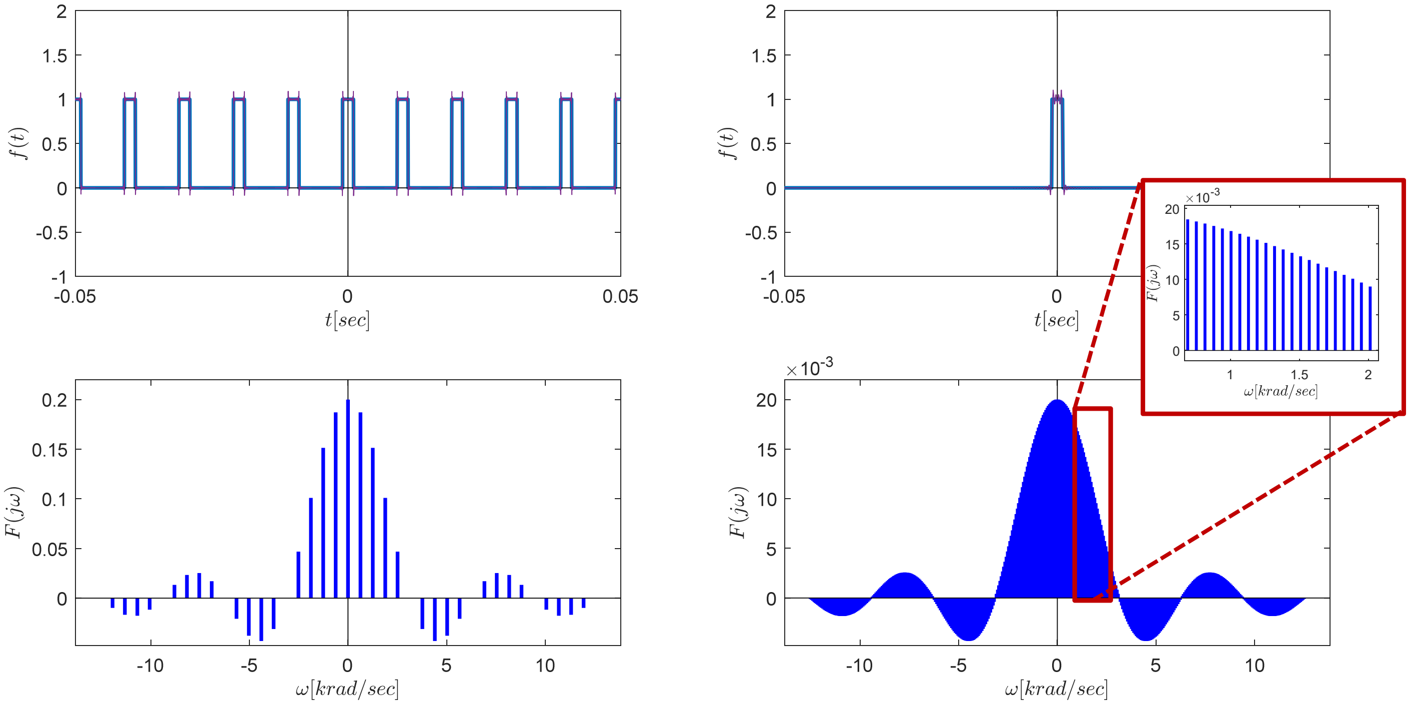
Alternate View

f = 100 Hz
T = 10 ms
τ = 2 ms

f = 10 Hz
T = 100 ms
τ = 2 ms



Non-periodic Waveforms: Fourier Transform

Fourier Series → works only for periodic waveforms

Fourier Transform → for non-periodic signals

Idea: treat any non-periodic signal as if it was periodic with $T \to \infty$

Fourier Series: $c_k = \frac{1}{T}\int_0^T f(t)\, e^{-jk\omega_0 t}\, dt$

Fourier Transform: $Tc_k = \boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$

Fourier Series: Summation $f(t) = \sum_{n=-\infty}^{\infty} c_k\, e^{jk\omega_0 t}$

Inverse Fourier Transform: $\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, dt}$

$f(t)$ can be expressed this way if

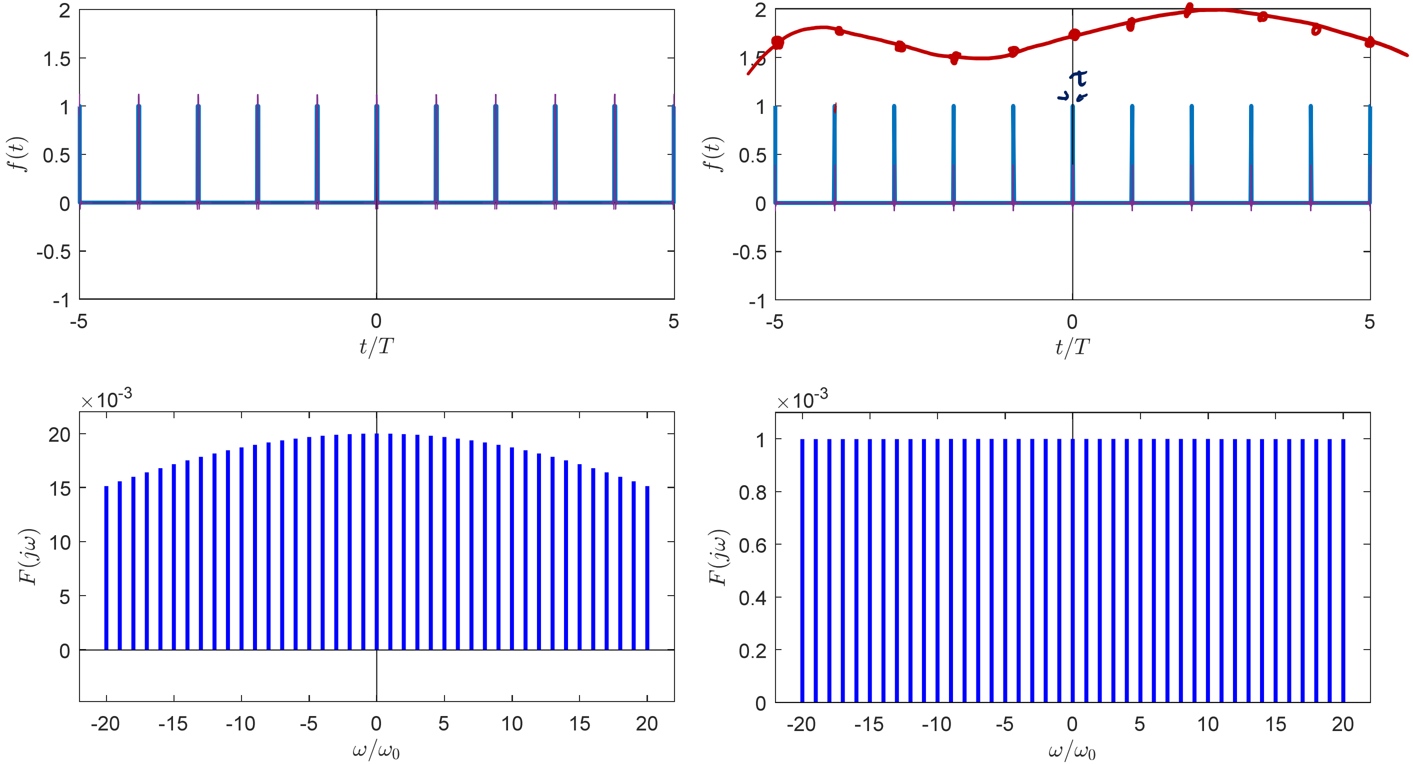
1. $f(t)$ is single-valued
2. $\int_{-\infty}^{\infty} |f(t)|\, dt$ exists
3. $f(t)$ had finite discontinuities and max/min in any closed interval

Fourier Series of Impulse Train

f = 10 Hz
T = 100 ms
τ = 2 ms

f = 1000 Hz
T = 1 ms
τ = .02 ms $\Big\}\ D = \frac{\tau}{T}$

in lim $D \to 0$
this is a sampler



Applications of Fourier Transform

- Imaging
  - Spectroscopy, x-ray crystallography
  - MRI, CT Scan
- Image analysis
  - Compression
  - Feature extraction
- Signal processing
  - Audio filtering
  - Spike detection
  - Modeling sampled systems (A/D & D/A)
  - Understanding aliasing
  - Speech recognition
- RF Communications
  - AM & FM Encoding

https://youtu.be/r6sGWTCMz2k?t=1298

Chapter 14

S-DOMAIN CIRCUIT ANALYSIS

Transform Domains

Phasor Transform

$$u(t) = A\cos(\omega_0 t + \varphi)$$

$$\begin{tikzcd}
\text{Circuit with inductor(s) and capacitor(s)} \arrow[r, "\text{Diff. EQs}"] \arrow[d, "\text{Phasor Transform}"] & \text{Sinusoidal steady-state solution} \\
\text{Phasor-domain circuit} \arrow[r, "\text{Algebraic EQs}"] & \text{Phasor-domain solution} \arrow[u, "\text{Inverse phasor Transform}"']
\end{tikzcd}$$

Fourier Transform

$$u(t) = \sum C_n \cos(n\omega_0 t + \varphi_n)$$

$$\begin{tikzcd}
\text{Circuit with inductor(s) and capacitor(s)} \arrow[r, "\text{Diff. EQs}"] \arrow[d, "\text{Fourier Transform}"] & \text{Steady-state solution} \\
\text{Frequency-domain circuit} \arrow[r, "\text{Algebraic EQs}"] & \text{Frequency-domain solution} \arrow[u, "\text{Inverse Fourier Transform}"']
\end{tikzcd}$$

Laplace Transform

$$u(t) = \sum K_n e^{s_n t}$$

$$\begin{tikzcd}
\text{Circuit with inductor(s) and capacitor(s)} \arrow[r, "\text{Diff. EQs}"] \arrow[d, "\text{Laplace Transform}"] & \text{Solution} \\
\text{s-domain circuit} \arrow[r, "\text{Algebraic EQs}"] & \text{s-domain solution} \arrow[u, "\text{Inverse Laplace Transform}"']
\end{tikzcd}$$

The Laplace Transform

Take Fourier Transform & replace $j\omega \to s = \sigma + j\omega$

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st}\,ds$$

σ_0 is any real number that works

Usually (always in ECE 202) we use Unilateral Laplace Transform ($f(t) = \emptyset$ for $t < 0$)

$$\boxed{F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt}$$

Laplace transform

$$\boxed{f(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s)\, e^{st}\,ds}$$

Inverse Laplace Transform

short-hand

$$F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{f(t)\}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{F(s)\}$$

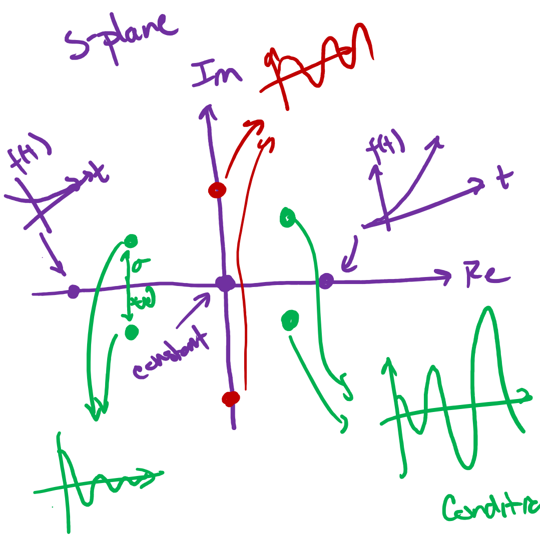
$f(t) \longrightarrow F(s)$

Time-domain		Frequency Domain		Laplace / s / complex Freq. Domain	
$f(t)$	ODEs	$F(j\omega)$	$H(j\omega)$	$F(s)$	$H(s)$
signals	systems	signals	systems	signals	systems

Complex Frequency

$s=\sigma+j\omega$ is a "complex frequency"

In laplace domain our signal $f(t)$ is made up of a superposition of signals that look like $k_a e^{st} = k_a e^{(\sigma+j\omega)t} = k_a e^{\sigma t} e^{j\omega t}$



if $\sigma=\varnothing$, $s=\varnothing+j\omega$ → sinusoids

($s=-j\omega$ will always also show up!)

if $\omega=\varnothing$ $s=\sigma+j\varnothing$ → exponentials

converging if $\sigma<\varnothing$

if $\omega=\varnothing$ & $\sigma=\varnothing$ → constants

if $\omega\neq\varnothing$ & $\sigma\neq\varnothing$ → exponentials * sinusoids

$e^{\sigma t}\cos(\omega t+\phi)$

Conditions for laplace Transform to exist

1. $f(t)$ is a function
2. $f(t)$ has a finite # of discontinuities & max/min over any finite time
3. $\int_0^\infty |e^{-at} f(t)|\,dt$ converges for some real a