Logarithm Review

In 202: $\log() \Leftrightarrow \log_{10}()$ $\qquad \ln() \Leftrightarrow \log_e()$

In MATLAB: log() function is natural logarithm, $\log_e()$

log10() function is $\log_{10}()$

$$x = \log_y(y^x)$$

Properties:

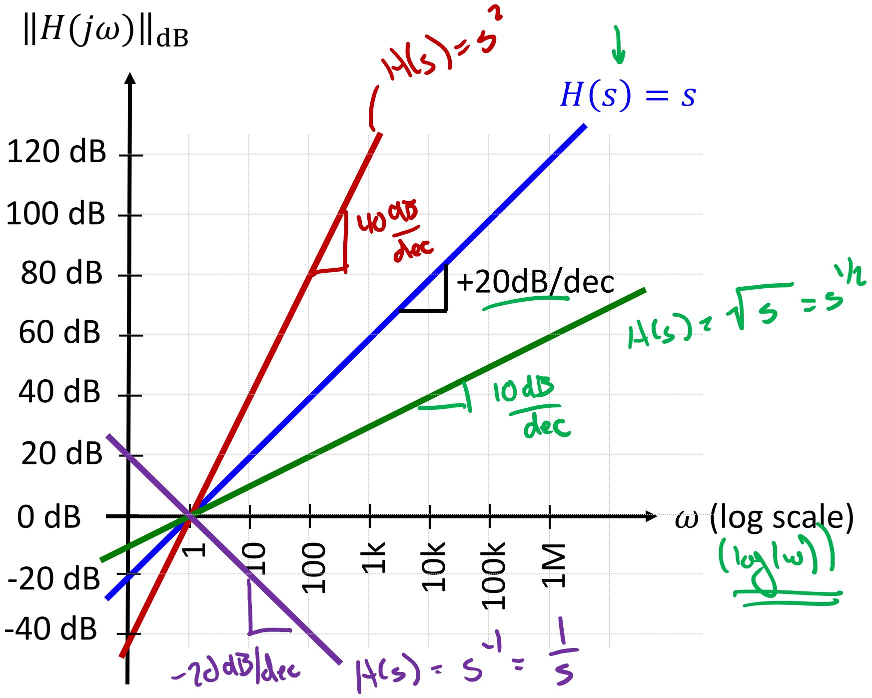
① $$\log_a(x) = \frac{\log_y(x)}{\log_y(a)}$$

② $$\log(x \cdot y) = \log(x) + \log(y)$$

③ $$\log(x^n) = n\log(x)$$

④ $$\log(x/y) = \log(x) - \log(y)$$

Plotting on Logarithmic Axes



$H(s) = s$

$H(j\omega) = j\omega$

$|H(j\omega)| = \omega$

$\|H(j\omega)\|_{dB} = 20\log(|H(j\omega)|) = 20\log(\omega)$

$H(s) = s^2$

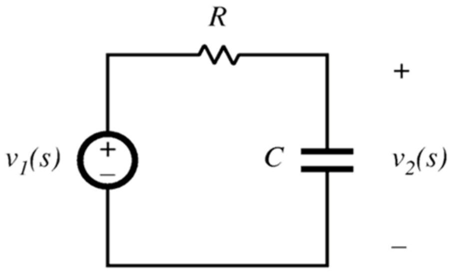
$H(j\omega) = (j\omega)^2 = -\omega^2$

$|H(j\omega)| = |-\omega^2| = \omega^2$

$\|H(j\omega)\|_{dB} = 20\log(\omega^2)$

$= 40\log(\omega)$

Single Pole Response



$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{RsC + 1}$$

Standard form for Frequency response (single pole)

$$H(s) = \frac{1}{\frac{s}{\omega_0} + 1} \qquad \omega_0 = \frac{1}{RC}$$

$$\|H(j\omega)\|_{dB} = 20\log\left(\left|\frac{1}{\frac{j\omega}{\omega_0} + 1}\right|\right)$$

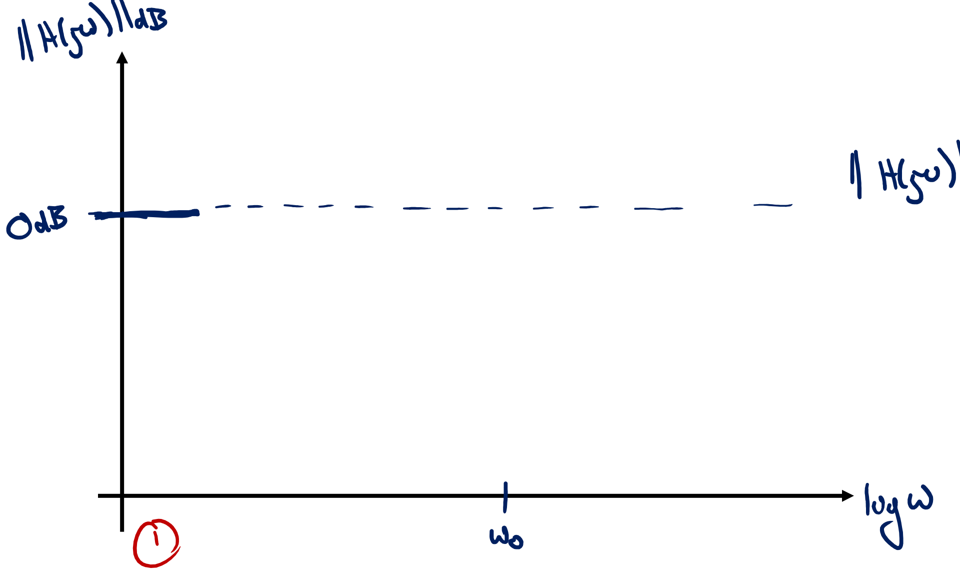
$$= 20\log(|1|) - 20\log\left(\left|\frac{j\omega}{\omega_0} + 1\right|\right)$$

$$= 0 - 20\log\left(\sqrt{1^2 + \left(\frac{\omega}{\omega_0}\right)^2}\right)$$

$$\boxed{\|H(j\omega)\|_{dB} = -20\log\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

Asymptotic Behavior

$$\|H(j\omega)\|_{dB} = -20\log\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}$$



$$\|H(j\omega)\|_{dB} \approx \begin{cases} 0\,dB & , \ \omega \ll \omega_0 \quad (1) \\ & , \ \omega = \omega_0 \\ & , \ \omega \gg \omega_0 \end{cases}$$

1 $\omega \ll \omega_0$: $\|H(j\omega)\|_{dB} \approx -20\log\sqrt{1} = 0\,dB$