PFE: Repeated Roots

Repeated Roots: Equating Coefficients

Repeated Roots: Differentiation

Complex Roots: Complex Math

Complex Roots: General Case

TABLE 14.2 Laplace Transform Operations

Operation	$f(t)$	$\mathbf{F(s)}$
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(s)$
Time differentiation	$\frac{df}{dt}$	$s\mathbf{F}(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2\mathbf{F}(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3\mathbf{F}(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{s}\mathbf{F}(s)$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$
Time shift	$f(t-a)u(t-a),\ a \geq 0$	$e^{-as}\mathbf{F}(s)$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(s+a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^{\infty} \mathbf{F}(s)\,ds$
Scaling	$f(at),\ a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
Initial value	$f(0^+)$	$\lim_{s\to\infty} s\mathbf{F}(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} s\mathbf{F}(s)$, all poles of $s\mathbf{F}(s)$ in LHP
Time periodicity	$f(t) = f(t+nT)$, $n = 1, 2, \ldots$	$\frac{1}{1-e^{-Ts}}\mathbf{F}_1(s)$, where $\mathbf{F}_1(s) = \int_{0^-}^{T} f(t)e^{-st}\,dt$

TABLE 14.1 Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}\{\mathbf{F}(s)\}$	$\mathbf{F}(s) = \mathcal{L}\{f(t)\}$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!}u(t), n = 1, 2, \ldots$	$\frac{1}{s^n}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), n = 1,2,\ldots$	$\frac{1}{(s+\alpha)^n}$
$\frac{1}{\beta-\alpha}(e^{-\alpha t} - e^{-\beta t})u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$\sin \omega t\, u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t\, u(t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)\, u(t)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)\, u(t)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-\alpha t}\sin \omega t\, u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t\, u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Complex Roots: Table Lookup