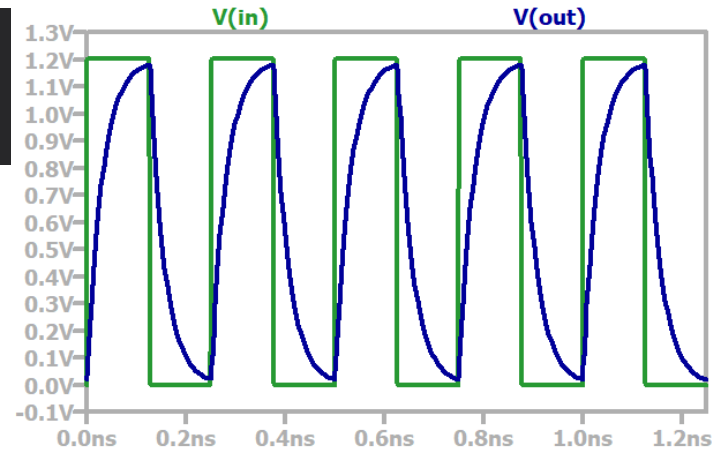
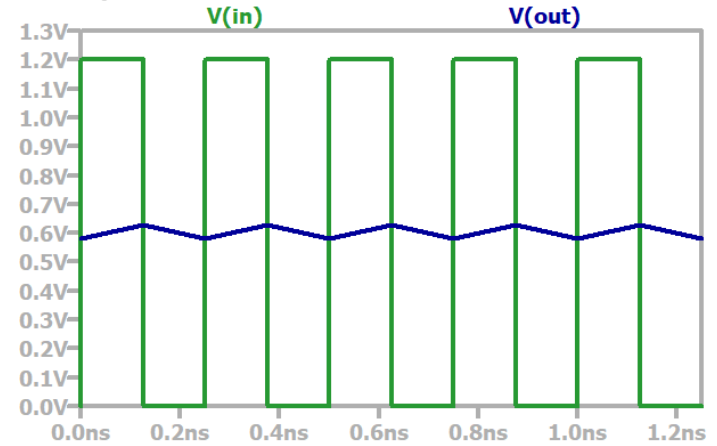
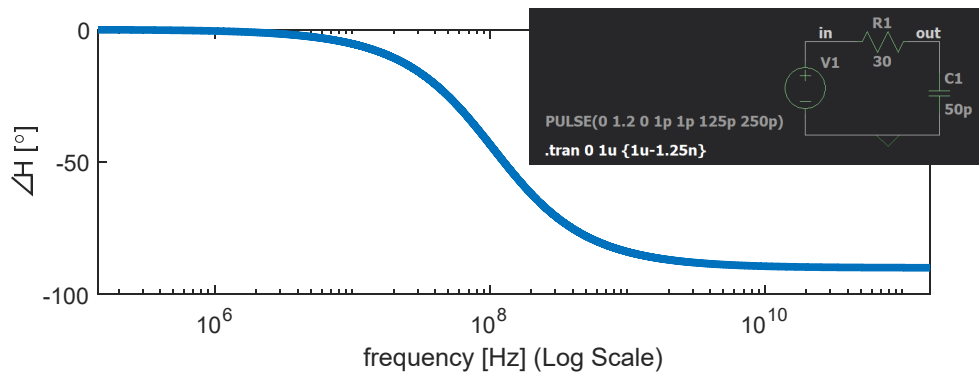
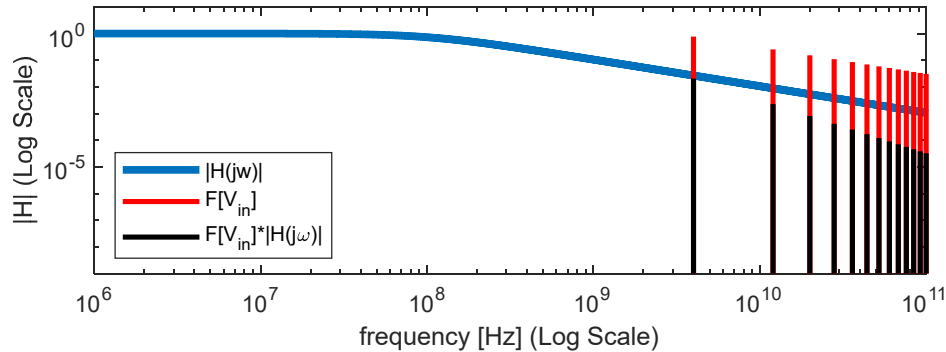


# End of Material for Midterm 2

- Midterm Exam #2 Friday Apr 26th
  - Lectures 20 -33
  - Homeworks 6 - 9
  - Quiz 3 - 4
  - Chapter 17 (17.1 -17.5) and Chapter 14 (all)
  - Experiment 3

# L22 - Frequency Response Interpretation



# Laplace Explanation

$$F(s) = \int_{0^-}^{+\infty} e^{-st} f(t) dt = \int_{0^-}^{+\infty} e^{-\sigma t} e^{-j\omega t} f(t) dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} e^{j\omega t} F(s) ds$$

L21

## Fourier Series

Assume we have some function  $f(t)$  which is periodic with period  $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(k\omega_0 t) + b_n \sin(k\omega_0 t)$$

- $f(t)$  can be expressed this way if
- $f(t)$  is single-valued
- $\int_{-\infty}^{+\infty} |f(t)| dt < \infty$
- $f(t)$  has finite discontinuities and maxima/minima per period

Need to find  $a_0, a_n, b_n$  for some function  $f(t)$   
for  $a_0$ :  $a_0 = \frac{1}{T} \int_0^T f(t) dt$   $a_0$  is average / DC value of  $f(t)$

For  $a_n$ :  $a_n \rightarrow$  but not  $\frac{1}{T} \int_0^T f(t) \cos(n\omega_0 t) dt$

plugging in Fourier Series for  $f(t)$ :

$$\frac{1}{T} \int_0^T f(t) \cos(n\omega_0 t) dt = \frac{1}{T} \int_0^T \left[ a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right] \cos(n\omega_0 t) dt$$

$$= \frac{1}{T} \int_0^T a_0 \cos(n\omega_0 t) dt + \frac{1}{T} \int_0^T \left[ \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) \cos(n\omega_0 t) + b_k \sin(k\omega_0 t) \cos(n\omega_0 t) \right] dt$$

<https://www.khanacad.com/a/fofourier/fofourier.html>

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L24

## Non-periodic Waveforms: Fourier Transform

Fourier Series  $\rightarrow$  work only for periodic waveforms

Fourier Transform  $\rightarrow$  for non-periodic signals

Idea: treat any non-periodic signal as if it was periodic with  $T \rightarrow \infty$

Fourier Series:  $C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt$   
Fourier Transform:  $T C_n = F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

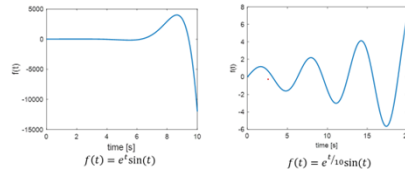
Fourier Series: Summation:  $f(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$   
Smart Fourier Transform:  $f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$

- $f(t)$  can be expressed this way if
- $f(t)$  is single-valued
- $\int_{-\infty}^{+\infty} |f(t)| dt < \infty$
- $f(t)$  has finite discontinuities and maxima/minima in any closed interval

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L30

## Unbounded Signals & Unstable Systems



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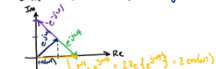
L23

## Complex Form of Fourier Series

Euler:  $e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$$



Plug into Fourier Series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2} (e^{jn\omega_0 t} + e^{-jn\omega_0 t}) + \frac{b_n}{2j} (e^{jn\omega_0 t} - e^{-jn\omega_0 t})$$

$$= a_0 + \sum_{n=1}^{\infty} \left( \frac{a_n}{2} - j \frac{b_n}{2} \right) e^{jn\omega_0 t} + \left( \frac{a_n}{2} + j \frac{b_n}{2} \right) e^{-jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t} \rightarrow C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt \quad C_n^* = C_{-n}$$

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L25

## Example Signal Laplace Transforms

$$f(t) = u(t) \quad \mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} u(t) dt = \int_0^{\infty} e^{-st} dt$$

$$= \left[ -\frac{1}{s} e^{-st} \right]_0^{\infty} = \left[ 0 - \left(-\frac{1}{s}\right) \right] = \frac{1}{s} \quad \text{if } \text{Re}\{s\} > 0$$

Region of convergence for  $\mathcal{L}\{u(t)\} \rightarrow \text{Re}\{s\} > 0$   
 $s = \sigma + j\omega \rightarrow \sigma > 0$

$$f(t) = e^{-at} u(t) \quad \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} e^{-at} u(t) dt = \int_0^{\infty} e^{-(s+a)t} dt$$

$$\mathcal{L}\{e^{-at} u(t)\} = \frac{1}{s+a} \quad \text{if } \text{Re}\{s+a\} > 0$$

Generalize:  $\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$   
(where  $F(s) = \mathcal{L}\{f(t)\}$ )

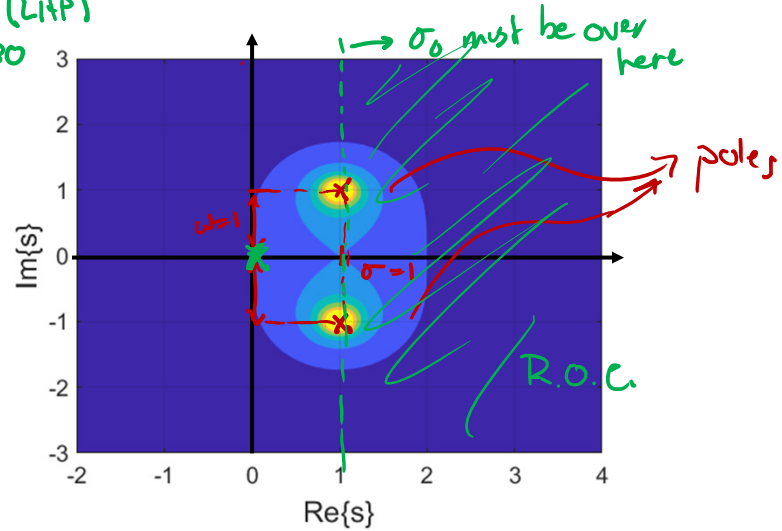
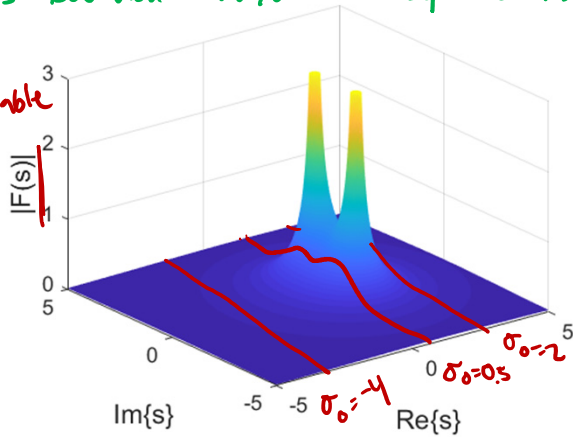
TENNESSEE T

# The s-plane

The Region of convergence of a Laplace transform is the complex plane to the right of all poles

If all poles are in the open left half plane, (LHP) the signal is bounded and/or the system is BIBO stable

RHP poles  $\Leftrightarrow$  unstable



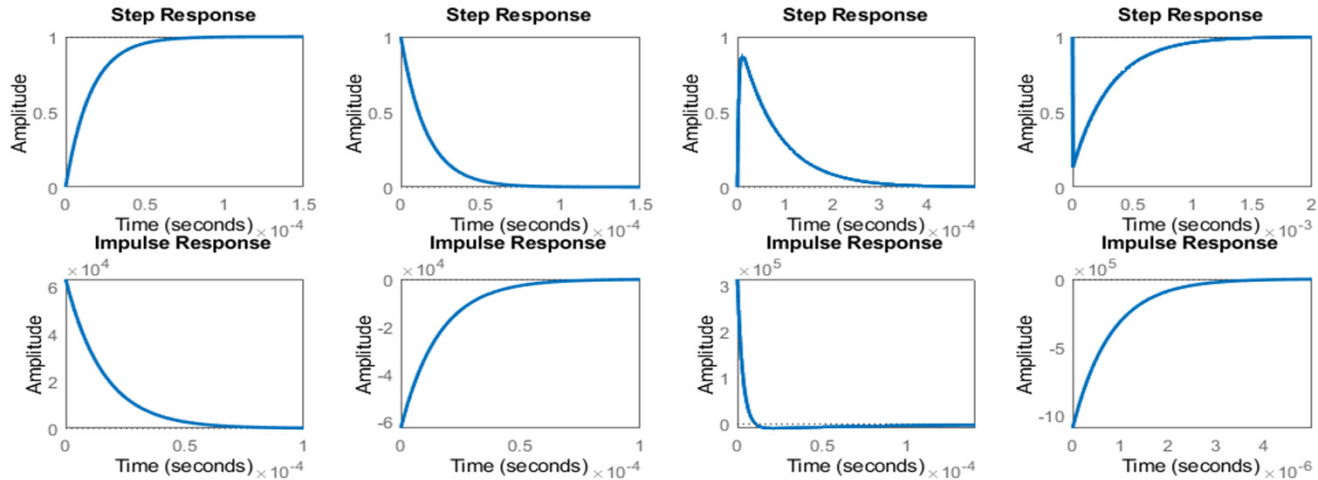
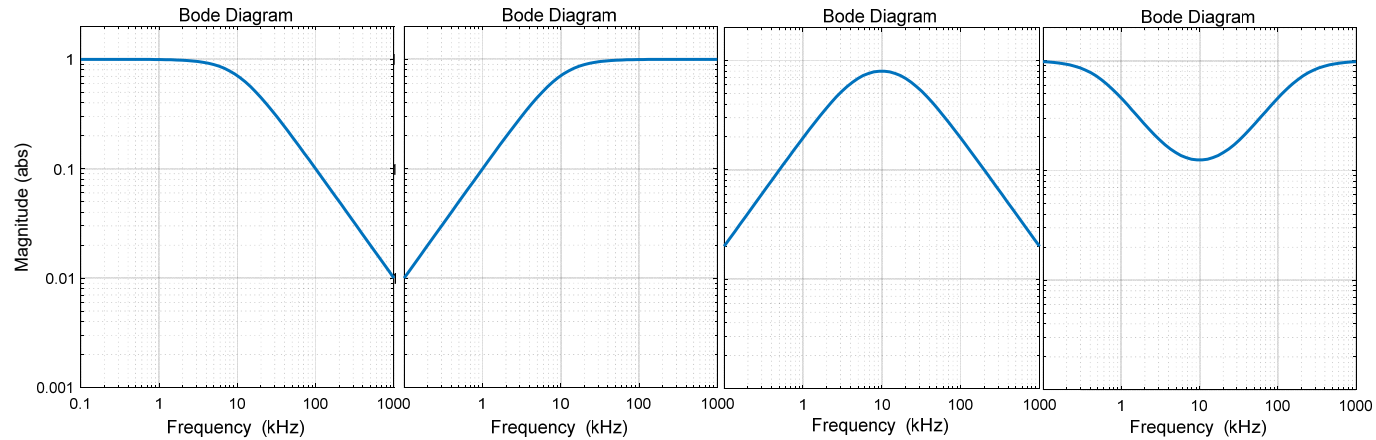
$$F(s) = \frac{1}{s^2 - 2s + 2} = \frac{1}{(s - (1 + j))(s - (1 - j))}$$

$$f(t) = e^t \sin(t)$$

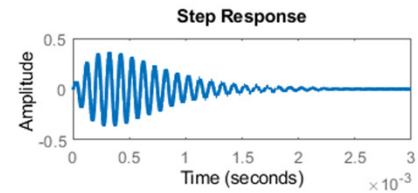
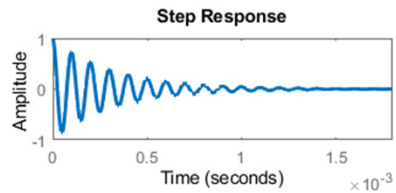
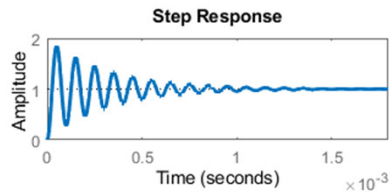
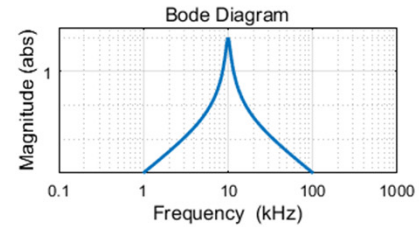
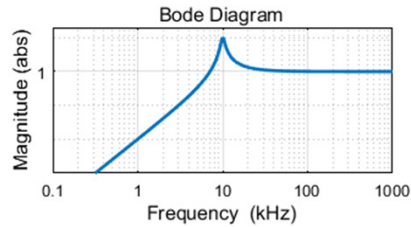
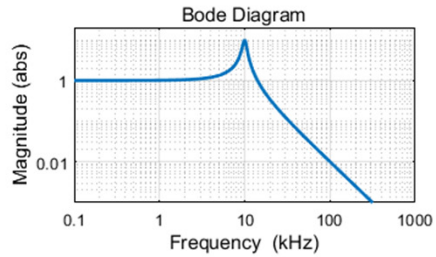
$$F(s) = \int_0^{\infty} e^{-st} e^t \sin(t) dt \rightarrow \text{Need } \text{Re}\{s\} > 1 \text{ for convergence}$$

ROC is  $\text{Re}\{s\} > 1$

# Frequency Response and Circuit Behavior



# Complex Poles



# Bode Plots

