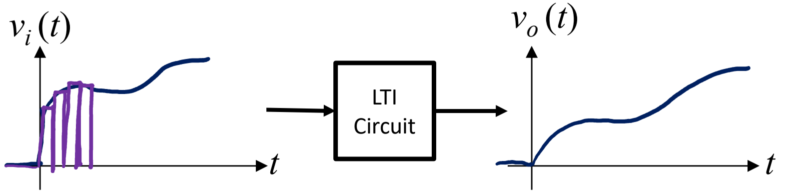
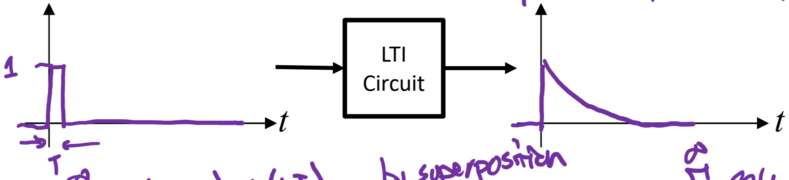
Convolution

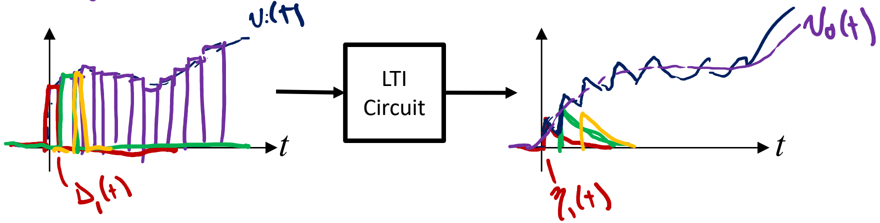


$\Delta(t) \rightarrow$ pulse

$\eta(t) \rightarrow$ pulse response



$v_i(t) = \sum_{k=0}^{\infty} \Delta(t-kT)\, v_i(kT)$ $\xrightarrow{\text{by superposition}}$ $v_o(t) = \sum_{k=0}^{\infty} \eta(t-kT)\, v_i(kT)$



Now, let $T \rightarrow \emptyset$

$$v_i(t) = \int_0^{\infty} \delta(t-\tau)\, v_i(\tau)\, d\tau$$

$\downarrow$ by sifting property of $\delta(t)$

$$\boxed{v_o(t) = \int_0^{\infty} h(t-\tau)\, v_i(\tau)\, d\tau}$$

convolution integral

The Convolution Integral

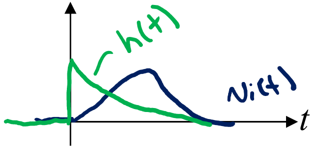
$$v_o(t) = \int_0^\infty \underline{h(t-\tau)\,v_i(\tau)}\,d\tau = \int_0^\infty v_i(t-\tau)\,h(\tau)\,d\tau = \underbrace{v_i(t) * h(t) = h(t) * v_i(t)}_{\text{short-hand}}$$

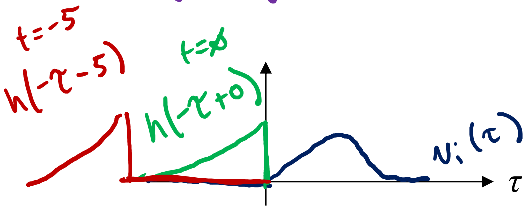
formally convolution integral $\underset{\text{output}}{y(t)} = \int_{-\infty}^{\infty} h(t-\tau)\,\underset{\text{input (not necessarily step fcn)}}{u(\tau)}\,d\tau$

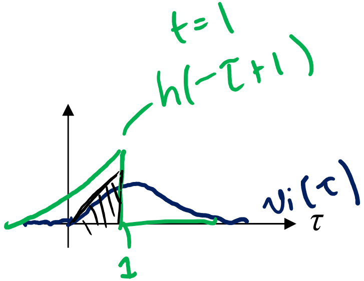
→ for causal systems $h(t)$ is zero for $t<0$ (real systems can't predict the future)

flip shift & integrate

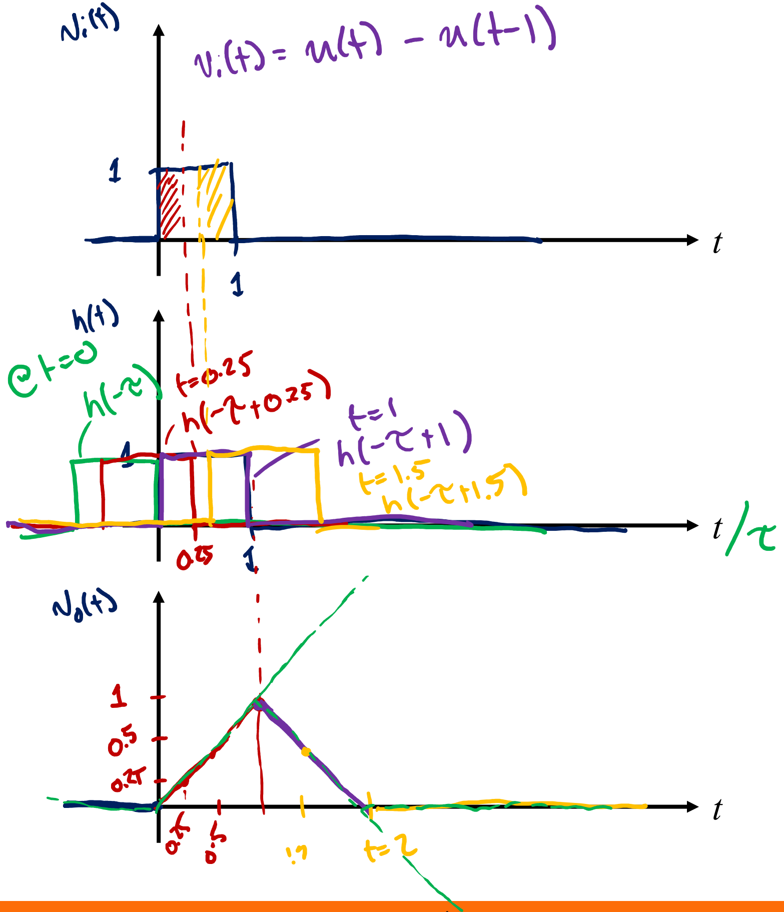
$$v_o(t) = \int_0^\infty h(-\tau+t)\,v_i(\tau)\,d\tau$$







Graphical Convolution



$$v_o(t) = \int_0^\infty h(t-\tau)\, v_i(\tau)\, d\tau$$

flip shift & integrate

$$V_I(s) = H(s) = \frac{1}{s} - \frac{1}{s}e^{-s}$$

$$V_o(s) = V_I(s)H(s) = \left(\frac{1}{s} - \frac{1}{s}e^{-s}\right)^2$$

$$= \frac{1}{s^2} - 2\frac{1}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s}$$

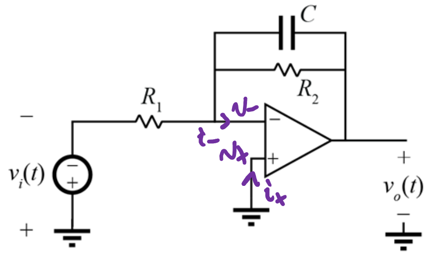
$$v_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = r(t) - 2r(t-1) + r(t-2)$$

✓

$$v_o(t) = r(t) - 2r(t-1) + r(t-2)$$

$$= tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2)$$

Example Problem



Ideal op-amp assumptions:

if there is negative feedback

(1) Virtual short : $v_+ = v_-$ → $V_+(s) = V_-(s)$

(2) $i_+ = i_- = \emptyset$ → $I_+(s) = I_-(s) = \emptyset$