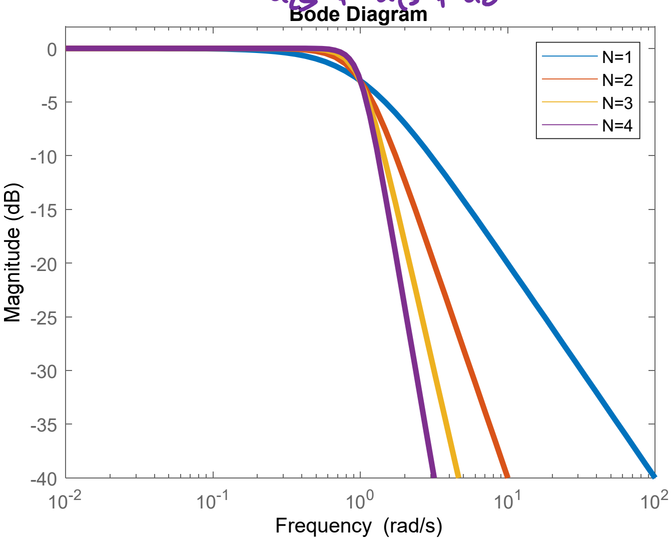
Canonical N^{th} Order Filter Designs (Section 15.9)

Butterworth

flat response in pass band

$$|\mathbf{H}(j\omega)| = \frac{K}{\sqrt{1 + (\omega/\omega_c)^{2n}}} \qquad n = 1, 2, 3, \ldots$$

@ $n=2$ $\quad \dfrac{K}{a_2 s^2 + a_1 s + a_0}$



Chebyshev

ripple in pass band & faster cutoff

$$|\mathbf{H}(j\omega)| = \frac{K}{\sqrt{1 + \beta^2 C_n^2(\omega/\omega_c)}} \qquad n = 1, 2, 3, \ldots$$

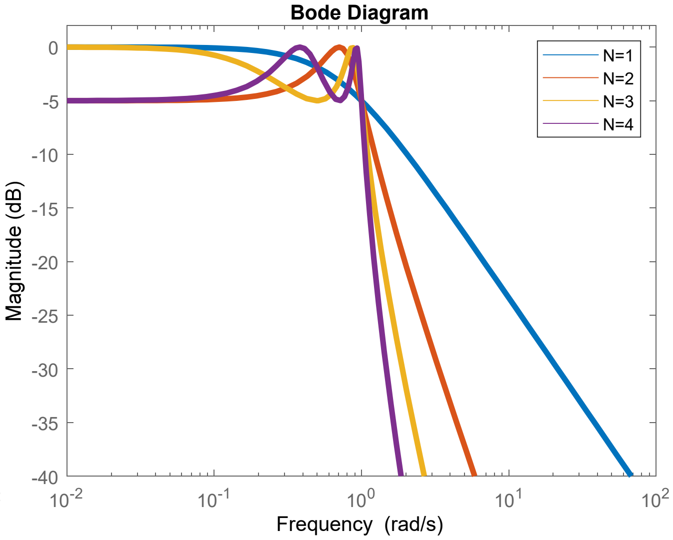


TABLE 15.2 Coefficients for Low-Pass Butterworth and Chebyshev (β = 0.9
Normalized to $\omega_c = 1$

n	a_0	a_1	a_2
Butterworth			
1	1.0000		
2	1.0000	1.4142	
3	1.0000	2.0000	2.0000
4	1.0000	2.6131	3.4142
5	1.0000	3.2361	5.2361
Chebyshev (β = 0.9976)			
1	1.0024		
2	0.7080	0.6449	
3	0.2506	0.9284	0.5972
4	0.1770	0.4048	1.1691
5	0.0626	0.4080	0.5489

EXAMPLE 15.15

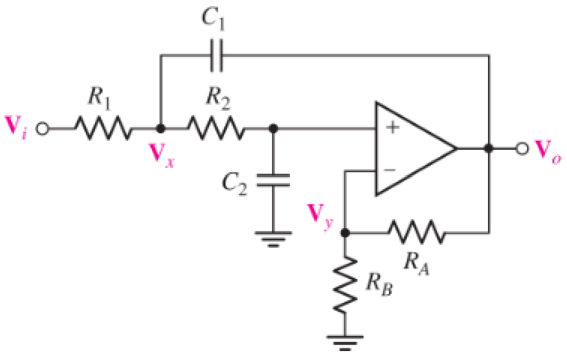
Design a second-order low-pass Butterworth filter having a gain of 4 and a corner frequency at 1400 rad/s.

In MATLAB

butter( )

cheby( )

The Sallen-Key Amplifier



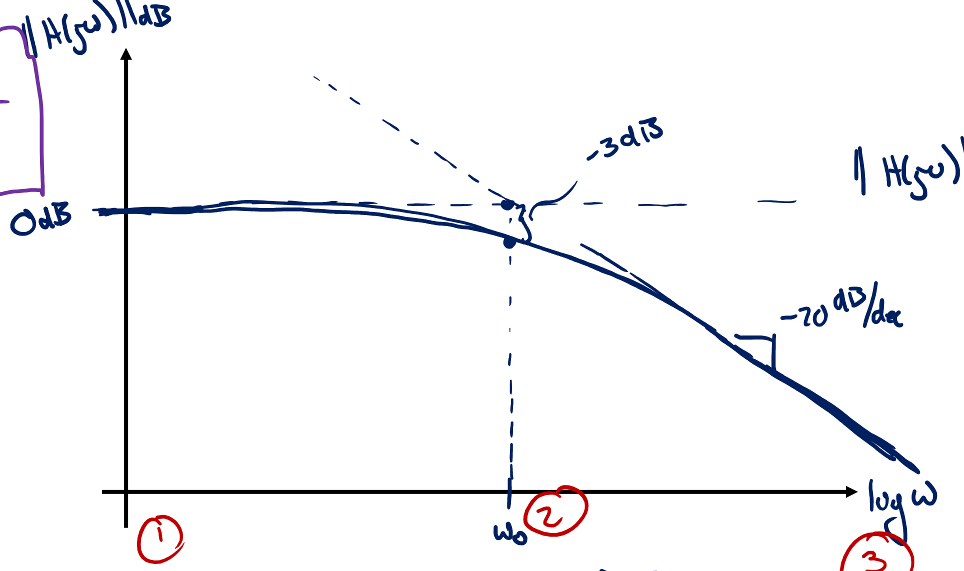
$$G \equiv \frac{R_A + R_B}{R_B}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{G/R_1R_2C_1C_2}{\mathrm{s}^2 + \left[\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2}\right]\mathrm{s} + \frac{1}{R_1R_2C_1C_2}}$$

Asymptotic Behavior

$$\|H(j\omega)\|_{dB} = -20\log\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}$$

$$H(s) = \frac{1}{\frac{s}{\omega_0}+1}$$



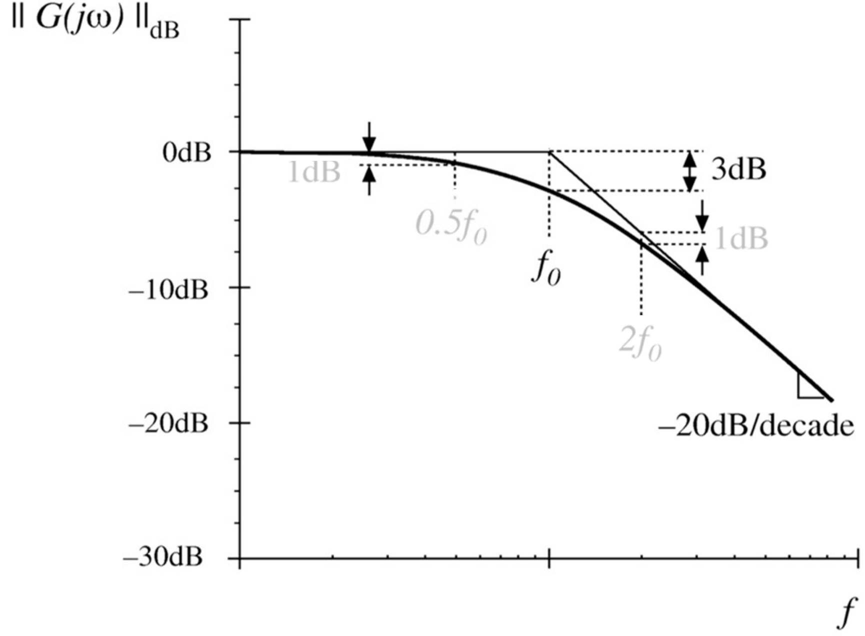
$$\|H(j\omega)\|_{dB} \approx \begin{cases} 0\text{ dB}, & \omega \ll \omega_0 \quad (1) \\ \quad , & \omega = \omega_0 \\ \quad , & \omega \gg \omega_0 \quad (3) \end{cases}$$

1 $\omega \ll \omega_0$: $\|H(j\omega)\|_{dB} \approx -20\log\sqrt{1} = 0\text{dB}$

2 $\omega = \omega_0$: $\|H(j\omega)\|_{dB} = -20\log\sqrt{1+1} = -20\log\sqrt{2} = -3\text{dB}$

3 $\omega \gg \omega_0$: $\|H(j\omega)\|_{dB} \approx -20\log\sqrt{\left(\frac{\omega}{\omega_0}\right)^2} = -20\log(\omega) + 20\log(\omega_0)$

Summary: Single Pole Magnitude



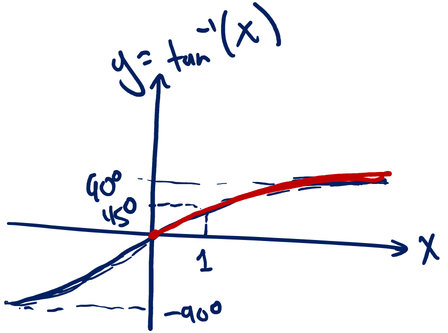
Phase of Single Pole

$$H(s) = \frac{1}{\frac{s}{\omega_0} + 1} \longrightarrow H(j\omega) = \frac{1}{\frac{j\omega}{\omega_0} + 1}$$

$$\angle H(j\omega) = \angle(1) - \angle\left(1 + j\frac{\omega}{\omega_0}\right)$$

$$= 0^\circ - \tan^{-1}\left(\frac{\omega/\omega_0}{1}\right)\left(\frac{180^\circ}{\pi}\right)$$

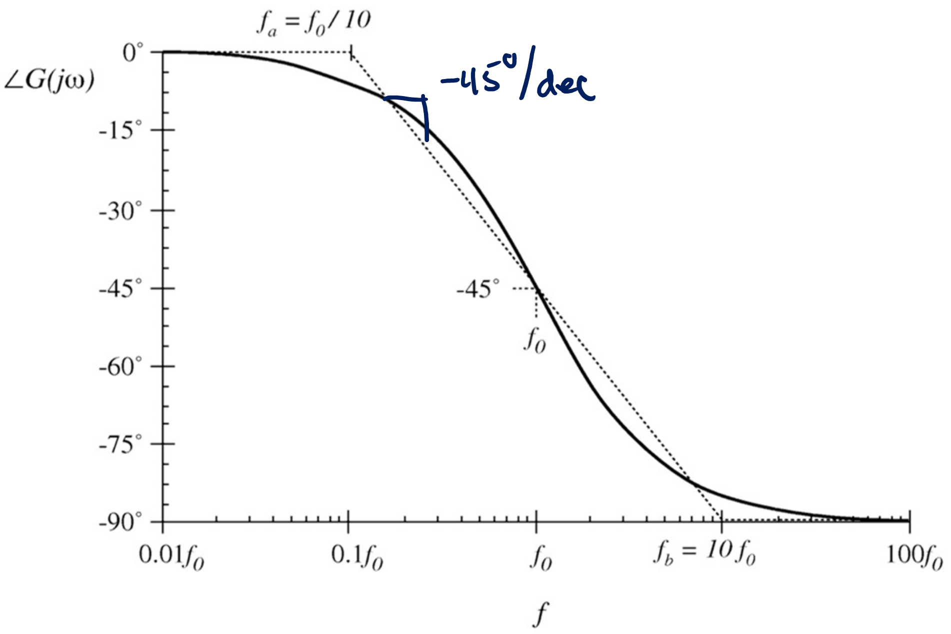
$$\boxed{\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)}$$



Approx

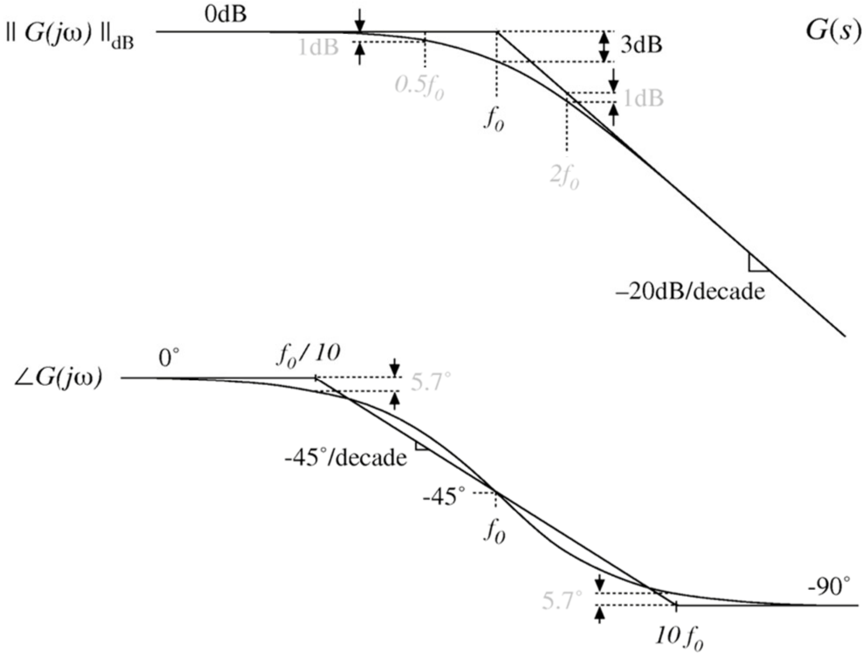
$$\angle H(j\omega) = \begin{cases} 0^\circ, & \omega \ll \omega_0 \\ -45^\circ, & \omega = \omega_0 \\ -90^\circ, & \omega \gg \omega_0 \end{cases}$$

Phase Asymptotes

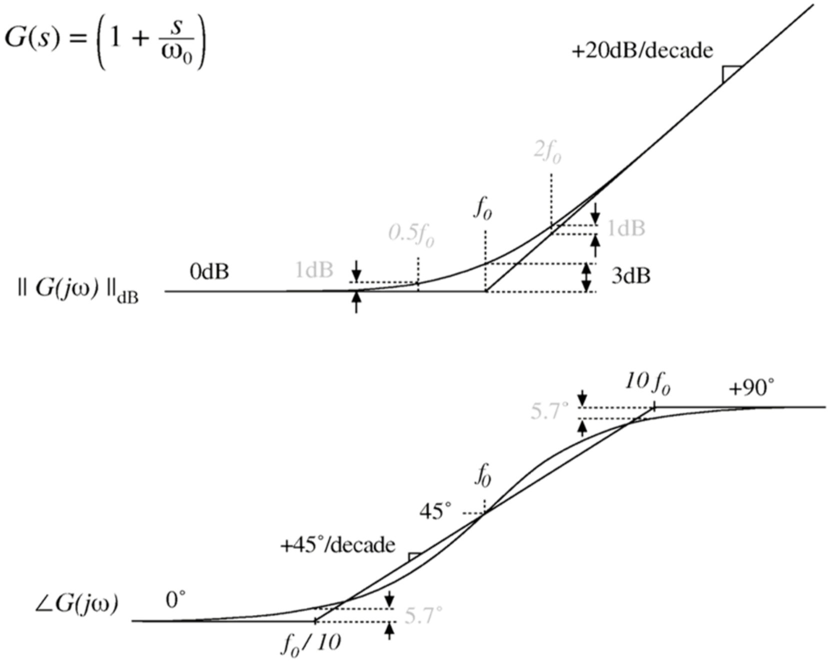


$f_a = f_0 / 10$
$f_b = 10\,f_0$

Summary: Single Real Pole

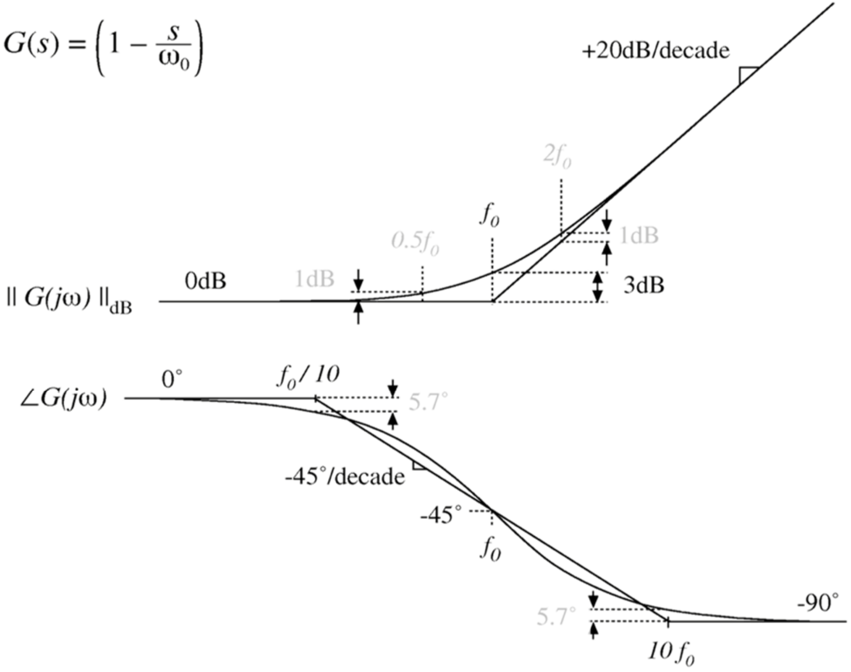


Bode Plot: Real Zero



RHP Zero

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$



RHP Pole

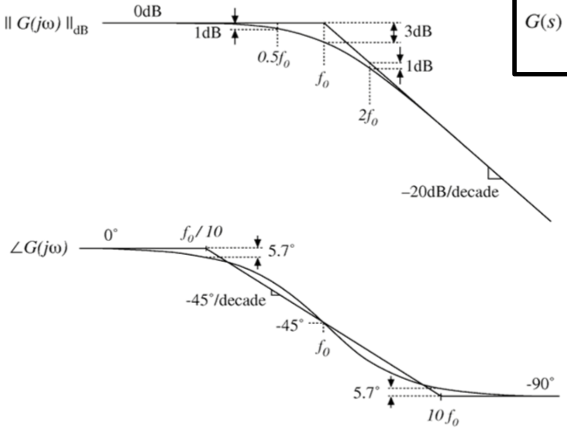
→ Bode plots not applicable

Frequency Response: if we have no RHP poles

$H(s)\big|_{s=j\omega} = H(j\omega)$ is the frequency response

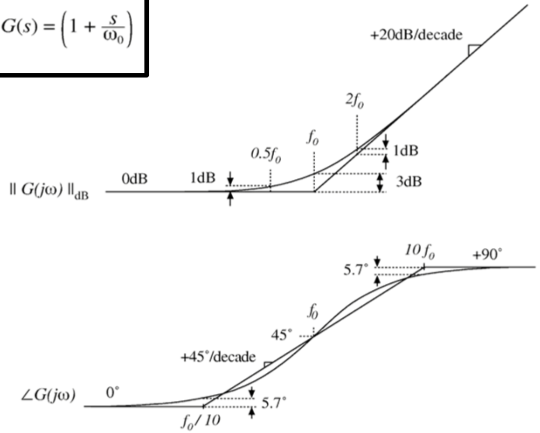
Real Pole

$$G(s) = \frac{1}{\left(1+\frac{s}{\omega_0}\right)}$$



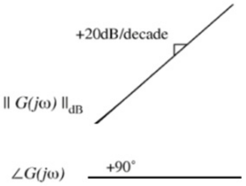
Real Zero

$$G(s) = \left(1+\frac{s}{\omega_0}\right)$$



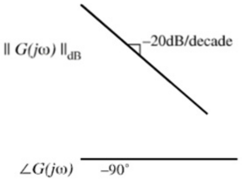
$$G(s) = s$$

LF Zero



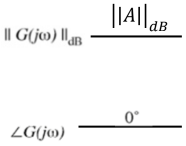
$$G(s) = \frac{1}{s}$$

LF Pole



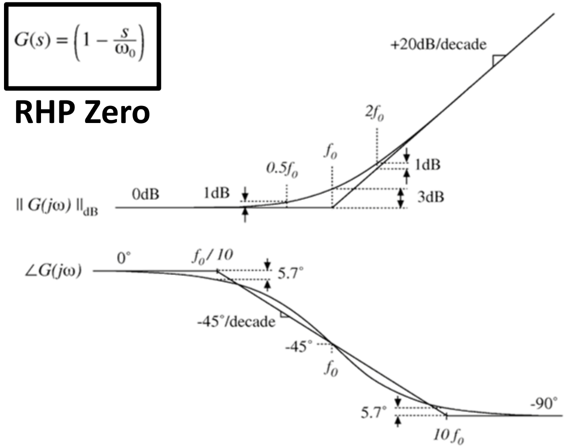
$$G(s) = A$$

Constant



$$G(s) = \left(1-\frac{s}{\omega_0}\right)$$

RHP Zero



Multiplying Transfer Functions

$$H(s) = A\,\frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots} = \underbrace{A}_{H_0}\cdot\underbrace{\left(1+\frac{s}{\omega_{z1}}\right)}_{H_1(s)}\cdot\underbrace{\left(1+\frac{s}{\omega_{z2}}\right)}_{H_2(s)}\cdot\underbrace{\left(\frac{1}{1+\frac{s}{\omega_{p1}}}\right)}_{H_3(s)}\underbrace{\left(\frac{1}{1+\frac{s}{\omega_{p2}}}\right)}_{H_4(s)}$$

$$H(s) = H_0\cdot H_1(s)\cdot H_2(s)\cdot \cdots$$

$$H(j\omega) = |H_0|e^{j\angle H_0}\,|H_1(j\omega)|e^{j\angle H_1(j\omega)}\,|H_2(j\omega)|e^{j\angle H_2(j\omega)}\cdots$$

$$= \underbrace{|H_0|\cdot|H_1(j\omega)|\cdot|H_2(j\omega)|\cdot(\ldots)}_{\text{Magnitudes of all factors multiply}}\; e^{j\overbrace{(\angle H_0+\angle H_1(j\omega)+\angle H_2(j\omega)+\cdots)}}$$

$\boxed{\text{phases of all factors add up}}$

If we plot magnitude in dB

$$|H(j\omega)|_{dB} = 20\log(|H(j\omega)|) = 20\log(|H_0|) + 20\log(|H_1(j\omega)|) + 20\log(|H_2(j\omega)|)\cdots$$

$\boxed{\text{In log scale, magnitudes add}}$

Example 1

$$H(s) = \frac{H_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

$H_0 = 40$

$\|H_0\|_{dB} = 20\log(40) = 32dB$

$f_1 = 100Hz$

$f_2 = 2kHz$