Circuit Solution Tips

- Order of analysis
  - Series/parallel equivalents
  - Source transformations
  - Transformer reflection
  - KVL/KCL equations + Element Equations
  - Node Voltage / Mesh Current Analysis (if necessary)
- Stay symbolic as long as possible
  - Define new equivalent variables as necessary

Form of the solution

$$\sum_{i=0}^{N} b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^{M} a_i \frac{d^i}{dt^i} v_i(t)$$

In phaser domain $\frac{d}{dt} \rightarrow (j\omega)$

$$\sum_{i=0}^{N} b_i (j\omega)^i \underline{V_o} = \sum_{i=0}^{M} a_i (j\omega)^i \underline{V_1}$$

from/to any voltage or current in the circuit $\longrightarrow$

$$\underline{V_o} = \underline{V_1} \left(\frac{\sum_{i=0}^{M} a_i (j\omega)^i}{\sum_{i=0}^{N} b_i (j\omega)^i} \right)$$

$\uparrow$

$H(\omega)$

Chapter 11

AC CIRCUIT POWER ANALYSIS

LTI Systems (L7)

LTI

For a function $f(\cdot)$

superposition applies

Linearity

if: $v_{o,1}(t) = f\left(v_{i,1}(t)\right)$ $\qquad v_{o,2}(t) = f\left(v_{i,2}(t)\right)$

then: $a v_{o,1}(t) + b v_{o,2}(t) = f\left(a v_{i,1}(t) + b v_{i,2}(t)\right)$

Time Invariance

if: $v_o(t) = f\left(v_i(t)\right)$

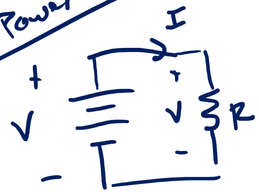
then: $v_o(t-T) = f\left(v_i(t-T)\right)$

LTI: $v_x(t) + v_y(t)$, $\alpha v_x(t)$, $\frac{dv_x(t)}{dt}$, $\int_0^t v_x(t)dt$ (neglecting ICs)

Not LTI: $v_x(t) \cdot v_y(t)$, $v_x(t)^2$, $|v_x(t)|$ → power isn't a linear calculation

Average Power

DC Power

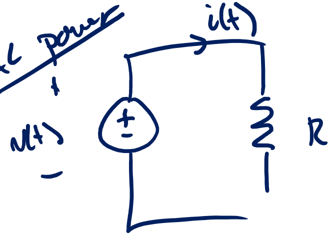


$P_R = V \cdot I \longrightarrow$ Generally true for any 2-terminal element

for a resistor $V = IR$

$$P_R = I^2 R = \frac{V^2}{R}$$

AC power



$P_R(t) = v(t) \cdot i(t) \longrightarrow$ Generally true of all 2-terminal elements

for a resistor $P_R(t) = i(t)^2 R = \dfrac{v(t)^2}{R}$

In any case, power calculation is not LTI

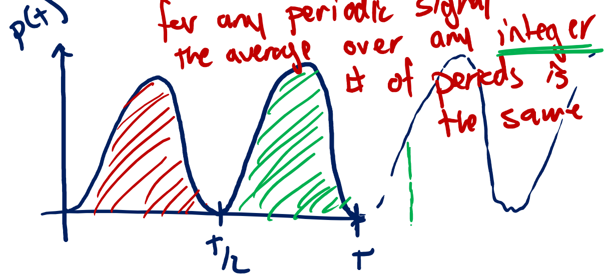
Average Power

Average power defined over some time interval $t \in [t_1, t_1 + T]$

capital "P" denotes average $\rightarrow$ $P = \dfrac{1}{T}\displaystyle\int_{t_1}^{t_1+T} p(t)\, dt$

Average power over all time

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t)\, dt$$



for any periodic signal the average over any integer # of periods is the same

Power in a Resistor

Average power in a resistor with periodic (e.g. sinusoidal) sources

$$P_R = \frac{1}{T}\int_0^T p_R(t)\,dt = \frac{1}{T}\int_0^T i(t)^2 R\,dt$$

$$= R\,\underbrace{\frac{1}{T}\int_0^T i(t)^2\,dt}$$

$$P_R = R\,\underbrace{\sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt}}^{\,2} \longleftrightarrow P_R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

rms is 'root mean squared'

Define rms $\quad X_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2\,dt}$

Note: Book calls this "effective" instead of rms

$$I_{eff} = I_{rms} \qquad V_{eff} = V_{rms}$$

RMS of a sinusoid

$\omega = 2\pi f \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}$

$i(t) = I_A \cos(\omega t + \varphi)$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(I_A \cos(\omega t + \varphi)\right)^2 dt}$$

trig identity

$\cos^2(\theta) = \frac{1}{2}(1 + \cos 2\theta)$

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_A^2 \cos^2(\omega t + \varphi)\, dt$$

$$I_{rms}^2 = \frac{1}{T} I_A^2 \frac{1}{2}\int_0^T 1 + \cos(2\omega t + 2\varphi)\, dt$$

$$I_{rms}^2 = \frac{1}{T} I_A^2 \frac{1}{2}\left[t + \sin(2\omega t + 2\varphi)\frac{1}{2\omega} \right]\Bigg|_0^{T = \frac{2\pi}{\omega}}$$

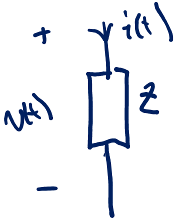
$$I_{rms}^2 = \frac{1}{T} I_A^2 \frac{1}{2}\left[\left(\frac{2\pi}{\omega} - 0\right) + \frac{1}{2\omega}\left(\cancel{\sin(4\pi + 2\varphi) - \sin(2\varphi)}\right)\right] \quad \phi$$

$$I_{rms}^2 = \frac{\cancel{\omega}}{\cancel{2\pi}} I_A^2 \frac{1}{2} \frac{\cancel{2\pi}}{\cancel{\omega}}$$

$$I_{rms}^2 = \frac{I_A^2}{2} \rightarrow \boxed{I_{rms} = \frac{I_A}{\sqrt{2}}}$$

for any sinusoidal signal

Power with Sinusoidal Sources



In steady-state with single-frequency sinusoidal sources @ ω

$$v(t) \rightarrow \underline{V} = V_A e^{j\varphi_V}$$
$$i(t) \rightarrow \underline{I} = I_A e^{j\varphi_I}$$

$$\underline{V} = \underline{I}\, Z \quad \rightarrow \quad Z = \frac{\underline{V}}{\underline{I}} = \frac{V_A e^{j\varphi_V}}{I_A e^{j\varphi_I}} = \frac{V_A}{I_A} e^{j(\varphi_V - \varphi_I)}$$

power: since power is <u>not</u> LTI, we don't know how to calculate power w/ phasors

<u>Not</u> $\underline{V} \cdot \underline{I}$ $\rightarrow$ go back to time domain to calculate

<u>Trig identity</u>: $2\cos\theta\cos\varphi = \cos(\theta+\varphi) + \cos(\theta-\varphi)$

$$p(t) = i(t)\cdot v(t) = I_A\cos(\omega t + \varphi_I)\cdot V_A\cos(\omega t + \varphi_V)$$
$$= \frac{I_A V_A}{2}\left(\underline{\cos(2\omega t + \varphi_I + \varphi_V)} + \underline{\cos(\varphi_I - \varphi_V)}\right)$$

sinusoid @ 2ω

constant (not time-varying)

<u>Average power</u>

$$\boxed{P_Z = \frac{I_A V_A}{2}\cos(\varphi_I - \varphi_V)} = \frac{I_A V_A}{2}\cos(\varphi_Z) = V_{rms} I_{rms}\cos(\varphi_Z)$$