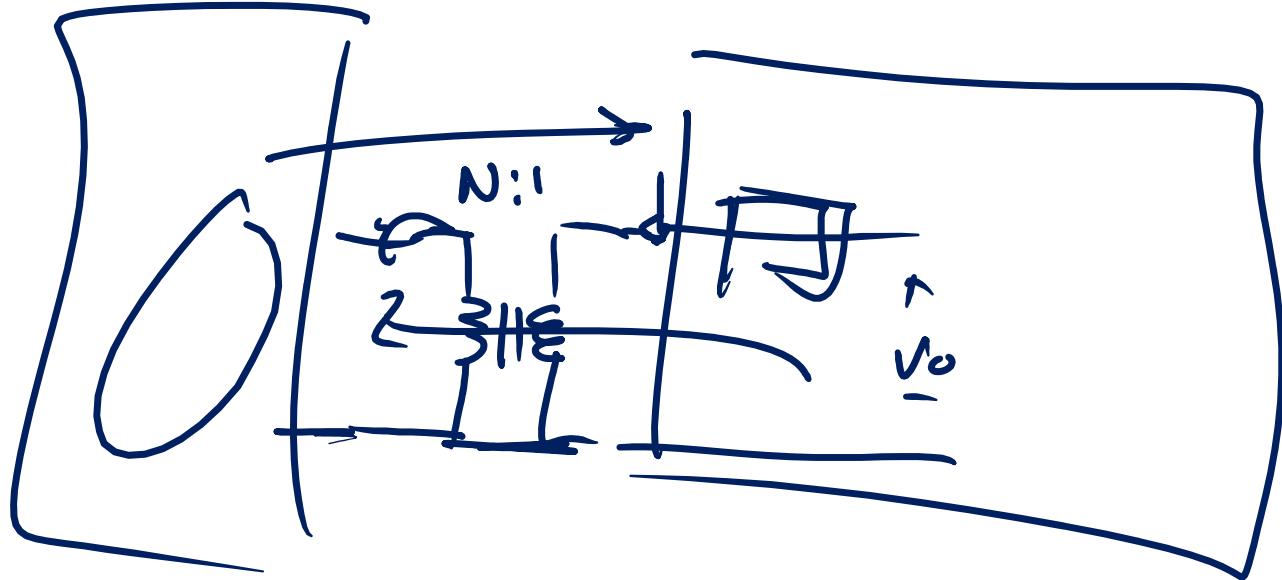
TABLE 14.1 Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}\{\mathbf{F}(s)\}$	$\mathbf{F}(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{\mathbf{F}(s)\}$	$\mathbf{F}(s) = \mathcal{L}\{f(t)\}$
$\delta(t)$	1	$\frac{1}{\beta - \alpha}(e^{-at} - e^{-\beta t})u(t)$	$\frac{1}{(s + \alpha)(s + \beta)}$
$u(t)$	$\frac{1}{s}$	$\underline{\sin \omega t u(t)}$	$\frac{\omega}{s^2 + \omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$
$\frac{t^n - 1}{(n - 1)!}u(t), n = 1, 2, \dots$	$\frac{1}{s^n}$	$\sin(\omega t + \theta)u(t)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\cos(\omega t + \theta)u(t)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^2}$	$e^{-\alpha t} \sin \omega t u(t)$	$\frac{\omega}{(s + \alpha)^2 + \omega^2}$
$\frac{t^n - 1}{(n - 1)!}e^{-\alpha t}u(t), n = 1, 2, \dots$	$\frac{1}{(s + \alpha)^n}$	$e^{-\alpha t} \cos \omega t u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$

$$z(t)e^{at} \cos(\omega t - \Phi)u(t) \quad s \frac{1}{s + (\omega_0^2)} + \frac{a}{s + (\omega_0^2)}$$

Operation	$f(t)$	$\mathbf{F}(s)$
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(s)$
Time differentiation	$\frac{df}{dt}$ $\frac{d^2f}{dt^2}$ $\frac{d^3f}{dt^3}$	$s\mathbf{F}(s) - f'(0^-)$ $s^2\mathbf{F}(s) - sf'(0^-) - f''(0^-)$ $s^3\mathbf{F}(s) - s^2f'(0^-) - sf''(0^-) - f'''(0^-)$
Time integration	$\int_0^t f(t) dt$ $\int_{-\infty}^t f(t) dt$	$\frac{1}{s}\mathbf{F}(s)$ $\frac{1}{s}\mathbf{F}(s) + \frac{1}{s} \int_{-\infty}^0 f(t) dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$
Time shift	$f(t - a)u(t - a), a \geq 0$	$e^{-as}\mathbf{F}(s)$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(s + a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty \mathbf{F}(s) ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} s\mathbf{F}(s)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} s\mathbf{F}(s)$, all poles of $s\mathbf{F}(s)$ in LHP
Time periodicity	$f(t) = f(t + nT), n = 1, 2, \dots$	$\frac{1}{1 - e^{-Ts}}\mathbf{F}_1(s)$, where $\mathbf{F}_1(s) = \int_{0^-}^T f(t) e^{-st} dt$



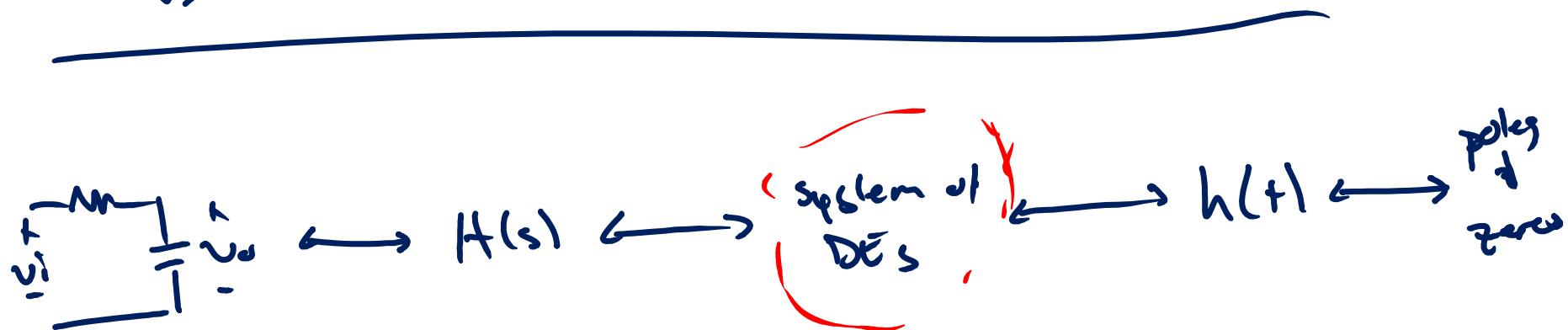


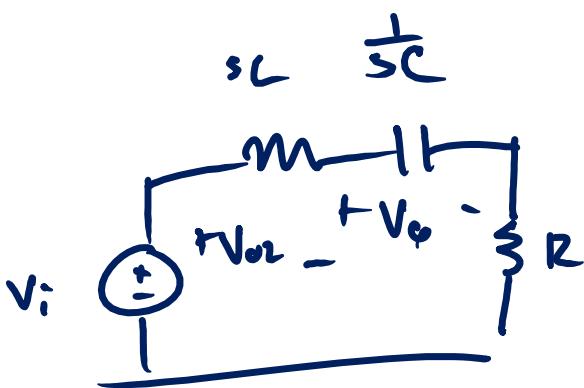
$$V_o(t) \rightarrow \frac{d}{dt} V_o(t) - V_o(t)$$

$$V_I(s) = sV_o(s) - \cancel{V_o(s)} - V_o(s)$$

$$V_I(s) = (s-1) V_o(s)$$

$$\frac{V_o}{s} = H = \frac{1}{s-1} \rightarrow \text{pole in RHP}$$





$$\frac{V_o}{V_i} = H(s) = \frac{\frac{1}{sC}}{sL + \frac{1}{sC} + R} = \frac{1}{s^2LC + sCR + 1}$$

$$\frac{1}{\omega_0^2} = LC \rightarrow \frac{1}{\sqrt{LC}} = \omega_0$$

$$\frac{1}{(\frac{s}{\omega_0})^2 + \frac{1}{Q\omega_0} + 1}$$

$$Q = \frac{1}{2R}$$

$$\frac{1}{Q\omega_0} = CR$$

$$\rightarrow Q = \frac{1}{\omega_0 CR} = \frac{\sqrt{LC}}{CR} = \frac{\sqrt{Lc}}{2} = \frac{R_0}{R}$$

$$H_2 = \frac{V_o}{V_i} = \frac{sL}{sL + \frac{1}{sC} + R} = \frac{s^2LC}{s^2LC + sCR + 1}$$

COURSE REVIEW

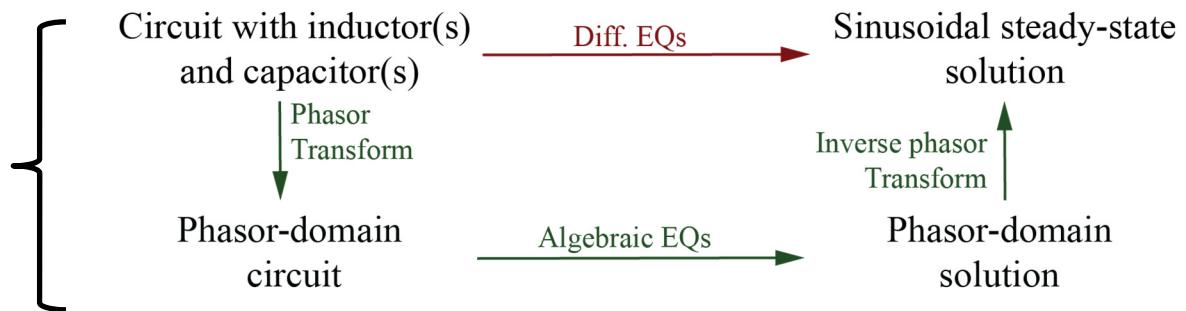
Course Content

- Magnetically Coupled Circuits (Ch 13)
- Sinusoidal Steady-State Analysis (Ch 10)
- AC Circuit Power Analysis (Ch 11)
- Circuit An Analysis in the s-Domain (Ch 14)
- Frequency Response (Ch 15)
- Fourier Circuit Analysis (Ch 17)
- Polyphase Circuits (Ch 12)
- Two-Port Networks (Ch 16)

Transform Domains

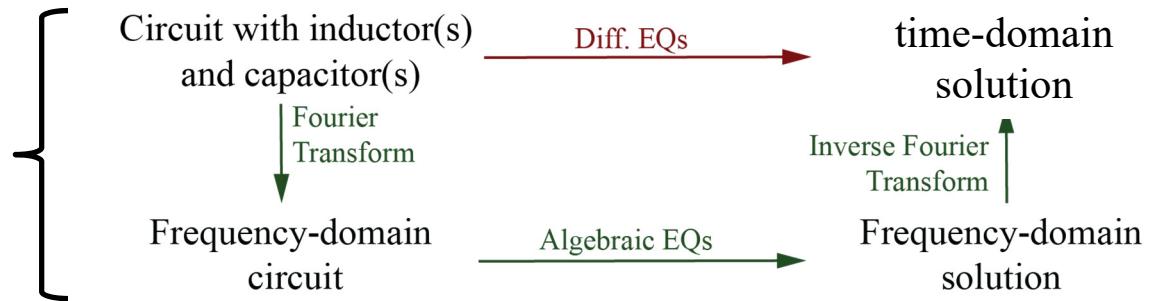
Phasor Transform

$$f(t) = A\cos(\omega_0 t + \varphi)$$



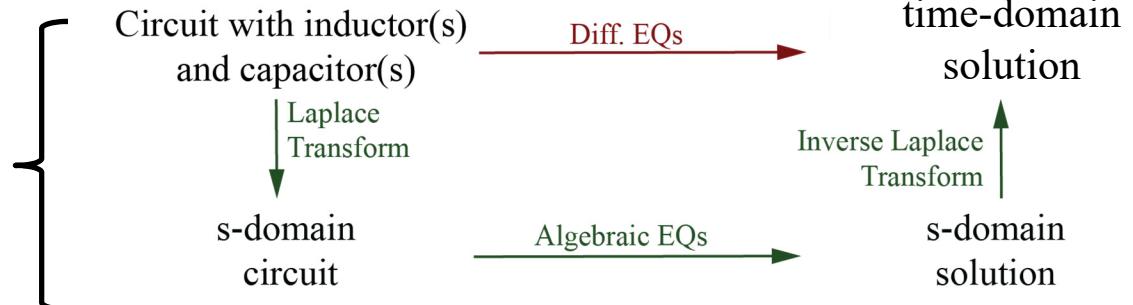
Fourier Transform

$$f(t) = \sum C_n \cos(n\omega_0 t + \varphi_n)$$



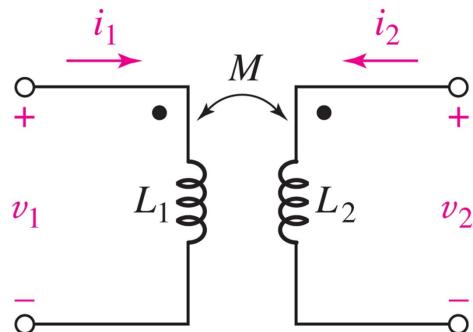
Laplace Transform

$$f(t) = \int e^{st} F(s) ds$$



Ch 13 – Magnetically Coupled Circuits

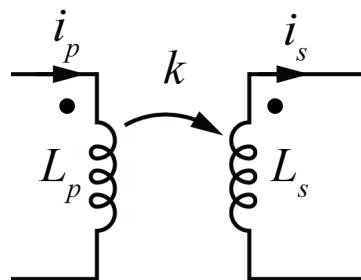
Coupled Inductors



Defining Equations

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



Coupling Coefficient

$$k = \frac{M}{\sqrt{L_p L_s}}$$

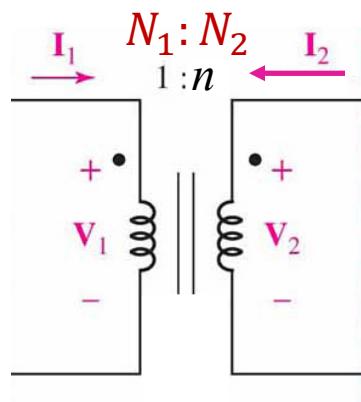
Dot convention

- Current into the dot on one terminal produces a positive open circuit voltage w.r.t. the dot on the other

Recall

- Equivalent circuits

Ideal Transformer



Defining Equations

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = \dots$$

$$0 = N_1 I_1 + N_2 I_2 + \dots$$

Coupled inductors with:

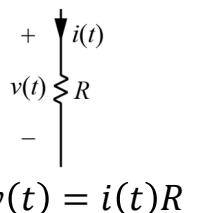
- no energy storage ($L \rightarrow \infty$)
- Perfect coupling ($k=1$)

Recall

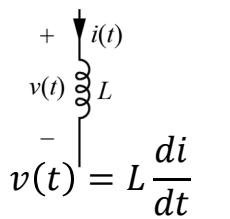
- Z/V/I reflection

Ch 10 – Sinusoidal Steady State

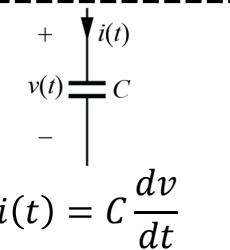
Time Domain



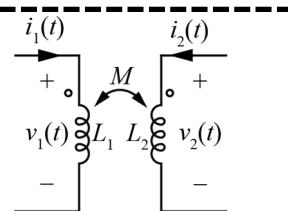
$$v(t) = i(t)R$$



$$v(t) = L \frac{di}{dt}$$

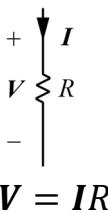


$$i(t) = C \frac{dv}{dt}$$

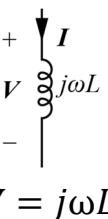


$$\begin{aligned} v_1(t) &= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ v_2(t) &= M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{aligned}$$

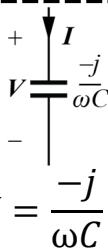
Phasor Domain



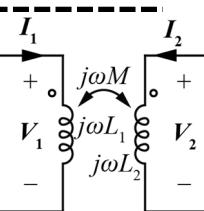
$$V = IR$$



$$V = j\omega LI$$



$$V = \frac{-j}{\omega C} I$$



$$\begin{aligned} V_1 &= j\omega L_1 I_1 + j\omega M I_2 \\ V_2 &= j\omega M I_1 + j\omega L_2 I_2 \end{aligned}$$

Phasor Notation

$$A\cos(\omega t + \varphi) \Leftrightarrow A\angle\varphi$$

$$= Re\{Ae^{j(\omega t+\varphi)}\}$$

$$= Re\{A\cos(\omega t + \varphi) + jA\sin(\omega t + \varphi)\}$$

Impedance and Admittance

$$Z = R + jX$$

↑

Impedance	Resistance	Reactance
-----------	------------	-----------

$$Y = \frac{1}{Z} = G + jB$$

↑

Admittance	Conductance	Susceptance
------------	-------------	-------------

Circuit Analysis

- Real circuits always have all real signals in the time domain
- All 201 analysis techniques apply
- Gives only forced/steady-state/particular response, for single sinusoidal source
- Phasor superposition

Ch 11 – AC Power Analysis

Average (DC) Power: $P = \int_{-\infty}^{\infty} p(t)dt$

For periodic signals: $P = \int_{t_0}^{t_0+T} p(t)dt$

Average power in a resistor: $P_R = \left[\sqrt{\int i(t)^2 dt} \right]^2 R$

$I_{rms} = I_{eff}$

For sinusoids: $I_{rms} = \frac{I_A}{\sqrt{2}}$

Sinusoidal Power

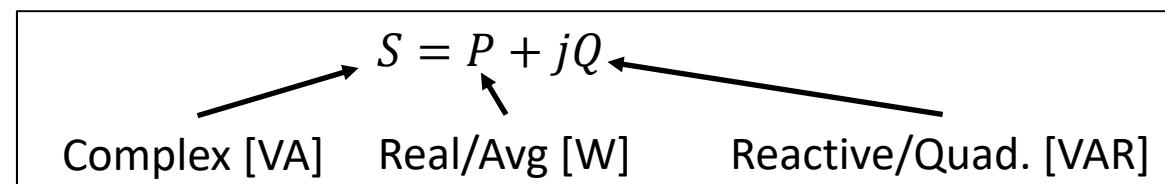
$$p(t) = [V_A \cos(\omega t + \varphi_V)][I_A \cos(\omega t + \varphi_I)]$$

$$= \underbrace{\frac{V_A I_A}{2} \cos(2\omega t + \varphi_V + \varphi_I)}_{\text{Double-frequency}} + \underbrace{\frac{V_A I_A}{2} \cos(\varphi_V - \varphi_I)}_{\text{DC = average}}$$

$$\frac{V_A I_A}{2} = V_{rms} I_{rms}$$

Complex Power

$$S = \frac{VI^*}{2} = V_{rms} I_{rms}$$



Apparent Power: $|S| = \frac{V_A I_A}{2} = V_{rms} I_{rms}$

Power Factor: $PF = \frac{P}{|S|}$ (leading/lagging)

Impedance match for max power transfer: $Z_L = Z_{th}^*$

Ch 14 – Laplace Transform

Unilateral Laplace Transform:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t) dt$$

Inverse Laplace Transform:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j}$$

, σ_0 in ROC

(to the right of all poles in complex plane)

Complex frequency = Laplace variable = $s = \sigma + j\omega$

Laplace transform is a **linear transformation**. Other properties and transforms in tables

Inverse Transforms: *Long Division* → *Factor* → *PFE* → *Tables*

PFE Special cases:

Repeated: $\frac{N(s)}{(s+5)^2} = \frac{k_1}{s+5} + \frac{k_2}{(s+5)^2}$ (Differentiation or coefficient matching)

Complex: $\frac{N(s)}{s^2+4} = \frac{k_1 s + k_2}{s^2+4} = \frac{k_1}{s-j2} + \frac{k_1^*}{s+j2}$

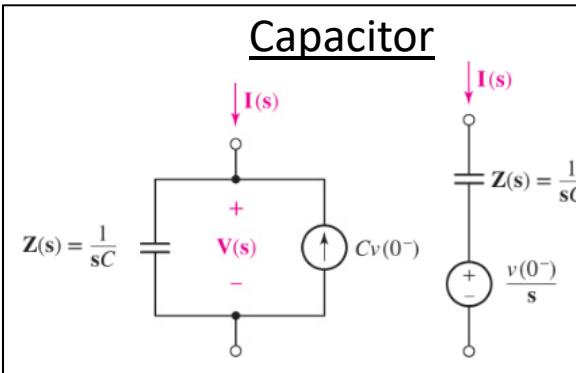
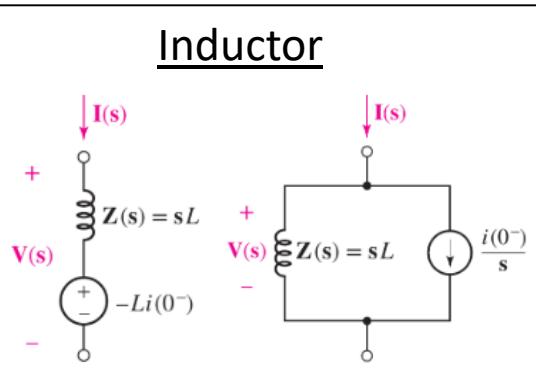
Ch 14 – Laplace Circuit Analysis

Poles and Zeros:

$$F(s) = \frac{N(s)}{D(s)}$$

Zeros: roots of $N(s)$
Poles: roots of $D(s)$

Circuit Transformation:

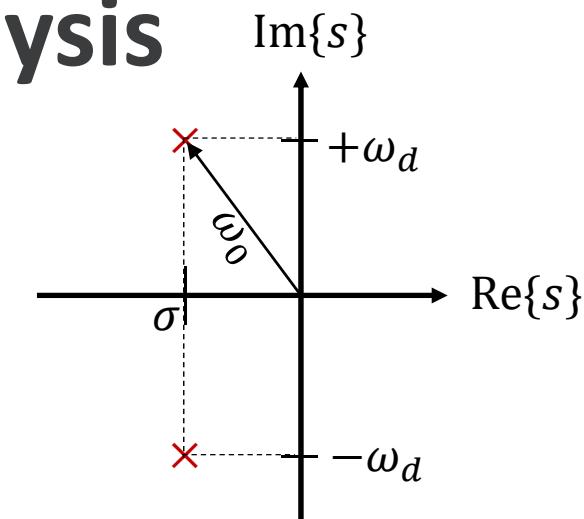
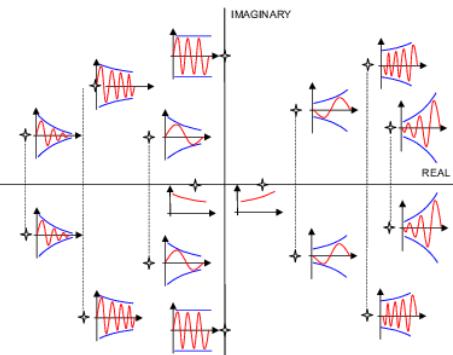


Transfer Functions:

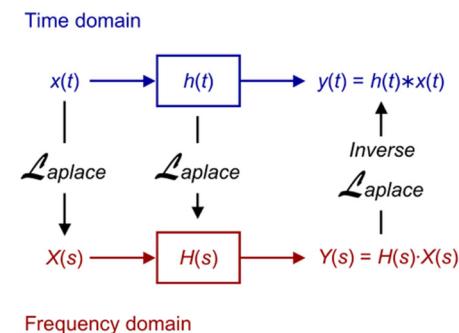
$$H(s) = \frac{V_o(s)}{V_i(s)}$$

Poles of $H(s)$ define “form” of terms in natural response of the circuit

Poles of $V_i(s)$ define “form” of terms in forced response of the circuit

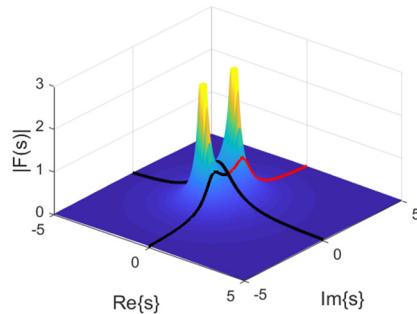


Convolution:



$$v_o(t) = v_i(t) * h(t) = \int_{-\infty}^{\infty} v_i(t - \tau)h(\tau)d\tau$$

Ch 15 – Frequency Response



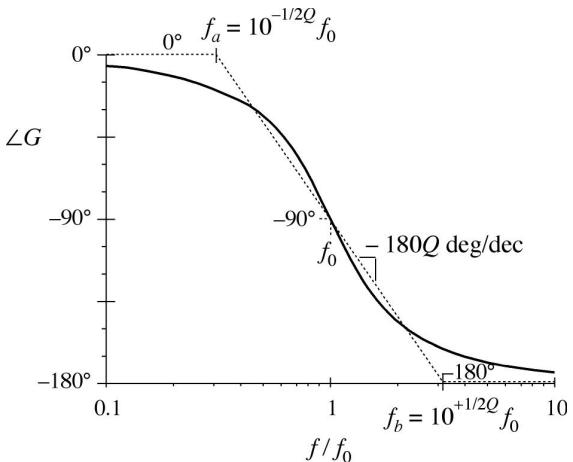
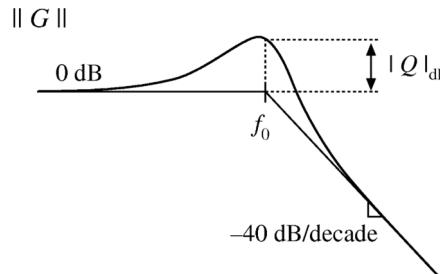
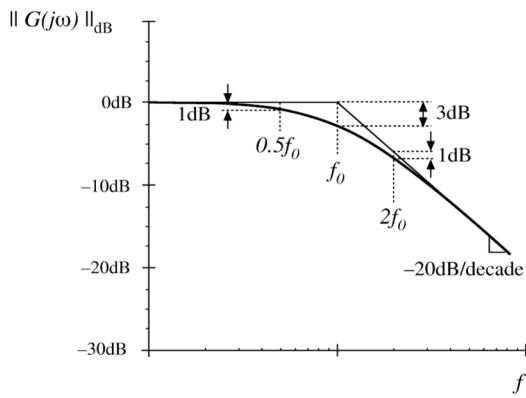
Frequency Response: $H(s \rightarrow j\omega)$

- Circuit response to sinusoidal inputs
- Valid if all poles in LHP

Bode plot: mag-phase plots on log-log axes

- $\|H(j\omega)\|_{dB} = 20 \log(|H(j\omega)|)$
- $\angle H(j\omega)$

Templates and Approximations:



Filter Design

- Bandwidth, graphical analysis, Chebyshev and Butterworth, Sallen-Key Amplifier
- Resonant circuits

Ch 17 – Fourier Series and Transform

Fourier Series:

For periodic $f(t)$ with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t) dt$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t) dt$$



https://en.wikipedia.org/wiki/Fourier_transform

Fourier Transform:

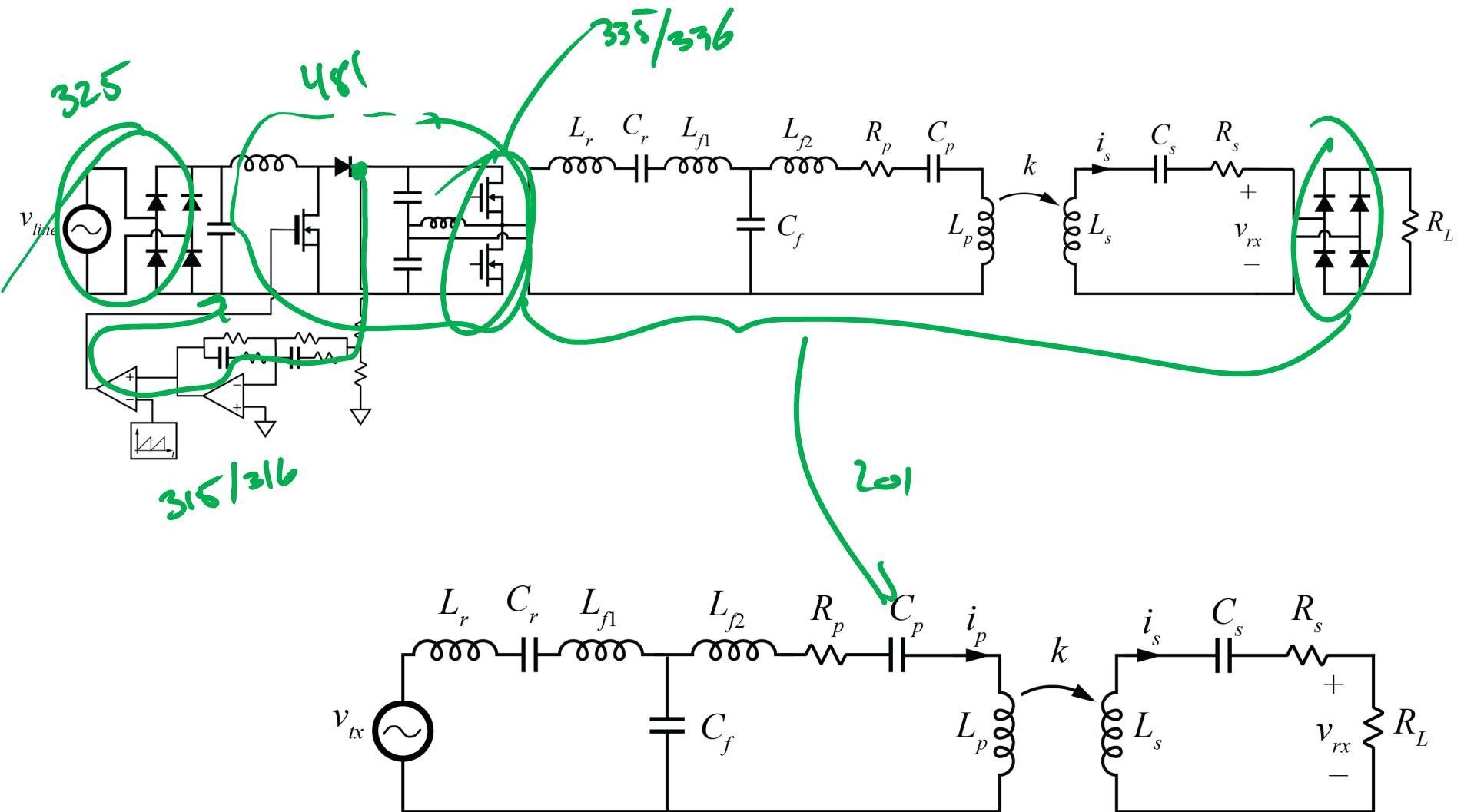
For periodic or non-periodic $f(t)$

$$F(\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} f(t) dt$$

$$f(t) = \mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} F(\omega) d\omega$$

- Frequency content of a signal
- Gives mag/phase of sinusoids that add up to original signal

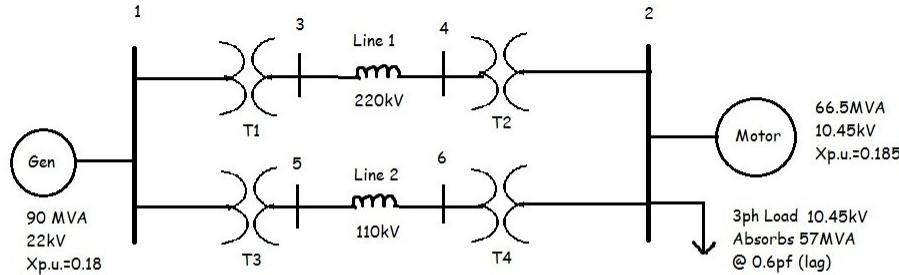
A Return to Lecture 1



FUTURE TOPICS

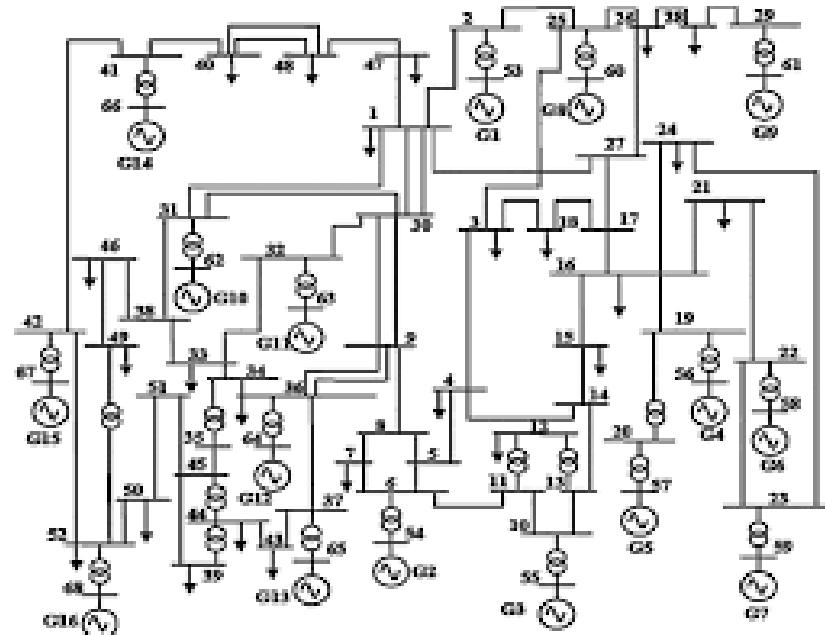
Power Systems

- ECE 325 -- Electric Energy System Components



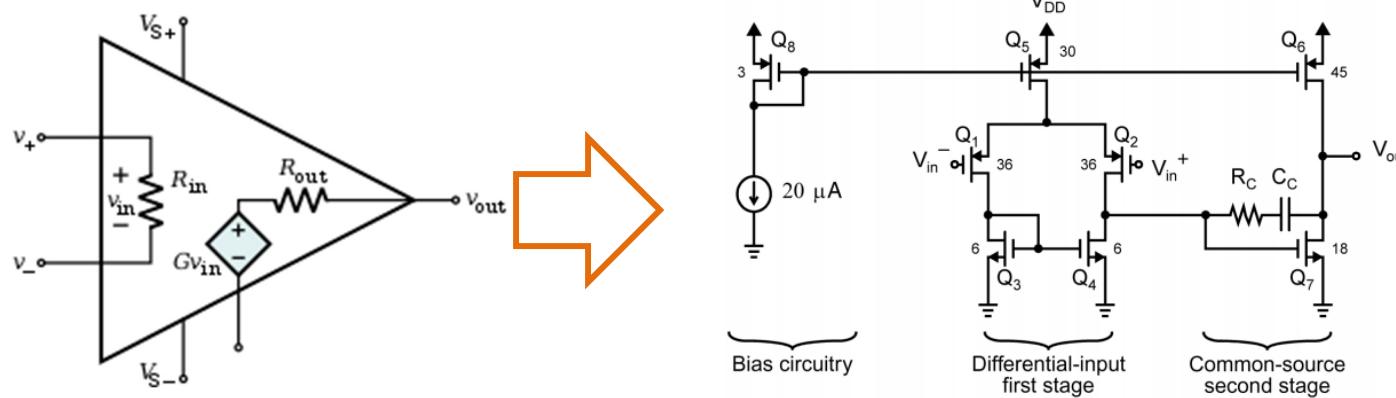
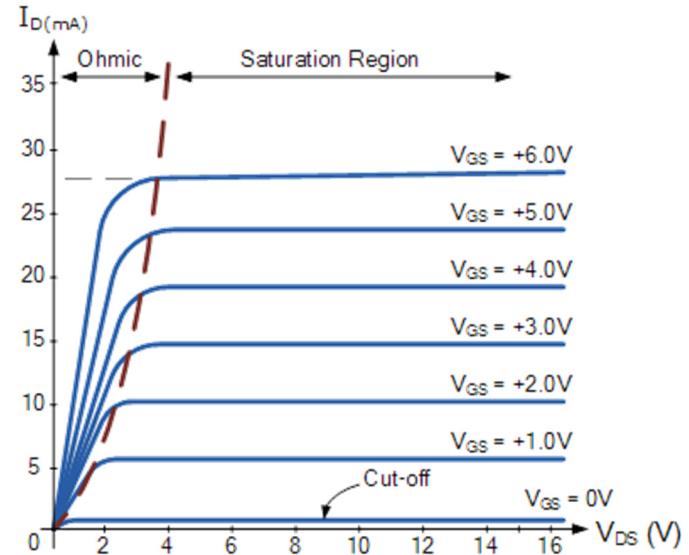
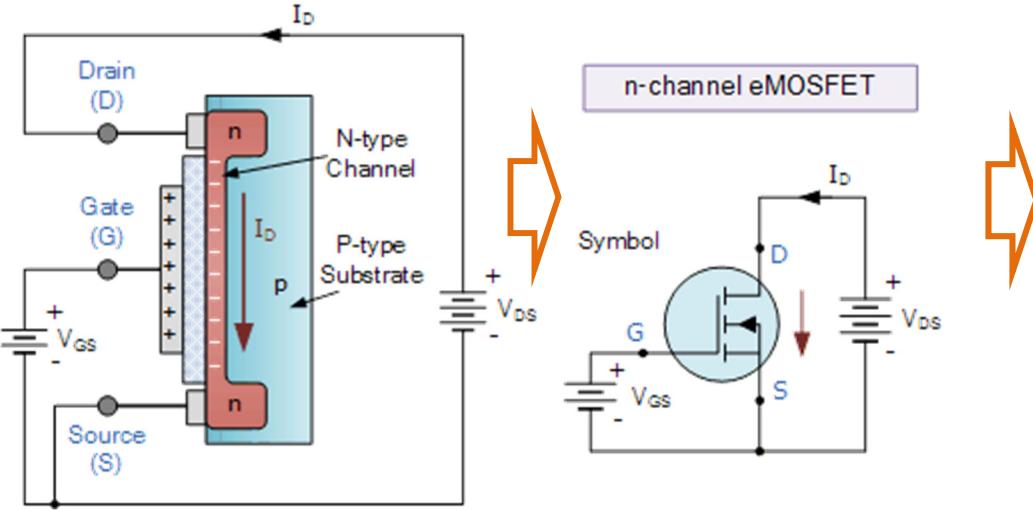
T1: 50MVA 22/220kV X_{p.u.} = 0.10
T2: 40MVA 220/11kV X_{p.u.} = 0.06
T3: 40MVA 22/110kV X_{p.u.} = 0.064
T4: 40MVA 110/11kV X_{p.u.} = 0.08
Line 1: 48.4Ohms (total)
Line 2: 65.43Ohms (total)

PEguru.com



Nonlinear Circuits

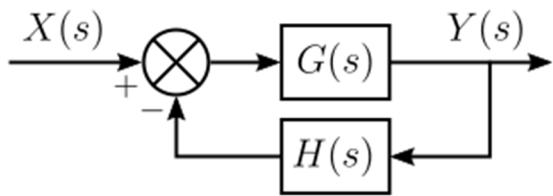
- ECE 335 & 336 – Electronic Devices



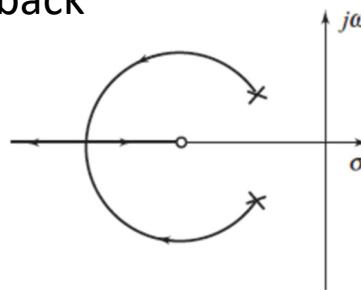
Closed-Loop Control

- ECE 316 – Signals and Systems

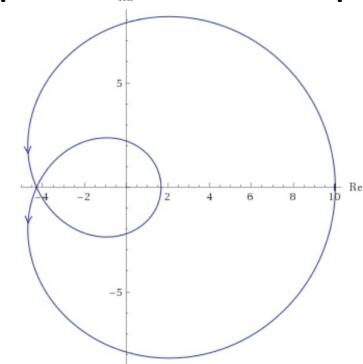
Classic Control



Root Locus: How poles and zeros move as you change feedback



Nyquist Plots: Frequency response in the complex plane



Modern Control

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)\end{aligned}$$

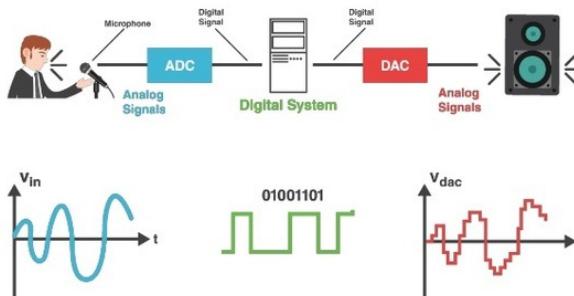
$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

Digital / Discrete Time Signals

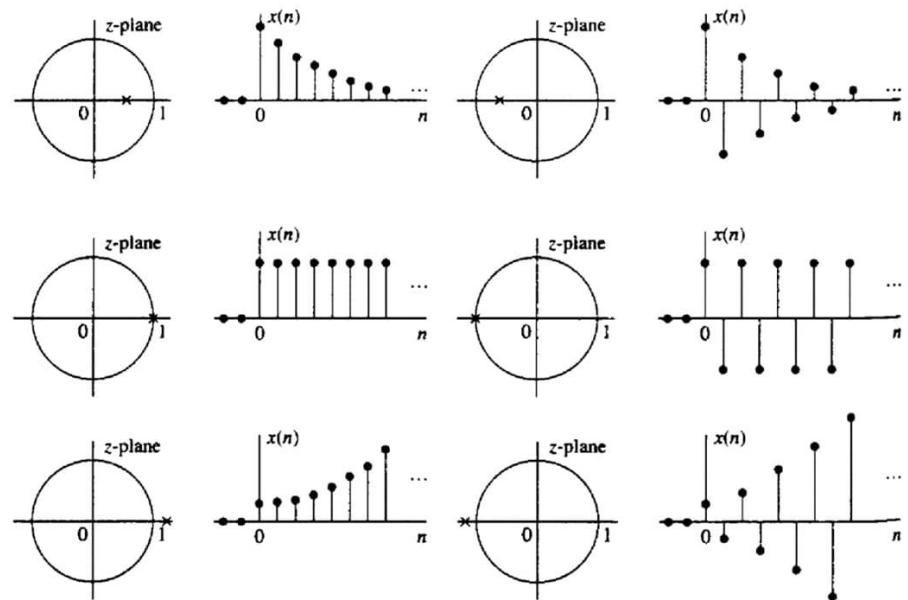
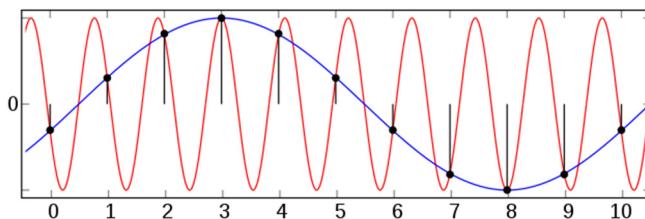
- ECE 315 – Signals and Systems

Cont. time $\sum_n A_n \frac{d^n}{dt^n} v_o(t) = \sum_m B_m \frac{d^m}{dt^m} v_i(t) \xrightarrow{\mathcal{L}} \sum_n A_n s^n V_o(s) = \sum_m B_m s^m V_i(s)$

Discrete time $\sum_n A_n y[k-n] = \sum_m B_m u[k-m] \xrightarrow{z} \sum_n A_n z^{-n} Y(z) = \sum_m B_m z^{-m} U(z)$



Sampling and aliasing:



FINAL REMARKS

Thank you for all your hard work

Good luck



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