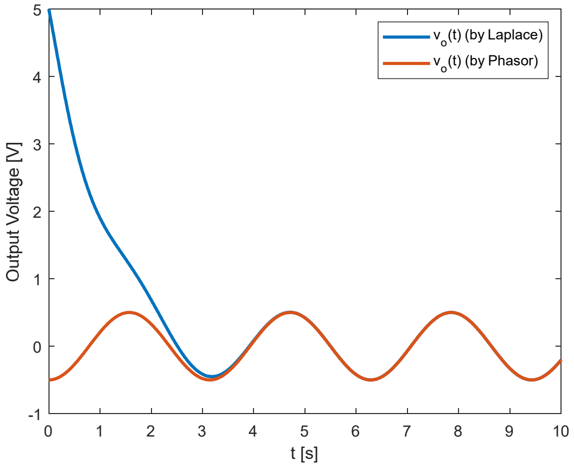
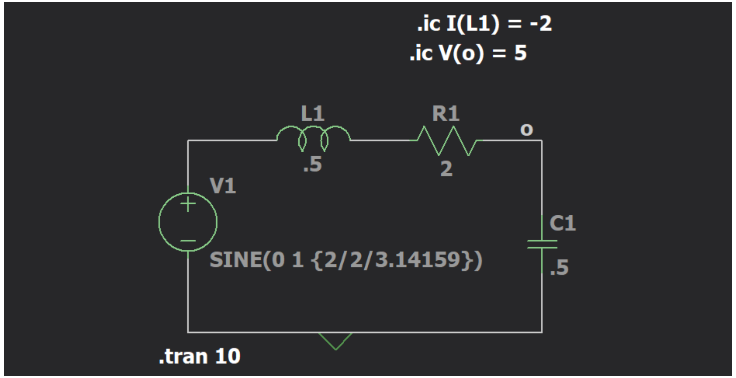
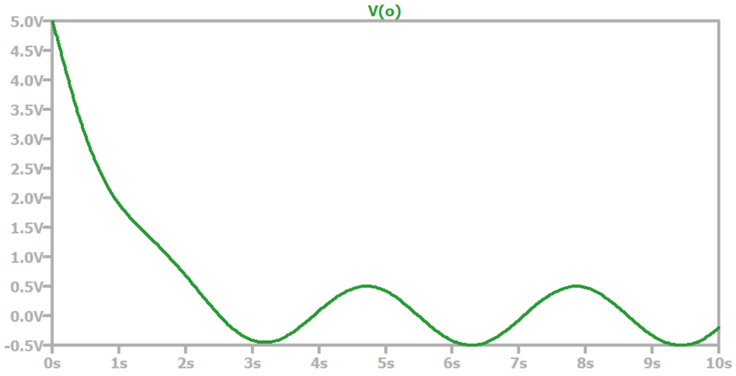
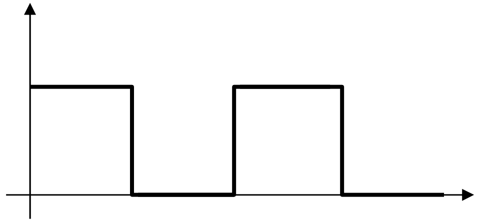
Comparison to Simulation







Laplace Transform of Periodic PWL Signals

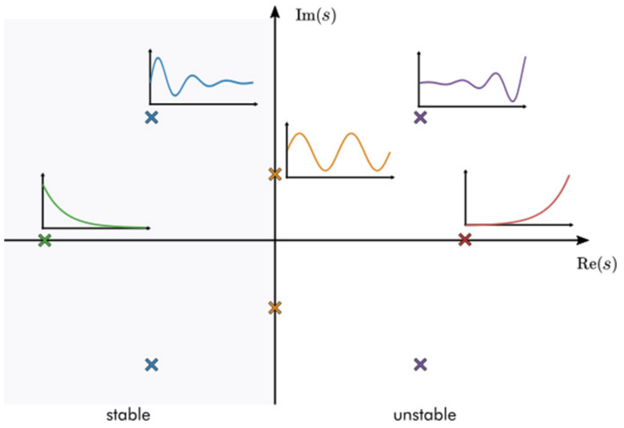


Pole Locations

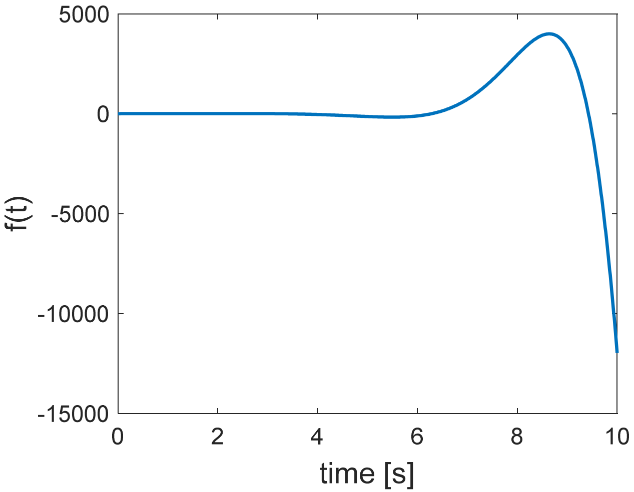
$$x = N_H + N_I$$

$$V_o(s) = V_I(s)H(s) = \left(\frac{\sum_{i=0}^{M_I} c_i s^i}{\sum_{i=0}^{N_I} d_i s^i}\right)\left(\frac{\sum_{i=0}^{M_H} a_i s^i}{\sum_{i=0}^{N_H} b_i s^i}\right) = \frac{(s-z_1)(s-z_2)\cdots(s-z_{M_H+M_I})}{(s-p_1)(s-p_2)\cdots(s-p_{N_H+N_I})}$$

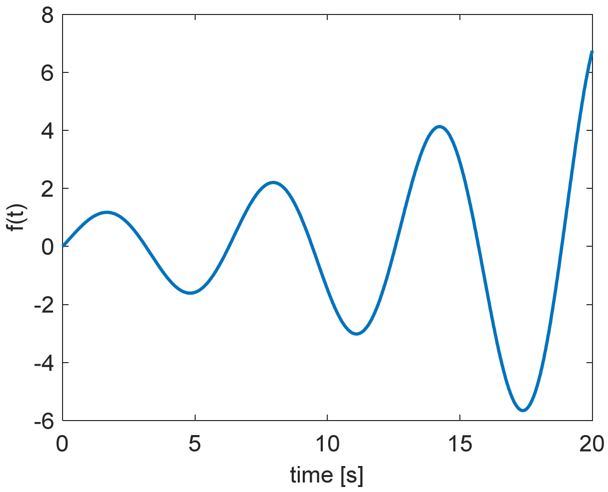
$$= \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots + \frac{k_x}{(s-p_x)}$$



Unbounded Signals & Unstable Systems



$$f(t) = e^{t}\sin(t)$$



$$f(t) = e^{t/10}\sin(t)$$

Laplace and Fourier Revisited

Fourier Transform:

$$F(j\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} f(t)\, dt$$

Laplace Transform (Bilateral):

$$F(s) = \int_{-\infty}^{+\infty} e^{-st} f(t)\, dt$$

Inverse Fourier Transform:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} d\omega$$

Inverse Laplace Transform (Bilateral):

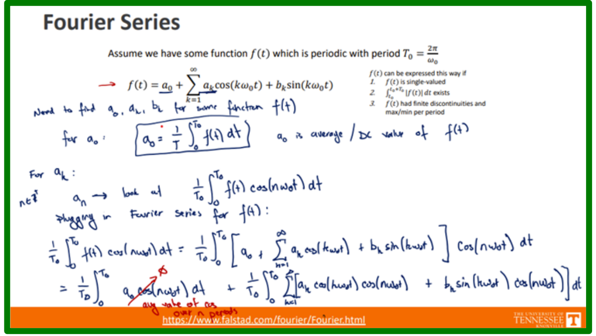
$$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s) e^{st} ds$$

Laplace Explanation

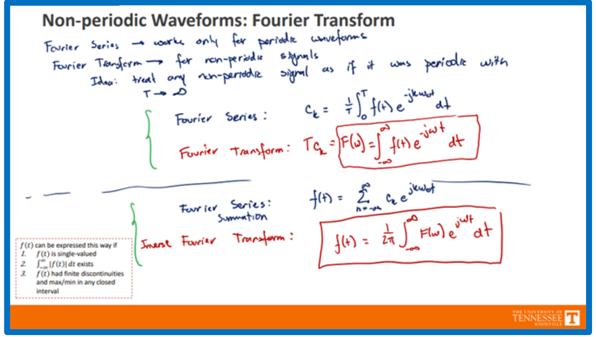
$$F(s) = \int_{0^-}^{+\infty} e^{-st} f(t)\, dt = \int_{0^-}^{+\infty} e^{-\sigma t} e^{-j\omega t} f(t)\, dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} e^{j\omega t} F(s)\, ds$$

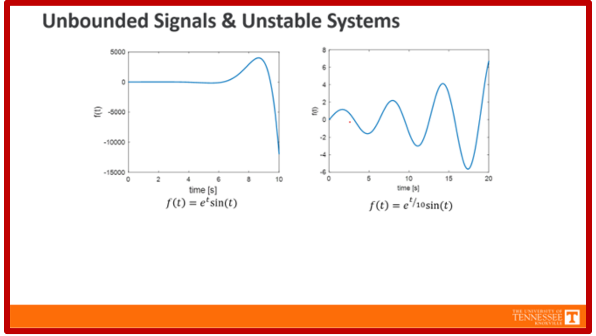
L21



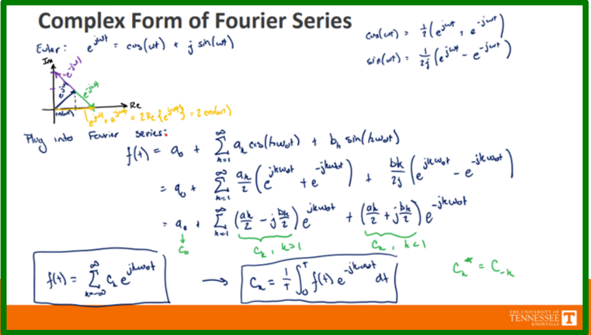
L24



L30



L23



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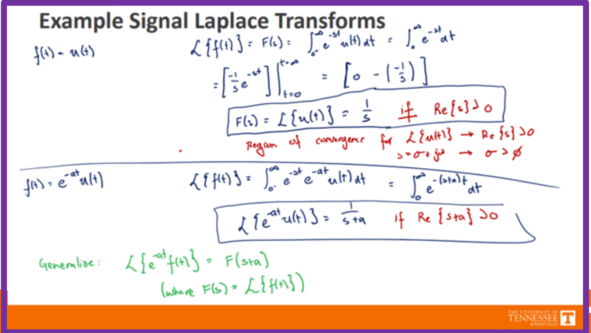
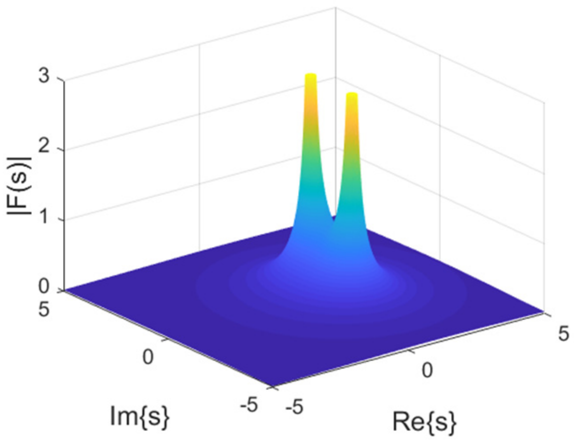
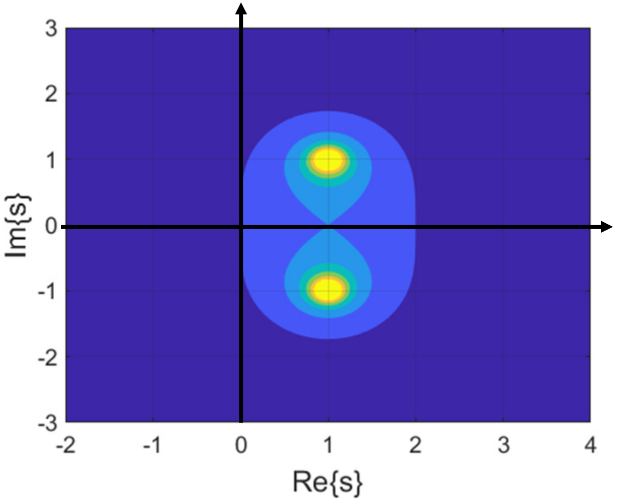


The s-plane





$$F(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{\big(s - (1 + j)\big)\big(s - (1 - j)\big)}$$