MATLAB

See Example in Section 14.8

- `roots`
- `residue`
- `conv`
- $\overset{u(t)}{\texttt{heaviside}}$ & $\overset{\delta(t)}{\texttt{dirac}}$

Symbolic Toolbox

- `syms`
- `laplace` & `ilaplace`
- `pretty`
- `Simplify`

Control Systems Toolbox

- `tf`

Example MATLAB Script

Code

```
syms s t

vi_t = heaviside(t);
VI_s = laplace(vi_t);

H_s = 1/(s + 1);

VO_s = H_s*VI_s;

vo_t = ilaplace(VO_s)
pretty(vo_t)
```

Results

```
VI_s = 1/s

VO_s =  1/(s*(s + 1))

vo_t = 1 - exp(-t)
```

PFE: Repeated Roots

e.g. $F(s) = \frac{5s}{(s+2)^2} = \frac{k_1}{(s+2)} + \frac{k_2}{(s+2)^2} = \frac{k_1(s+2) + k_2}{(s+2)^2} = \frac{k_1 s + (k_2 + 2k_1)}{(s+2)^2}$

for $F(s) = \frac{N(s)}{(s-p_0)(s-p_1)^M} = \frac{k_0}{s-p_0} + \overbrace{\frac{k_1}{s-p_1} + \frac{k_2}{(s-p_1)^2} + \cdots}^{} + \frac{k_M}{(s-p_1)^M}$

k_0 can be found by coverup method

k_M can be found by coverup method

$k_1 \cdots k_{M-1}$ cannot use coverup method

find by $\left\{ \begin{array}{l} \text{equating coefficients} \\ \text{differentiation} \end{array} \right.$

Repeated Roots: Equating Coefficients

ex/

$$F(s) = \frac{32s(s+1)}{(s+2)(s+10)^2} = \frac{k_1}{s+2} + \frac{k_2}{s+10} + \frac{k_3}{(s+10)^2}$$

$$k_1 = \left.\frac{32s(s+1)}{(s+10)^2}\right|_{s=-2} = 1$$

$$k_3 = \left.\frac{32s(s+1)}{(s+2)}\right|_{s=-10} = \frac{-320(-9)}{-8} = -360$$

Multiply both sides by full denominator & equate coeffients of powers of s

$$32s(s+1) = k_1(s+10)^2 + k_2(s+2)(s+10) + k_3(s+2)$$

$$32s^2 + 32s = k_1(s^2+20s+100) + k_2(s^2+12s+20) + k_3(s+2)$$

s^2: $32 = k_1 + k_2 = 1 + k_2 \rightarrow \boxed{k_2 = 31}$

s: $32 = k_1 20 + k_2 12 + k_3 = 20 + 12k_2 - 360$ ✓

s^0: $0 = k_1 100 + k_2 20 + 2k_3 = 100 + 20k_2 - 720$ ✓

$$F(s) = \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2} \longrightarrow f(t) = \mathcal{L}^{-1}\{F(s)\} = \left[1e^{-2t} + 31e^{-10t} - 360te^{-10t}\right]u(t)$$

Repeated Roots: Differentiation

$$\frac{1}{(s+p)^3} = \underset{\uparrow \frac{d^2}{ds^2}}{\frac{k_1}{s+p}} + \underset{\uparrow \frac{d}{ds}}{\frac{k_2}{(s+p)^2}} + \underset{\uparrow \text{coverup}}{\frac{k_3}{(s+p)^3}}$$

ex $F(s) = \frac{32s(s+1)}{(s+2)(s+10)^2}$ $k_1 = 1$, $k_3 = -360$ as before by coverup

Multiply both sides by repeated root with its full multiplicity, then take derivative(s) with respect to s before plugging in $s = p_i$

$$\frac{d}{ds}\left[\frac{32s(s+1)}{(s+2)\cancel{(s+10)^2}}\cancel{(s+10)^2}\right]\Bigg|_{s=-10} = \frac{d}{ds}\left[\frac{k_1}{s+2}(s+10)^2 + \frac{k_2}{\cancel{s+10}}\cancel{(s+10)^2} + \frac{k_3}{\cancel{(s+10)^2}}\cancel{(s+10)^2}\right]$$

$$\left[\cancel{\frac{k_1 2(s+10)}{s+2}} \emptyset + k_2 + \emptyset\right]\Bigg|_{s=-10}$$

$$k_2 = \frac{d}{ds}\left[\frac{32s(s+1)}{(s+2)}\right]\Bigg|_{s=-10} = \frac{(64s+32)(s+2) - (32s^2+32s)(1)}{(s+2)^2}\Bigg|_{s=-10} = 31$$

$$\boxed{k_2 = 31}$$

MATLAB Check

$N(s) = 32s(s+1) = 32s^2 + 32s + 0$

$D(s) = (s+2)(s+10)^2$

$s^2 \downarrow \quad s \downarrow \quad s^0 \downarrow$

$\underbrace{s+2}$ $\quad \underbrace{(s+10)^2}$

```
>>[R,P,K] = residue([32 32 0],conv([1 2],[1 20 100]))
>>
R = residues

    31.0000 k2 }
  -360.0000 k3 }
     1.0000 k1


P = poles
   -10.0000
   -10.0000
    -2.0000


K =
       []
```

MATLAB Check 2

```
>> syms s
>> ilaplace(32*s*(s+1)/(s+2)/(s+10)^2)

ans =
   exp(-2*t) + 31*exp(-10*t) - 360*t*exp(-10*t)
```

Complex Roots: Complex Math

ex $F(s) = \frac{1}{s^2 - 2s + 2}$

$p_{1,2} = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = 1 \pm j$

$$F(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{(s - (1+j))(s - (1-j))} = \frac{k_1}{(s - (1+j))} + \frac{k_2}{(s - (1-j))}$$

complex roots will always occur in conjugate pairs $(s-p)(s-p^*)$ and their residues will always be complex conjugates $k_1 = k_2^*$, for any real signals & systems