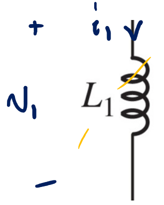
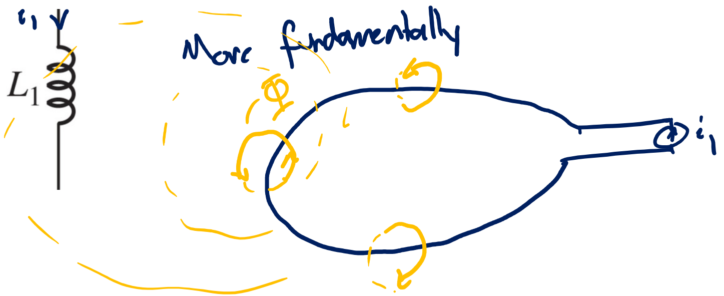
Inductance: Review

We know inductor has circuit behavior $v_1 = L_1 \frac{di_1}{dt}$



More fundamentally



Ampere's law $\oint_c B \cdot dl = \mu_0 I_{encl}$

total flux

$\Phi = \alpha i_1$

↑ parameter depends on geometry & physical constants

for an inductor

$\Phi = \Phi_r$

$v_1 = \alpha \frac{di_1}{dt}$

↑ $= L$ for single-turn

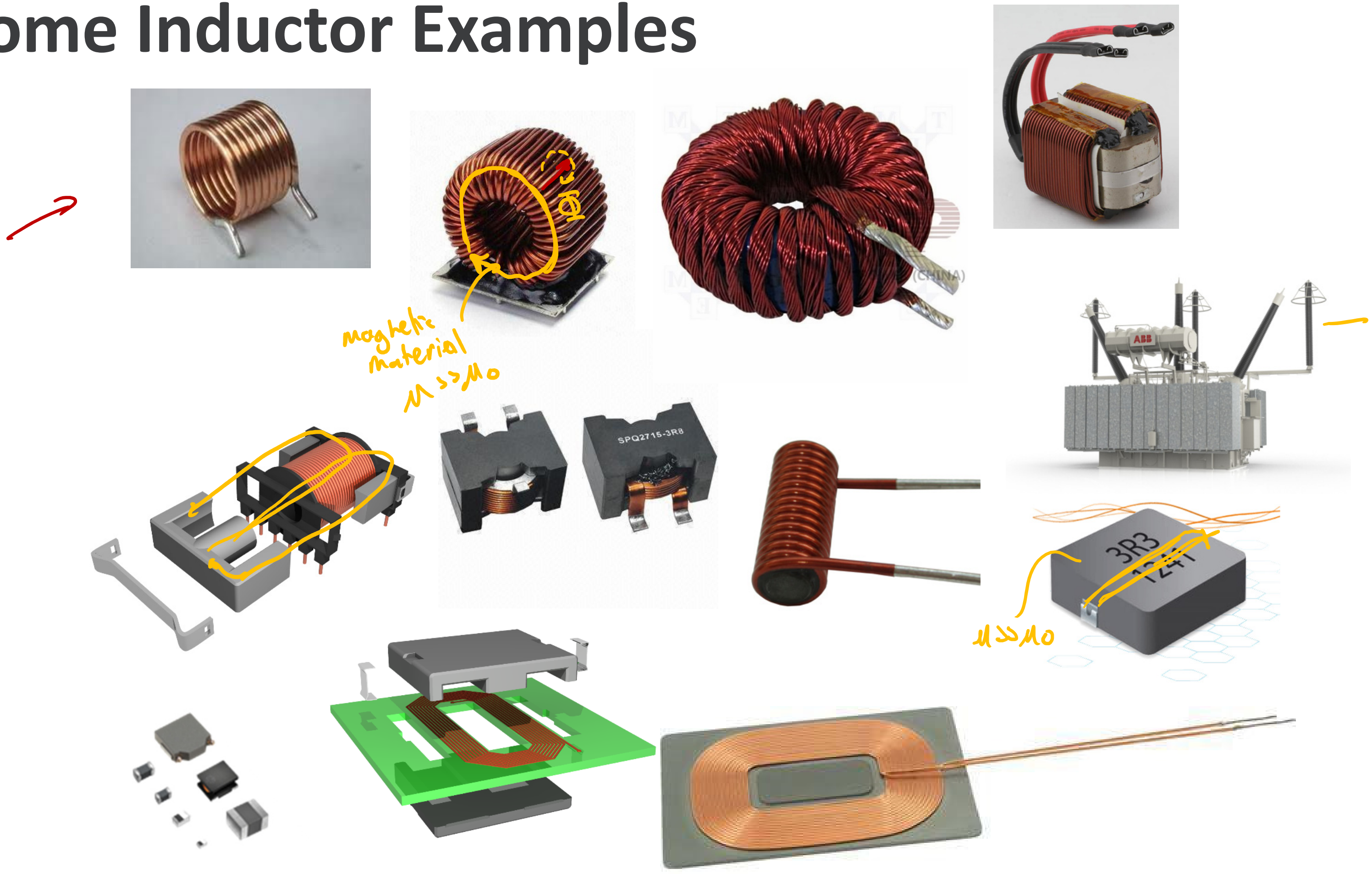
Faraday's law

$v_1 = \frac{d\Phi_r}{dt}$

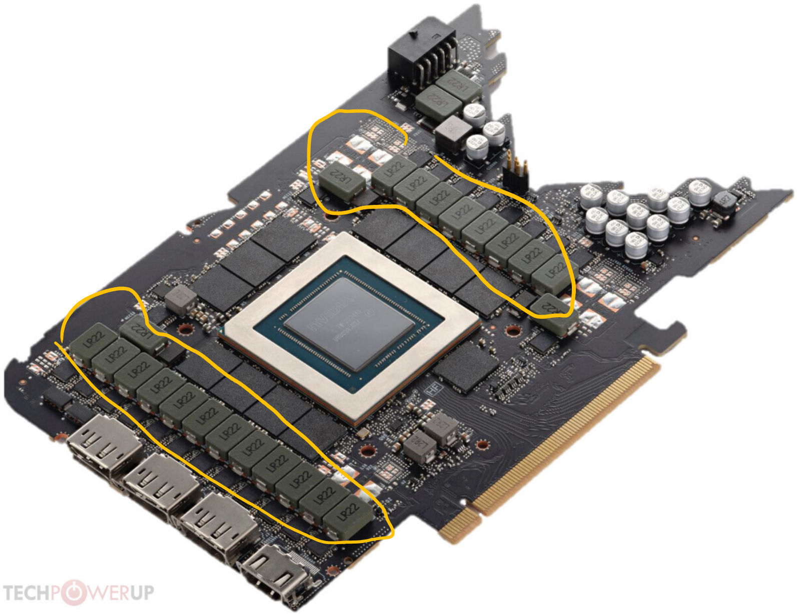
For an N-turn inductor

$\Phi = N\alpha i_1$, $v_1 = N\frac{d\Phi}{dt}$, $v_1 = N^2 \alpha \frac{di_1}{dt}$

Some Inductor Examples

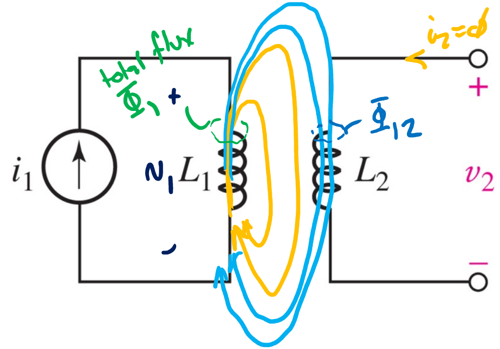


Example Desktop Motherboard and GPU



Magnetic Coupling

→ flux from one inductor goes through coils of another inductor



Case 1: $i_1 > 0 \qquad i_2 = 0$

L_1 behaves as before

$$\Phi_1 = \alpha_1 N_1 i_1 \qquad v_1 = N_1 \frac{d\Phi_1}{dt}, \qquad \boxed{v_1 = \alpha_1 N_1^2 \frac{di_1}{dt}}$$

$$L_1 = \alpha_1 N_1^2$$

Because Φ_{12} flows through L_2

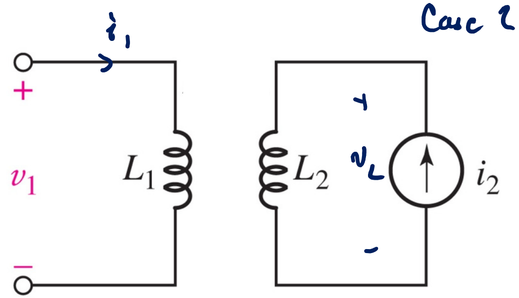
$$v_2 = N_2 \frac{d\Phi_{12}}{dt}, \qquad \Phi_{12} = \alpha_{12} N_1 i_1 \rightarrow \boxed{v_2 = \alpha_{12} N_1 N_2 \frac{di_1}{dt}}$$

$$M = \alpha_{12} N_1 N_2$$

mutual Inductance

$$\boxed{\begin{aligned} v_1 &= L_1 \frac{di_1}{dt} \\ v_2 &= M \frac{di_1}{dt} \end{aligned}}$$

Magnetic Coupling



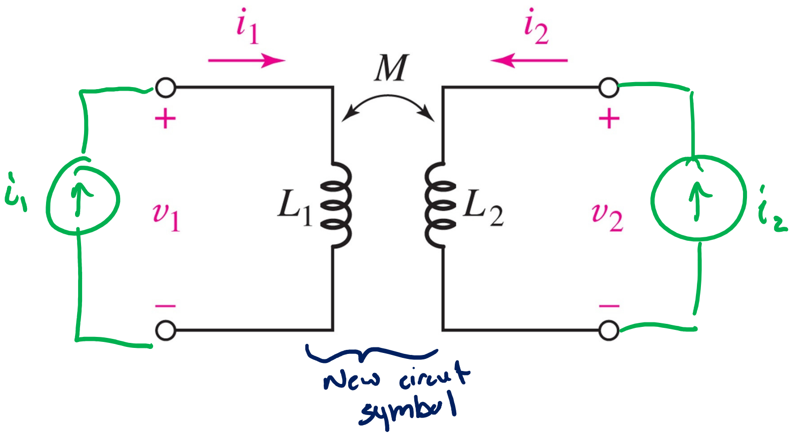
Case 2: $i_1 = \emptyset$, $i_2 \neq \emptyset$

same circuit just mirrored

$$\begin{cases} v_2 = L_2 \dfrac{di_2}{dt} \\ v_1 = M \dfrac{di_2}{dt} \end{cases}$$

$\alpha_{12} = \alpha_{21} \rightarrow$ so M is the same in both sets of equations

Mutual Inductance



Apply superposition

with $i_2 = 0$

$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} \\ v_2 = M \dfrac{di_1}{dt} \end{cases}$$

with $i_1 = 0$

$$\begin{cases} v_1 = M \dfrac{di_2}{dt} \\ v_2 = L_2 \dfrac{di_2}{dt} \end{cases}$$

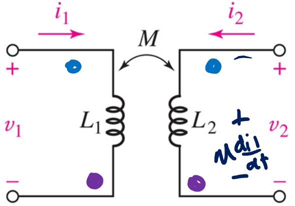
By superposition

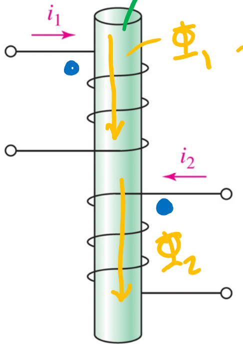
$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} \pm M \dfrac{di_2}{dt} \\ v_2 = \pm M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

Symbols and Dot Convention

★ Make sure to use passive sign convention

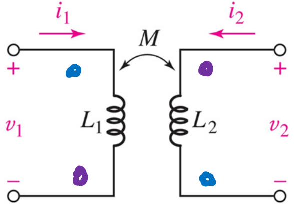
$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{cases}$$

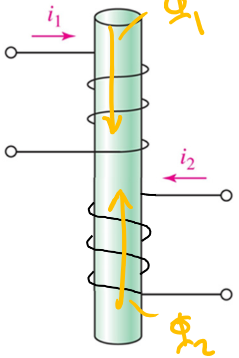




Physical: If both i_1 & i_2 enter the dotted terminals of L_1 & L_2, they produce additive flux

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{cases}$$





Circuit: current flowing into the dotted terminal will produce a positive voltage relative to the dotted terminal of the other winding (in open circuit)