

Experiment 3: DC-DC Voltage Conversion

Fourier and Laplace Transforms, Frequency Response

ECE 202: Circuits II

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I. Introduction

Power Electronics is the study of the design, construction, and use of circuits to convert or condition energy. The use of power electronics is prolific in modern society. Laptop chargers, computer power supplies, solar photovoltaic systems, electric vehicle powertrains, audio amplifiers, and cardiac pacemakers are just a few examples of applications requiring high performance power electronics.

Power converters designed for these functions may convert between various types and magnitudes of voltages or currents between their input and output. Some of the most common power converters are dc-dc step-down converters such as the Buck converter which convert a high voltage at their input to a well-regulated low-voltage output. Any electronic device today, such as cell phones, laptops, tablets, or televisions will contain dozens of dc-dc step-down converters to produce the small and tightly-regulated voltages required by digital processors from higher voltages produce by batteries or grid-connected power supplies.

In this experiment, we will investigate the design of a dc-dc step down converter as a means to apply knowledge related to Fourier, Laplace, and Frequency Response.

The goals of this experiment are

- Applying Fourier Transform to analytical and measured signals
- Examining Frequency Response of a circuit
- Observing resonance in second-order filters
- Measuring power in wide-bandwidth circuits
- Investigating design techniques in power converters

II. Background

The complete analysis and design of power converter circuits is covered in ECE 481, building upon knowledge of semiconductors from ECE 335 and energy components from ECE 325. As such, power converter design is out-of-scope for our course. The section below covers some basics to establish how the experiment relates to these future application areas

a. Fundamentals of dc-dc voltage conversion

Consider the design of a voltage supply for a modern processor in a computer, as shown in Fig. 1. The function of this supply is to convert the input voltage, commonly 5V or 12V, to a precisely regulated 1.5V supply required by the processor.

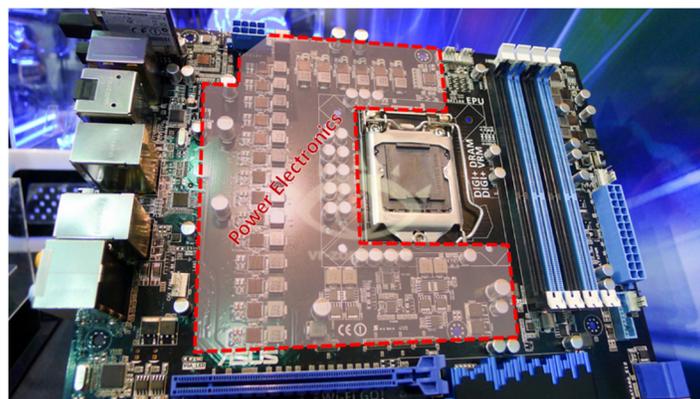


Figure 1: PC Motherboard with CPU voltage regulator power electronics highlighted

From a circuit perspective, this application is shown in Fig. 2. V_g is a 12V input supply, and the load resistance models the CPU itself. In a certain operating condition, the processor consumes 15W of power at 1.5V, so we can model its behavior as a resistance of value R_{eq} , where

$$\frac{V_{out}^2}{R_{eq}} = 15W \quad (1)$$

Therefore, $R_{eq} = 0.15 \Omega$.



Figure 2: Circuit for designing dc-dc converter

The job of the dc-dc converter (in the grey dashed box) is to transfer power from the 12V supply to the CPU, while keeping $V_{out} = 1.5V$ and, ideally, wasting as little energy as possible in the process. One simple way to generate the 1.5 V output voltage is a voltage divider, as shown in Fig. 3.

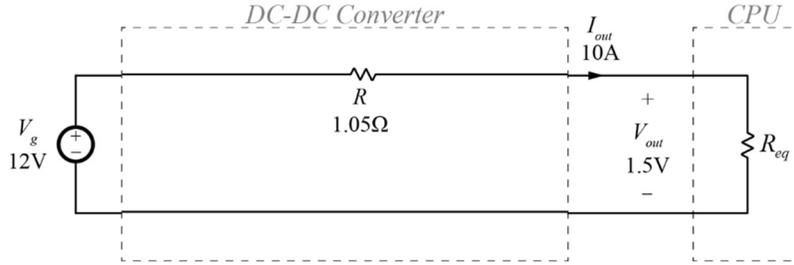


Figure 3: Voltage divider circuit used to generate 1.5 V

We know that

$$V_{out} = V_g \frac{R_{eq}}{R_{eq} + R} \quad (2)$$

so therefore, to get 1.5V output voltage, R is selected to be

$$R = \left(\frac{V_g}{V_{out}} - 1 \right) R_{eq} \quad (3)$$

For the case previously detailed, where the processor is modeled by $R_{eq} = 0.15 \Omega$, this gives $R = 1.05 \Omega$. In isolation, this design works. The output voltage is $V_{out} = 1.5 V$. However, this approach has two major flaws: control and efficiency.

If the power consumption of the CPU changes, as commonly occurs when the computational load of the computer changes, the circuit of Fig. 3 cannot maintain a regulated output voltage. From (1), R_{eq} is determined by how much power the processor is consuming, and will vary depending on the computational load. Because R depends on R_{eq} , as shown in (3), the series resistance will need to change in order to keep the output voltage constant, otherwise V_{out} will vary. As an example, if the power consumption of the CPU

changes from 15W to 7.5W, but R remains 1.05Ω , the output voltage will increase to 6.75 V, high enough to damage the processor.

Additionally, if we examine the power consumption of the circuit in Fig. 3, we find

$$P_R = I_{out}^2 R = 105 \text{ W}$$

$$P_{R_{eq}} = I_{out}^2 R_{eq} = 15 \text{ W}$$

$$P_g = I_{out} V_g = 120 \text{ W}$$

Thus, this circuit requires 120 W of power at the input to deliver just 15 W of power to the CPU. The remaining $P_R = 105 \text{ W}$ is dissipated in the series resistance R . This would cause many issues, among them the resistor heating up and melting, higher electricity bill, and wasteful use of energy.

Rather than the circuit of Fig. 3, dc-dc converters for this application are often designed using the circuit shown in Fig. 4, also known as the Buck converter. Notice that this converter uses only an ideal switch, and inductor, and a capacitor. There are no resistors (apart from the load resistance modeling the CPU), and therefore no real (average) power consumed by the circuit in the ideal case. The circuit is also controllable to that the output voltage can be adjusted in real time to stay constant even as the load varies.

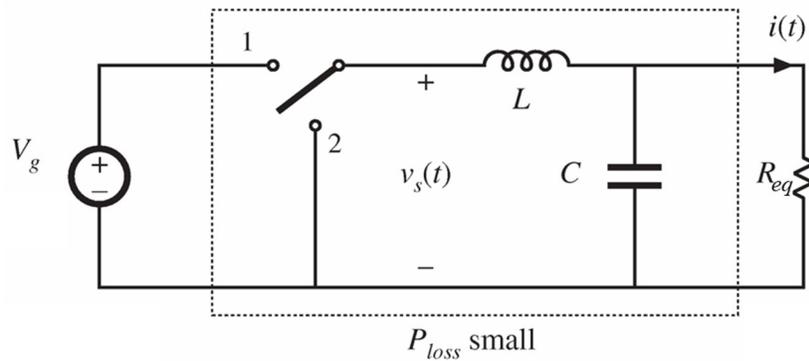


Figure 4: Basic Buck converter for dc-dc step-down conversion.

The ideal switch is controlled to periodically switch back and forth between the connections labeled “1” and “2”. The result is a voltage $v_s(t)$ as shown in Fig. 5. The voltage $v_s(t)$ is equal to the input V_g when the switch is in position 1, and is equal to zero when the switch is in position 2.

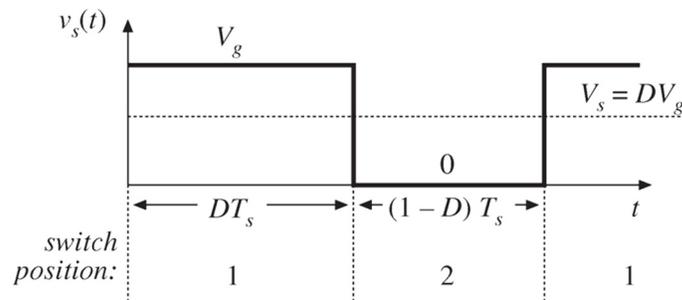


Figure 5: waveform of $v_s(t)$ with parameters labeled.

The switch commutates back and forth with period T_s and frequency $f_s = 1/T_s$. The “Duty Cycle” D is the ratio of the time the switch is in position 1 to the overall period, $0 \leq D \leq 1$. The ideal switch is implemented with semiconductor elements (transistor(s) and diode) allowing switching frequencies f_s in the hundreds of kilohertz up to a few megahertz.

Note that the average value of the waveform in Fig. 5 is

$$V_{s,avg} = \frac{1}{T_s} \int_0^{T_s} v_s(t) dt = \frac{1}{T_s} [DT_s V_g] = DV_g \quad (4)$$

With $0 \leq D \leq 1$, this means that $0 \leq V_{s,avg} \leq V_g$, i.e. the average value of this voltage can be anywhere between zero and the input voltage. The remainder of the circuit, comprised of L and C , is used to “pick off” this average value using a low-pass filter characteristic.

To understand how this works, we need to examine $v_s(t)$ and the behavior of the filter through the frequency domain.

b. Fourier Series

Many *periodic* waveforms, such as $v_s(t)$ in Fig. 5, can be represented as an infinite sum of sinusoids of the following form, known as a Fourier Series

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \quad (5)$$

where $\omega_0 = 2\pi f_0 = 2\pi/T$ is the frequency about which the original signal is periodic and a_0 , a_k , and b_k are real, constant coefficients. Not every periodic function can be represented as a Fourier series, but most functions that are relevant to circuit analysis can. The conditions for the Fourier series to exist for a given function are

1. $f(t)$ must be single-valued
2. $\int_{t_0}^{t_0+T} |f(t)| dt$ must exist and be finite for all values of t_0
3. $f(t)$ must have a finite number of discontinuities and maximum/minimums per period

The Fourier series represents a periodic function with an infinite sum of sinusoids. Given what we have already studied in phasor analysis, we know that sinusoidal signals have some convenient properties in circuit analysis that can make solving circuits easier, and superposition can extend those properties to multiple different-frequency sinusoids.

The coefficient a_0 is the portion of $f(t)$ which is not oscillating about any frequency, and is thus the average value of the function,

$$a_0 = \frac{1}{T} \int_{-\infty}^{\infty} f(t) dt \quad (6)$$

and the remaining coefficients are found by multiplying $f(t)$ by a sinusoid at the same frequency, then taking the average value

$$a_k = \frac{2}{T} \int_0^T f(t) \cos(k\omega_0 t) dt \quad (7)$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin(k\omega_0 t) dt \quad (8)$$

Alternate, equivalent forms of the Fourier series include the magnitude-phase form,

$$f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k), \quad \text{where} \quad \begin{cases} A_k = \sqrt{a_k^2 + b_k^2} \\ \varphi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right) \end{cases} \quad (9)$$

and the complex form

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \quad \text{where} \quad \begin{cases} c_k = \frac{1}{2}(a_k - jb_k) \\ c_{-k} = \frac{1}{2}(a_k + jb_k) \\ c_0 = a_0 \end{cases} \quad (10)$$

alternately, the coefficients c_k can be found directly by

$$c_k = \frac{1}{T} \int_0^T f(t) e^{-jk\omega_0 t} dt \quad (11)$$

and noting that $c_k = c_{-k}^*$, i.e. the complex conjugate.

Conceptually, in any form, the Fourier series coefficient tell us how much of every frequency of sinusoid is inside the signal $f(t)$. These coefficients comprise an alternate way of describing the function. Originally, $f(t)$ is the *time-domain* description of the function. In the time domain, the function has a specified value at any time, t . Now, a_k and b_k are a *frequency-domain* description of the function. The function instead has a specified value at every index k , corresponding to a frequency $k\omega_0$. In this way, calculating a_k and b_k from (7) and (8) is a transformation from time-domain to frequency-domain, and recombining them using (5) is a transformation from frequency-domain to time-domain. Fig. 6 diagrams the relationship between these two descriptions.

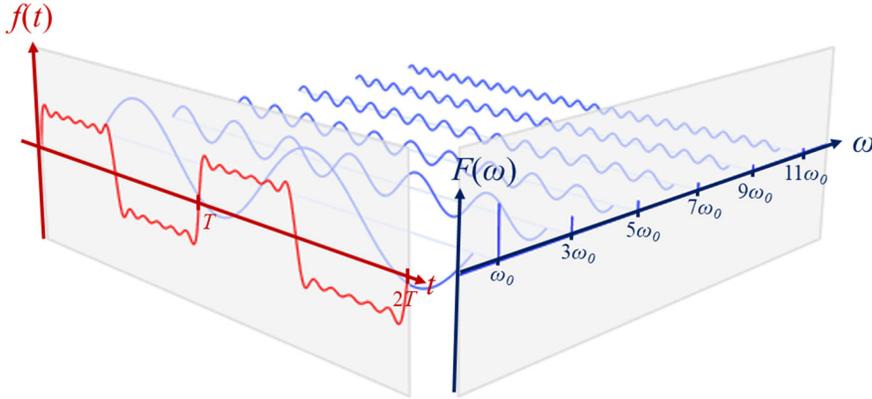


Figure 6: Relationship between time and frequency-domain descriptions of a function

c. Fourier Transform

The Fourier Transform extends the Fourier series to cover non-periodic signals. The Fourier transform is derived from (10) and (11) by taking the limit as the period T approaches infinity. The result is the transformation

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad (12)$$

and inverse transformation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \quad (13)$$

Unlike the Fourier series, the Fourier Transform can cover both periodic and nonperiodic functions. Sufficient (but not necessary) conditions for the Fourier Transform to exist are

1. $f(t)$ is single-valued
2. $\int_{-\infty}^{\infty} |f(t)| dt$ exists and is finite
3. $f(t)$ has a finite number of discontinuities and maximum/minimums over any finite interval

In most cases, the types of signals that we want to occur in real-world circuits have a Fourier transform that exists. However, certain signals that are common in circuit analysis and design do not have a Fourier transform; for example, $f(t) = e^t$ violates the second condition, above.

In the Fourier series, a periodic signal is decomposed into sinusoids at an infinite number of discrete frequencies $k\omega_0$. In the Fourier transform, the signal is decomposed into an infinite number of sinusoids at *all* frequencies.

d. Laplace Transform

The Laplace transform modifies the Fourier transform by pre-multiplying the signal $f(t)$ by a real-valued exponential. For example, the Fourier transform in (12) becomes

$$F(\sigma, \omega) = \int_{-\infty}^{\infty} e^{-\sigma t} f(t) e^{-j\omega t} dt \quad (14)$$

where the insertion of $e^{-\sigma t}$ is the only change from the Fourier transform. Recall that the Fourier transform of $f(t) = e^t$ does not exist due to violation of the second existence condition. Using the Laplace transform, as long as $\sigma \geq 1$, the integral $\int_{-\infty}^{\infty} |e^{-\sigma t} f(t)| dt$ exists and is finite, allowing the Laplace transform to exist within the region of convergence $\sigma \geq 1$.

The usual form of the Laplace transform simplifies (14) by combining the two exponentials and replacing $s = \sigma + j\omega$,

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt \quad (15)$$

and inverse Laplace transformation

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st} ds \quad (16)$$

where σ_0 is any value within the region of convergence. In general, the region of convergence is the region with σ greater than the real part of all poles of $F(s)$; however, we will rarely, if ever, apply (16) and instead use tables for the inverse Laplace transformation. Because a transformation table already gives transformation pairs within the region of convergence, this detail is handled inherently.

Commonly, the “unilateral” or “single-ended” Laplace transform is used for signal and systems which have are zero-valued prior to $t = 0$,

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} f(t) e^{-st} dt \quad (17)$$

Useful transformation pairs and properties are given in Tables 14.1 and 14.2 in the course textbook. Of particular note is the derivative property,

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = \int_{0^-}^{\infty} \frac{df}{dt} e^{-st} dt = (f(t)e^{-st})|_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t)(-s)e^{-st} dt \quad (18)$$

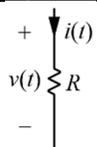
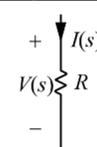
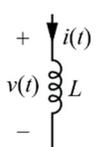
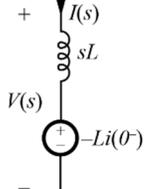
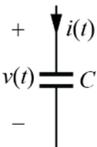
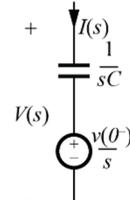
$$\mathcal{L}\left\{\frac{df}{dt}\right\} = sF(s) - f(0^-) \quad (19)$$

where (18) applies integration by parts (commonly useful in solving Laplace transform integrals) and evaluates it in (19). This result tells us that a derivative in the time domain becomes a multiplication by s in the s -domain, along with a subtraction of the initial condition.

e. Transfer Functions

Like the phasor transformation, the Laplace transformation is linear, and basic circuit elements can be transformed into s -domain impedances using the derivative property of the Laplace transformation, as shown in Table I. Unlike the phasor transformation, the Laplace transform is not limited to steady-state analysis, so the initial conditions of the circuit are incorporated into the transformation as additional sources.

Table I: Basic Circuit Elements in the Time and Laplace Domains

| Element | Time-Domain Symbol | Time-Domain Equation(s) | Laplace-Domain Symbol | Laplace -Domain Equation |
|-----------|---|-----------------------------|--|---|
| Resistor |  | $v(t) = i(t)R$ |  | $V(s) = RI(s)$ |
| Inductor |  | $v(t) = L \frac{di(t)}{dt}$ |  | $V(s) = sLI(s) - Li(0^-)$ |
| Capacitor |  | $i(t) = C \frac{dv(t)}{dt}$ |  | $V(s) = \frac{1}{sC}I(s) + \frac{1}{s}v(0^-)$ |

Note that initial conditions are transformed into independent sources, and thus can be treated as a separate input and incorporated into the circuit solution using superposition. After transforming all sources and signals into the s -domain, the circuit can be solved for signals of interest using standard circuit analysis techniques, with the defining equations for inductors and capacitors now being Ohm's Law-like algebraic equations rather than differential equations. Therefore, all impedances can be treated like resistors, in the time-domain. Generally, neglecting initial conditions (via the above), for an input voltage $v_i(t)$ and output voltage $v_o(t)$, the circuit solution will be of the form

$$\frac{V_o(s)}{V_i(s)} = H(s) \quad (20)$$

where $H(s)$ is known as a "transfer function". When initial conditions are included, they will have their own transfer function, all of which can be combined by superposition. Additional details of the form of the transfer function can be ascertained from its time-domain derivation. Consider an arbitrary time-domain circuit described by

$$b_N \frac{d^N}{dt^N} v_o(t) + \dots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) = a_M \frac{d^M}{dt^M} v_i(t) + \dots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t) \quad (21)$$

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t) \quad (22)$$

where all the a_i and b_i coefficients are functions of the passive elements in the circuits. Taking the Laplace transform of (22), using the derivative property from (19) gives

$$\sum_{i=0}^N b_i s^i V_o(s) = \sum_{i=0}^M a_i s^i V_i(s)$$

or, rearranging

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} \quad (23)$$

which confirms the form of (20), but also tells us information about the form of the transfer function, i.e. that we expect the transfer function to be a ratio of polynomials of s . For causal systems (circuits which cannot predict the future) the order of the numerator must be equal or lower than the denominator, $M \leq N$. Knowing this, we can simplify the form of $H(s)$ in two steps, first factoring the numerator and denominator to get into factored (or pole-zero form),

$$\frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} \xrightarrow{\text{factoring}} A \frac{(s-z_1)(s-z_2)\dots(s-z_M)}{(s-p_1)(s-p_2)\dots(s-p_N)} \quad (24)$$

In factored form, the roots of the numerator and denominator are shown explicitly. The roots of the numerator, z_i , are called “zeros” and the roots of the denominator, p_i , are called “poles”. Both are constant and may be real or complex. For real signals and systems (things that could actually be built and measured), anytime a pole is complex, its conjugate must also be a pole, i.e. complex poles always show up in conjugate pairs. The same is true of zeros.

From this form, using partial fraction expansion puts the transfer function into the form of the same name.

$$\frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} \xrightarrow{\text{factoring}} A \frac{(s-z_1)(s-z_2)\dots(s-z_M)}{(s-p_1)(s-p_2)\dots(s-p_N)} \xrightarrow{\text{PFE}} \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \dots + \frac{k_N}{(s-p_N)} \quad (25)$$

where k_i are called “residues” and are constants that may be real or complex. Like poles and zeros, residues that are complex will always show up in conjugate pairs for real signals and systems. If any repeated poles exist, their partial fraction expansion takes a slightly different form

$$A \frac{(s-z_1)(s-z_2)\dots(s-z_M)}{(s-p_1)\dots[(s-p_i)^k]\dots(s-p_N)} \xrightarrow{\text{PFE}} \frac{k_1}{(s-p_1)} + \dots + \left[\frac{k_{i1}}{s-p_i} + \frac{k_{i2}}{(s-p_i)^2} + \dots + \frac{k_{ik}}{(s-p_i)^k} \right] + \dots + \frac{k_N}{(s-p_N)} \quad (26)$$

In the partial fractions form, the expression is split into a sum of simple terms. Then, because the Laplace transform is linear, the inverse transformation of the overall expression is found by taking the inverse transform of each term

$$\mathcal{L}^{-1} \left\{ \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} \right\} \xrightarrow{\text{factor + PFE}} \mathcal{L}^{-1} \left\{ \frac{k_1}{(s-p_1)} \right\} + \mathcal{L}^{-1} \left\{ \frac{k_2}{(s-p_2)} \right\} + \dots + \mathcal{L}^{-1} \left\{ \frac{k_N}{(s-p_N)} \right\} \quad (27)$$

Note the significance of the poles in (27). The poles directly determine the terms in the time-domain, while the zeros determine the constants that multiply them, k_i . Thus, by only looking at the poles of the system, we can determine whether a signal will have oscillations, or whether it will be unbounded. Analysis of

system poles is fundamental to examining stability of control systems. Fig. 7 shows a variety of pole locations in the s-plane (complex plane) and the time-domain response that they will cause. Note that poles in the open left half-plane (LHP) result in time-domain responses that converge to zero as time progresses. Poles in the open right half-plane (RHP) result in time-domain responses that explode to $\pm\infty$ as time progresses. Poles directly on the imaginary axis may result in signals that oscillate indefinitely without growing or shrinking in magnitude, or signals that still grow unbounded when repeated poles occur (e.g. $1/s^2 \Leftrightarrow tu(t)$).

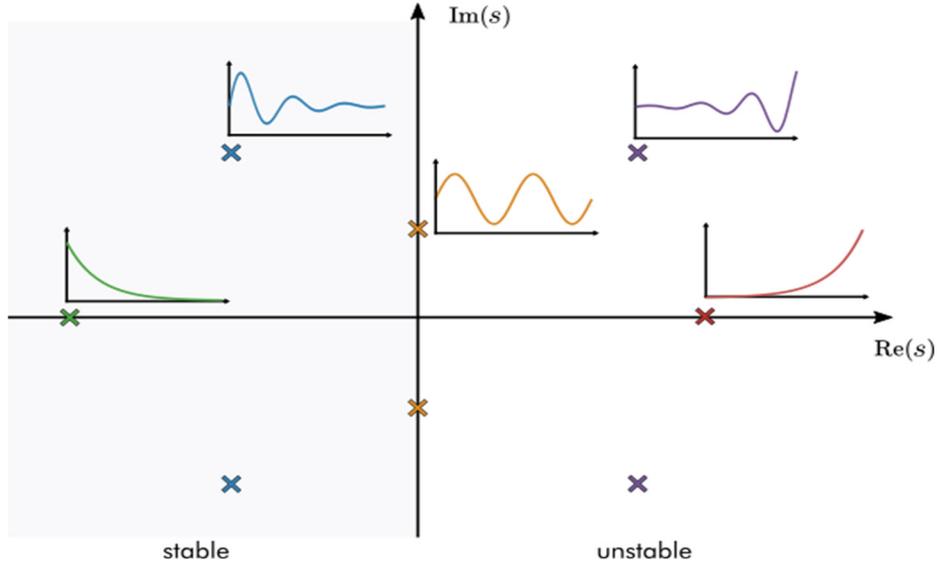


Figure 7: Example poles and their resulting time-domain response.¹

Rearranging (23), we see that the poles in any output signal that we want to examine are the union of the poles from the transfer function and the poles from the input source

$$V_o(s) = H(s)V_i(s) = \frac{\sum_{i=0}^M a_i s^i \sum_{i=0}^{M_2} c_i s^i}{\sum_{i=0}^N b_i s^i \sum_{i=0}^{N_2} d_i s^i} \xrightarrow{\text{factoring}} A \frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)} \frac{(s-z_1)(s-z_2)\cdots(s-z_{M_2})}{(s-p_1)(s-p_2)\cdots(s-p_{N_2})} \quad (28)$$

If there are non-zero initial conditions, they can each be treated as a separate independent source and incorporated into the output by superposition. Then, the above will have multiple terms of the same form.

Referring back to the definition of the inverse Laplace transform in (16), and our original motivation in extending the Fourier Transform, signals in the RHP that grow unbounded need to be pre-multiplied by a decaying exponential to make sure that the integral will converge. In (16), we need to select σ_0 greater than the real part of all poles to ensure that any unbounded signals are forced to converge. If all poles are in the LHP, $\sigma_0=0$ is a valid choice; eliminating the real part is equivalent to replacing $s = j\omega$, i.e., returning to the Fourier Transform.

f. Frequency Response

If all poles of a transfer function are in the open LHP, the system is called “Bounded Input, Bounded Output (BIBO) Stable”. This means that, as long as the input signal is bounded, the output will also be bounded. Usually, we want any circuit that we might build to possess this property as the alternative could be catastrophic.

¹ From: <<https://www.mathworks.com/matlabcentral/fileexchange/94635>>

As stated at the end of the previous section, as long as all poles are in the open LHP, then $\sigma_0=0$ is within the region of convergence of the Laplace Transform, and we can completely define the signal by looking solely at the values of $H(s)$ on the imaginary axis. The “frequency response” of a circuit is its transfer function evaluated on the line $s = j\omega$.

$$H(s)|_{s=j\omega} = H(j\omega) \quad (29)$$

The significance of the frequency response is the same as it was in the Fourier domain: at any frequency ω_x , if we apply an input to the system $v_i(t) = A\cos(\omega t)$, the output will be

$$v_o(t) = |H(j\omega)|A\cos(\omega t + \angle H(j\omega)) \quad (30)$$

i.e. $H(j\omega)$ tells us the gain and phase of the system at all frequencies.

III. Prelab Exercises

Complete the following exercises prior to coming to the lab. Turn-in your prelab work on canvas prior to starting the Laboratory Exercises.

The prelab will examine the signal and power performance of a few alternative circuit implementations attempting to convert a 5V input to a 1V output.

PE1 Prelab Exercise 1: Resistor Divider Voltage Conversion

First, we will examine using a resistor divide for our 5:1 voltage conversion, as shown in Fig. 8.

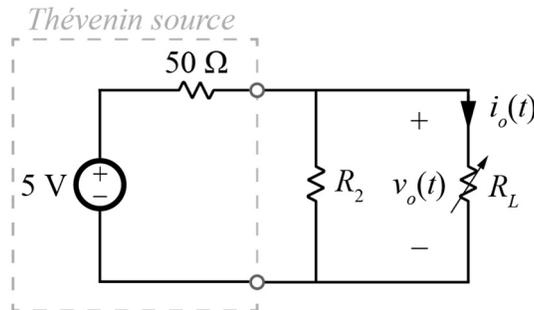


Figure 8: Fixed resistor divider used as a dc-dc converter

To begin, assume R_L is very large and solve for the value of R_2 that will result in $V_o=1V$. In LTSpice, recreate the circuit setting R_2 to the value you solve. Sweep R_L from 10 k Ω to 500 Ω , and turn in plots of both the output voltage $v_o(t)$ and the efficiency $\eta = P_{out}/P_{in}$ as the load resistance is varied. Comment on any issues with this design.

PE2 Prelab Exercise 2: Linear Regulator Voltage Conversion

Next, we will consider a linear regulator. A full model of a linear regulator includes transistors, but a simplified functional model of the linear regulator is shown in Fig. 9. The behavior of the linear regulator is that of a variable resistance R_{var} which is controlled to be the correct value such that the output voltage remains 1V.

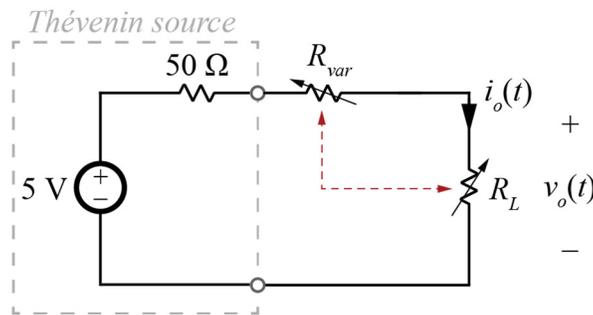


Figure 9: Simplified equivalent model of a linear regulator.

Solve the circuit for an expression $R_{var} = f(R_L)$ for what value R_{var} needs to take at any load resistance to keep $V_o = 1V$. In LTSpice, recreate the circuit setting R_{var} to the expression you solve. Sweep R_L from 10 k Ω to 500 Ω , and turn in plots of both the output voltage $v_o(t)$ and the efficiency $\eta = P_{out}/P_{in}$ as the

load resistance is varied. Comment on any issues with this design, and any advantages over the resistor divider in PE1.

PE3 Prelab Exercise 3: DC-DC Voltage Converter

Now, we will model the full switching converter. We'll simplify the ideal switch by just modeling $v_s(t)$ as a squarewave source, as shown in Fig. 10

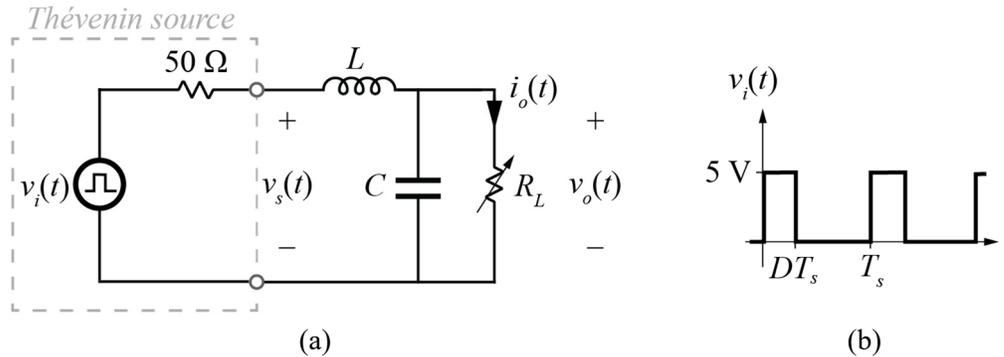


Figure 10: Simplified equivalent model of a Buck converter (a) and waveform of $v_s(t)$ (b)

Use the starting values from Table II to build an LTSpice model of the circuit.

Table II: Starting Values for System Design

| $f_s = 1/T_s$ | L | C |
|---------------|--------|--------------|
| 100 kHz | 100 mH | 0.22 μ F |

Solve, using (4) for the necessary duty cycle $D = f(R_L)$ to generate 1.2 V average value at the output (don't forget about the $50\ \Omega$ resistor, and remember that you only care about average (DC) component).

In LTSpice, recreate the circuit setting D to the expression you solve. Set the rise and fall times of the squarewave source to 1ns. Run transient simulations for 50ms or more to let the circuit reach steady-state but only save data for a few periods. For example, saving the last five periods, your LTSpice simulation command is

```
.tran 0 50m {50m-5/100k}
```

Sweep R_L from $500\ \Omega$ to $10\ \text{k}\Omega$ and turn in plots of both the average output voltage V_o and the efficiency $\eta = P_{out}/P_{in}$ as the load resistance is varied. Use `.meas` commands for the averages of V_o , P_{out} , and P_{in} and plotting from the spice error log to generate the plots. Comment on any issues with this design, and any advantages over the resistor divider in PE1 and/or linear regulator in PE2.

PE4 Prelab Exercise 4: Signal Frequency Content

For the operating point where $R_L = 500\ \Omega$, calculate (by hand) the Fourier Series coefficients of the input squarewave in Fig. 10(b) for the duty cycle you solved in the previous section. In LTSpice, view the FFT of this voltage (from a waveform window View>FFT). Turn in this plot. Compare the waveform generated by LTSpice to your calculation, noting any discrepancy.

PE5 Prelab Exercise 5: Circuit Frequency Response

Using AC analysis in LTSpice, examine the frequency response of the circuit, $H(j\omega)$. Turn a plot of $H(j\omega)$ for load resistances of $10\ \text{k}\Omega$, $1\ \text{k}\Omega$, and $500\ \Omega$ (one curve each) Comment on how the frequency response

and the signal Fourier transform from the previous exercise combine to give the (approximate) behavior of a dc-dc converter.

IV. Laboratory Experiment

The following experimental exercises require the ECE201/202 parts kit (one per group) and access to oscilloscope and waveform generator either through the on-campus lab or using an Analog Discovery Studio. No additional components are needed beyond the content of the parts kit.

Note: If using the Analog Discovery Studio, make sure to update the software through Help->Check for Update before beginning. A recent update patched some errors in the Spectrum functionality.

LE1 Lab Exercise 1: Squarewave Spectrum Measurement

Using a waveform generator, produce a squarewave with 500 kHz frequency, 5 V maximum, 0 V minimum, and 20% duty cycle. measure this waveform on the oscilloscope. Using the FFT (“Spectrum”) function of the scope, measure the Fourier transform (Frequency content) of the signal. Set the frequency range to “10 MHz to 12.21 kHz”, frequency scale to “Logarithmic”, and on your measurement channel, set scaling to “Logarithmic” units to “Peak (V)”. Turn in screen captures of the oscilloscope time-domain waveforms and the measured FFT. Using the measurement command, take measurements of the fundamental (“FF”) through 10th harmonic and compare the spectrum to your predictions from the prelab.

LE2 Lab Exercise 2: Converter Assembly and Testing

Using the 100 mH, 0.22 μ F capacitor (blue), and 1 k Ω potentiometer (blue) from your kit, assemble the converter of Fig. 10(a) on your breadboard. Note that the 50 Ω resistor is internal to the function generator. Set the potentiometer to present $R_L = 500 \Omega$, and the duty cycle of the function generator to the value solved in PE3. Measure the output voltage and turn in a screen capture of the oscilloscope showing both $v_o(t)$ and $v_s(t)$ at the same time.

LE3 Lab Exercise 3: Discrete Frequency Response

In this exercise, we will measure the frequency response of the circuit in one configuration by looking at individual frequencies one at a time. Keeping $R_L = 500 \Omega$, change the input to a sinusoidal signal with zero offset. For a range (at least 10) frequencies spanning 100 Hz to 100 kHz, alter the sinusoidal source to each frequency, then measure the gain and phase of the output voltage $v_o(t)$ relative to the input sinusoid at $v_s(t)$. Turn in a plot with, as two separate curves, the gain and phase of the circuit as measured by these points. Compare this measurement to the frequency response simulated in PE5.

LE4 Lab Exercise 4: Continuous Frequency Response

In this exercise, we will measure the frequency response of the circuit using the built-in network analyzer functionality of the Analog Discovery Studio. Keeping scope 1 and wevegen 1 on $v_s(t)$ and scope 2 on $v_o(t)$, use the “Network” tool of the Analog Discovery Studio to measure the transfer function $H(j\omega)$ of the circuit. Measure from 100 Hz to 1 MHz, and include plots of the magnitude in units of “Gain (X)” and phase in units of degrees. Comment on how these plots correspond to both PE5 and LE3.

Remove R_L from the circuit and capture the new gain and phase plots. Comment on the differences in the circuit without a load resistance to provide damping.

LE5 Lab Exercise 5: Frequency Sweep

Place R_L back into the circuit and return the waveform generator to a 5 V maximum squarewave with the same duty cycle from LE2. Change the frequency of the squarewave to 10 kHz, 5 kHz, and 2 kHz. For each of these frequencies, turn in a screen capture of the oscilloscope showing both $v_o(t)$ and $v_s(t)$ at the same time. Comment on how they differ, and why this is happening.