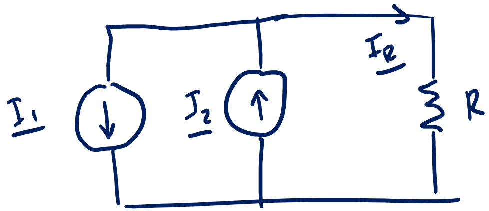
Power Spectrum



$I_1 = I_2$

By inspection $I_R = \emptyset$

Therefore $P_R = \emptyset$

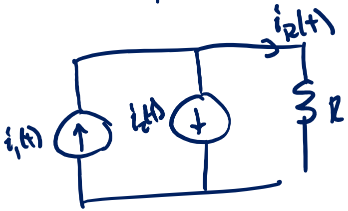
Correctly apply superposition:

$I_R = I_{R1} + I_{R2} = (-I_1) + I_2 = \emptyset$

Incorrect to apply superposition to power (because it's nonlinear)

If I did it anyway: $P_R = P_1 + P_2 = I_{1rms}^2 R + I_{2,rms}^2 R \neq \emptyset$ (wrong)

However, this will work if I have two sources at different frequencies



$i_1(t) = I_{A1}\cos(\omega_1 t)$ $\quad i_2(t) = I_{A2}\cos(\omega_2 t)$, $\quad \omega_1 \neq \omega_2$

$P_R(t) = i_R(t)^2 R = (i_1(t) - i_2(t))^2 R$

$P_R = \frac{1}{T}\int_0^T i_R(t)^2 R\,dt = \frac{1}{T}\int_0^T (I_{A1}\cos(\omega_1 t) + I_{A2}\cos(\omega_2 t))^2 R\,dt$

$= \frac{R}{T}\int_0^T I_{A1}^2\cos^2(\omega_1 t) + I_{A2}^2\cos^2(\omega_2 t) + \underbrace{2 I_{A1} I_{A2}\cos(\omega_1 t)\cos(\omega_2 t)}_{I_{A1}I_{A2}(\cos(\omega_1 t+\omega_2 t) + \cos(\omega_1 t-\omega_2 t))}\,dt$

$\rightarrow \emptyset$ due to averaging

$= I_{1rms}^2 R + I_{2rms}^2 R$ (only for $\omega_1 \neq \omega_2$)

Limitations of Phasor Analysis

① single frequency

② sinusoids only

③ only steady-state response

④ LTI systems only

Remainder of the course: develop techniques to address ① - ③

Approaches:

① use superposition in the time-domain

→ Ch 15 or Frequency Response

② Express arbitrary signal as a sum of (infinite) sinusoids

→ Ch 17 Fourier Series / Transform

③ Include exponentials with our sinusoids

→ Ch 14 Laplace Transform

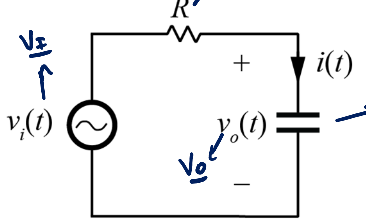
Limitations of Phasor Analysis

Frequency Response

phasor Analysis:

$$\underline{V_O} = \underline{V_I}\,\frac{Z_C}{Z_C + Z_R} = \underline{V_I}\,\frac{-j/\omega C}{-j/\omega C + R} = \underline{V_I}\left(\frac{1}{1 - j\omega CR}\right)$$

$Z_R = R$



$Z_C = \frac{-j}{\omega C}$

$$\underline{V_O} = \underline{V_I}\underbrace{\left(\frac{1}{1 - j\omega CR}\right)} = \underline{V_I}\,\underbrace{H(j\omega)}$$

Frequency Response

→ tells us, at any ω, how does circuit alter input at the output

$$H(j\omega) = \frac{1}{1 - j\omega RC}$$

Gain: $|H(j\omega)| = \frac{1}{\sqrt{1^2 + (\omega RC)^2}}$

Phase: $\measuredangle H(j\omega) = 0 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$

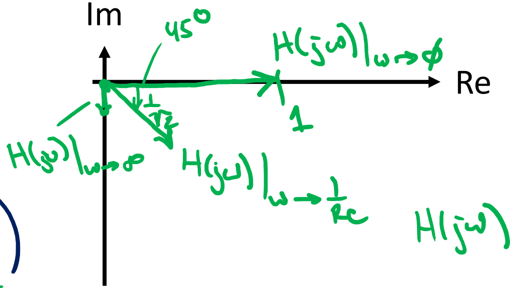
$= -\tan^{-1}(\omega RC)$

Any LTI circuit has $\underline{V_O} = \underline{V_I}\,H(j\omega)$

In polar form

$$V_{OA}\measuredangle\varphi_{VO} = (V_{IA}\measuredangle\varphi_{Vin}) \cdot |H(j\omega)|\measuredangle\varphi(H(j\omega))$$

$$= \underbrace{(V_{IA}\,|H(j\omega)|)}_{\text{Magnitudes multiply}}\ \measuredangle\ \underbrace{\varphi_{Vin} + \varphi(H(j\omega))}_{\text{phases add}}$$



Frequency Response

example

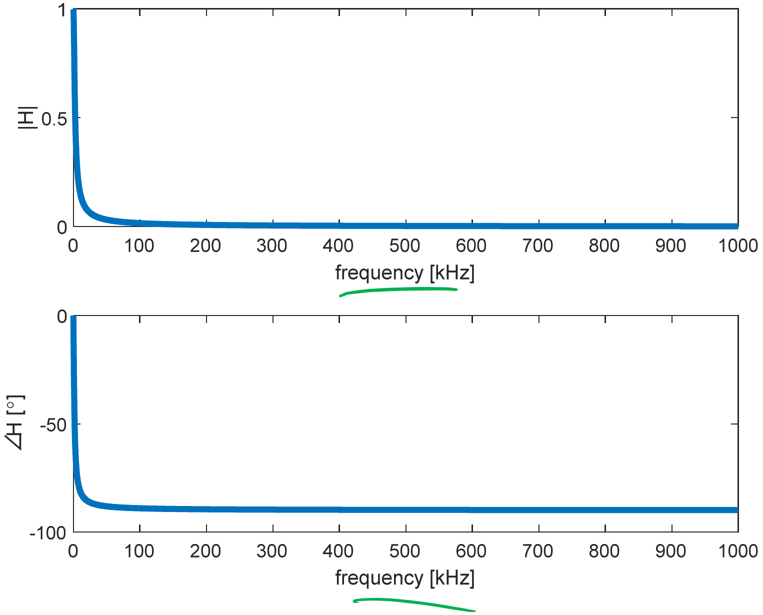
$R = 1\Omega$

$C = 10\mu F$

$\frac{1}{RC} = 10k\ \text{rad/sec} = 1.6\text{kHz}$

Gain

Phase



Bode Plot – Frequency Response