

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t)$$

let's guess $v_i(t) = V_I \cos(\omega t)$

$$v_{o,p}(t) = A \cos(\omega t + \varphi) =$$

$$v'_{o,p}(t) = -A\omega \sin(\omega t + \varphi) =$$

$$v''_{o,p}(t) = -A\omega^2 \cos(\omega t + \varphi) =$$

$$v'''_{o,p}(t) = A\omega^3 \sin(\omega t + \varphi) =$$

$$v^{(IV)}_{o,p}(t) = A\omega^4 \cos(\omega t + \varphi) =$$

every $\frac{d}{dt}$
 • +90° phase
 • mult by ω

$$\begin{aligned}
 & A \cos(\omega t + \varphi) \\
 & A \omega \cos(\omega t + \varphi + 90^\circ) \\
 & A\omega^2 \cos(\omega t + \varphi + 180^\circ) \\
 & A\omega^3 \cos(\omega t + \varphi + 270^\circ) \\
 & A\omega^4 \cos(\omega t + \varphi + 360^\circ)
 \end{aligned}$$

Trig Identities (Review)

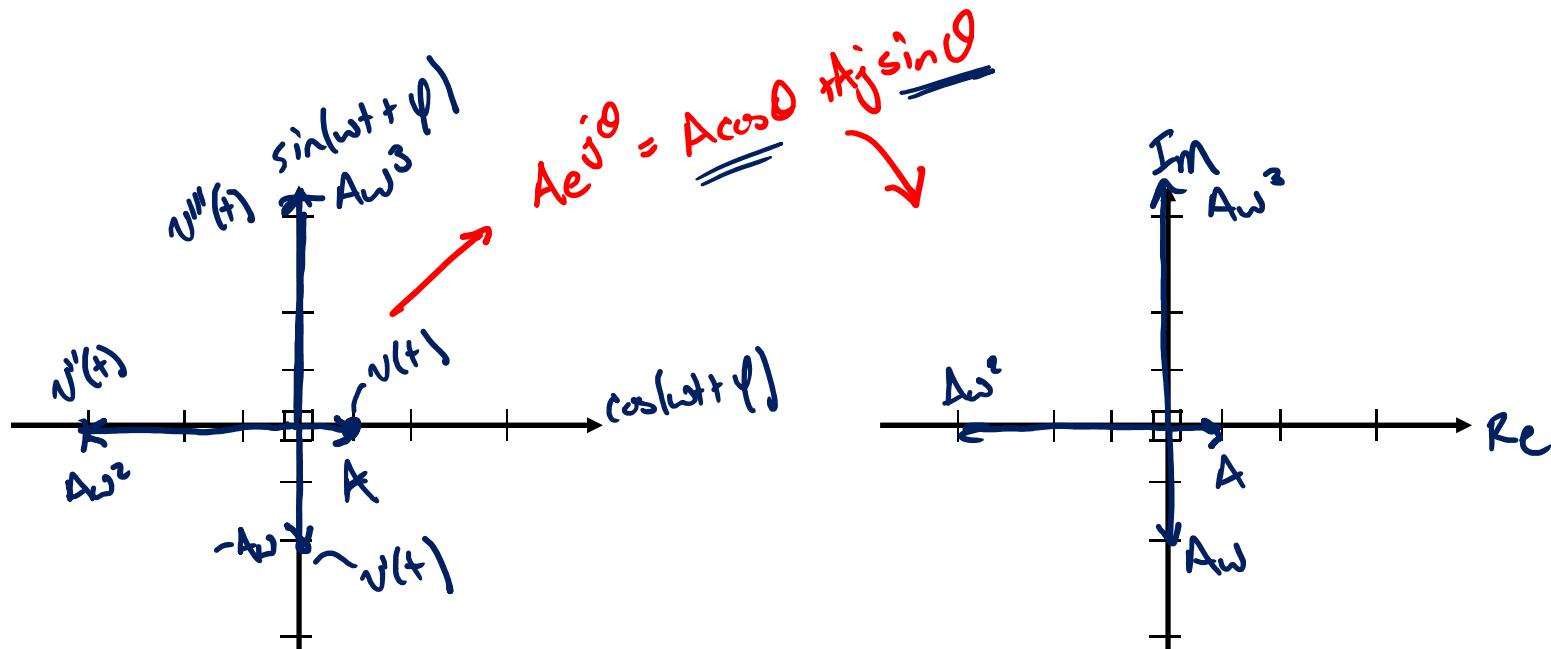
$$\begin{aligned}\rightarrow \quad \sin(\theta) &= \cos(\theta - 90^\circ) \\ -\cos(\theta) &= \cos(\theta \pm 180^\circ)\end{aligned}$$

$$\rightarrow A\cos(\omega t) + B\sin(\omega t) = \sqrt{A^2 + B^2} \cos\left(\omega t - \tan^{-1}\left(\frac{B}{A}\right)\right)$$

$$Ae^{j\theta} = A\cos(\theta) + jA\sin(\theta)$$

Sinusoidal Steady State

$$\begin{aligned}v(t) &= A \cos(\omega t + \varphi) &= A \cos(\omega t + \varphi) \\v'(t) &= -A\omega \sin(\omega t + \varphi) &= A\omega \cos(\omega t + \varphi + 90^\circ) \\v''(t) &= -A\omega^2 \cos(\omega t + \varphi) &= A\omega^2 \cos(\omega t + \varphi + 180^\circ) \\v'''(t) &= \underline{A\omega^3 \sin(\omega t + \varphi)} &= A\omega^3 \cos(\omega t + \varphi + 270^\circ) \\v''''(t) &= \underline{A\omega^4 \cos(\omega t + \varphi)} &= A\omega^4 \cos(\omega t + \varphi + 360^\circ)\end{aligned}$$



Complex Numbers (Review)

$$z = x + jy \xrightarrow{\text{Rectangular form}} r e^{j\theta} \xrightarrow{\text{Polar}}$$

Annotations:

- Real part: x
- Imaginary part: y
- amplitude: r
- phase: θ

$$\begin{cases} |z| = r = \sqrt{x^2 + y^2} \\ \angle z = \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$$

$$\begin{cases} \operatorname{Re}\{z\} = x = r \cos \theta \\ \operatorname{Im}\{z\} = y = r \sin \theta \end{cases}$$

Euler:

$$r e^{j\theta} = r \cos \theta + j r \sin \theta$$

Complex Conjugate:

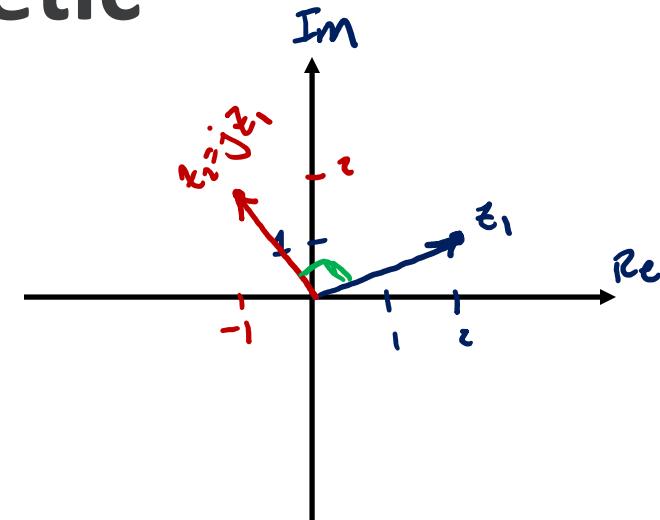
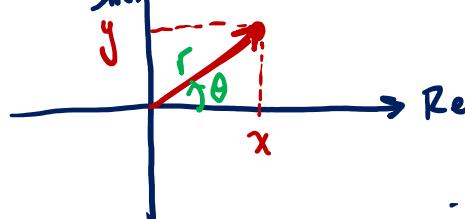
$$\text{for } z = x + jy$$

$$z^* = x - jy$$

Complex Number Arithmetic

Complex #'s are vectors in 2D space

$$z = x + jy \quad = r e^{j\theta}$$



ex $z_1 = 2 + j1 = \sqrt{5} e^{j\tan^{-1}(\frac{1}{2})}$

$$z_2 = j z_1 = j^2 + j^2 1 = -1 + j2$$

Multiplication by j is a 90° phase shift

$$j = 1 e^{j\pi/2}$$
$$z_1 j = \sqrt{5} e^{j\tan^{-1}(\frac{1}{2})} \cdot 1 e^{j\pi/2} = \sqrt{5} e^{j(\tan^{-1}(\frac{1}{2}) + \frac{\pi}{2})}$$

Usually:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Multiplication

$$z_1 \cdot z_2 = (r_1 r_2) e^{j(\theta_1 + \theta_2)}$$

Sinusoids as Complex Numbers

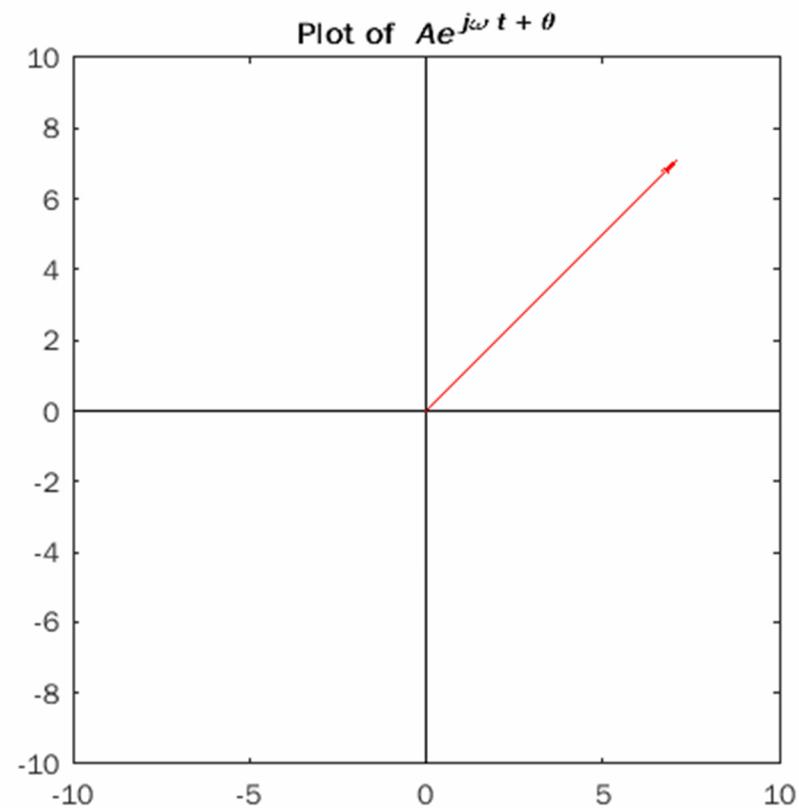
$$\begin{aligned} v(t) &= A \cos(\omega t + \phi) \\ &= \operatorname{Re} \{ A e^{j(\omega t + \phi)} \} \\ &= \operatorname{Re} \{ A e^{j\omega t} e^{j\phi} \} \end{aligned}$$

←
magnitude sinusoid phase

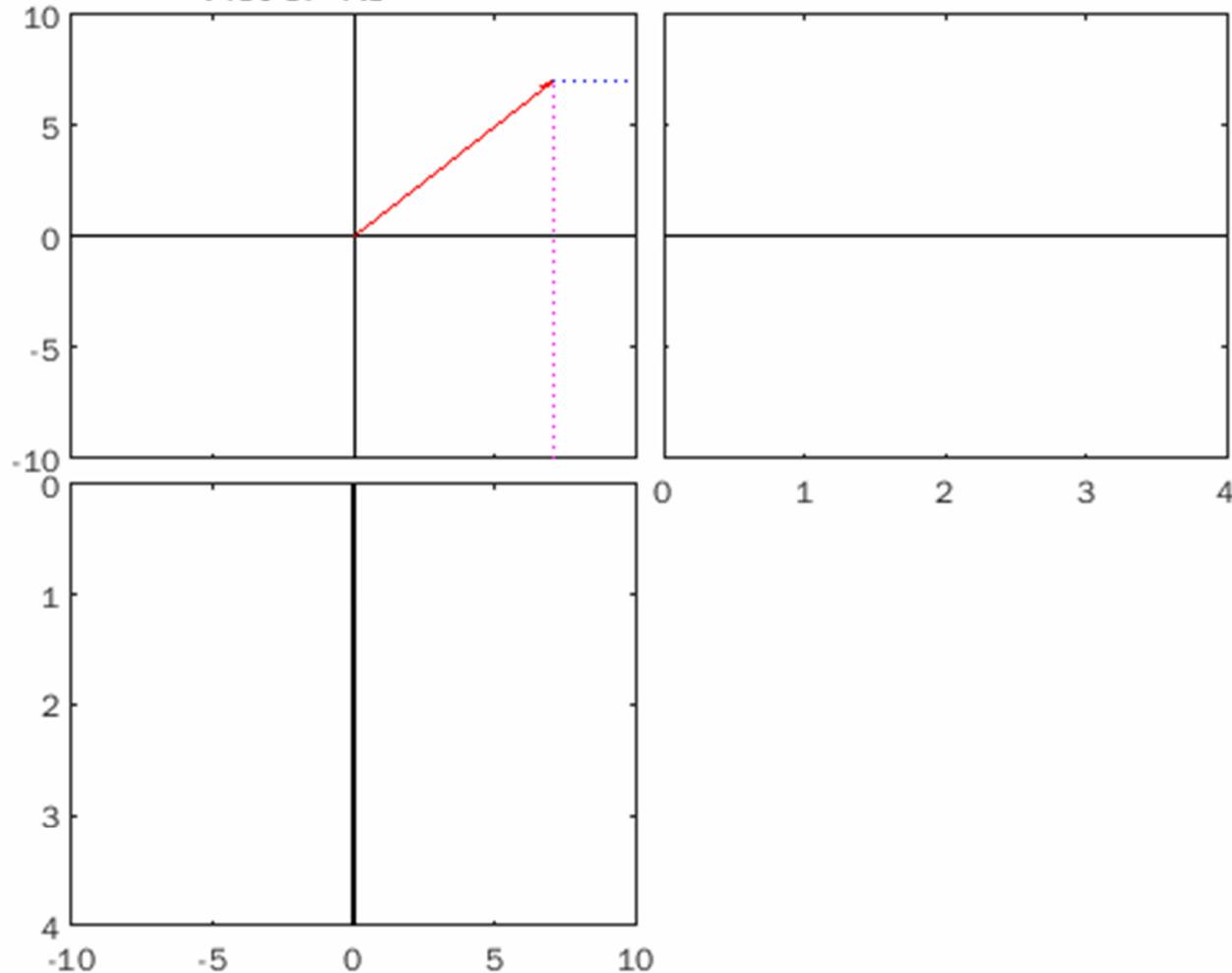
cosine

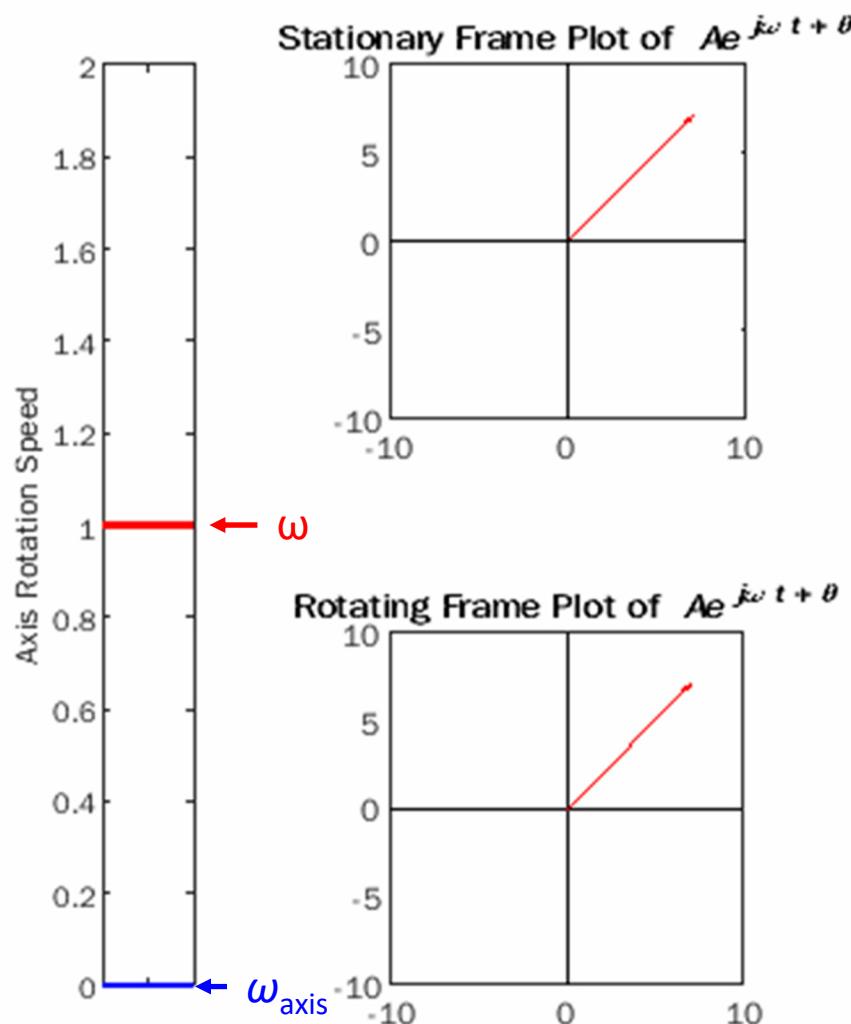
$$\begin{aligned} \frac{d}{dt} v(t) &= -A\omega \sin(\omega t + \phi) = A\omega \cos(\omega t + \phi + 90^\circ) \\ &= \operatorname{Re} \{ A\omega e^{j(\omega t + \phi + \pi/2)} \} \\ &= \operatorname{Re} \{ A\omega e^{j\omega t} e^{j\phi} e^{j\pi/2} \} \\ &= \operatorname{Re} \{ A[j\omega] e^{j\omega t} e^{j\phi} \} \end{aligned}$$

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Plot of $Ae^{k\omega t + \theta}$





Phasor Transformation

"Phasor" is a complex number that represents a sinusoid
useful in single-frequency sinusoidal circuits & solving steady-state output

$$A \cos(\omega t + \phi)$$

{
A amplitude
 ϕ phase
 ω frequency
cos function
t time

→ for single-freq problem
→ its (constn) → by convention always use
desire
→ steady-state only

$$v(t) = A \cos(\omega t + \phi) = \cancel{\operatorname{Re}\{A e^{j\omega t} e^{j\phi}\}}$$

$A e^{j\phi}$

|
phasor transform