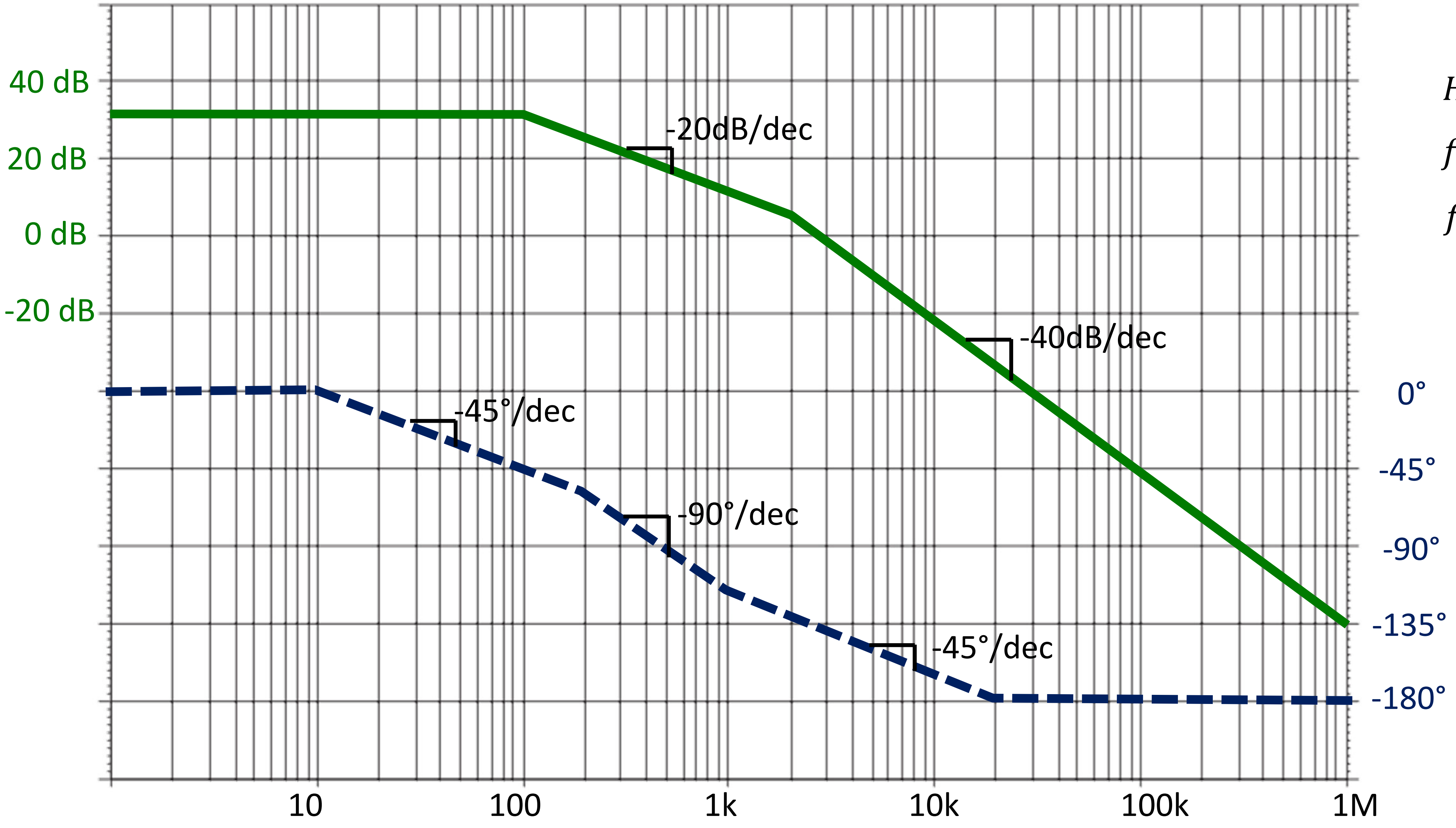
Example 1

$$H(s) = \frac{H_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

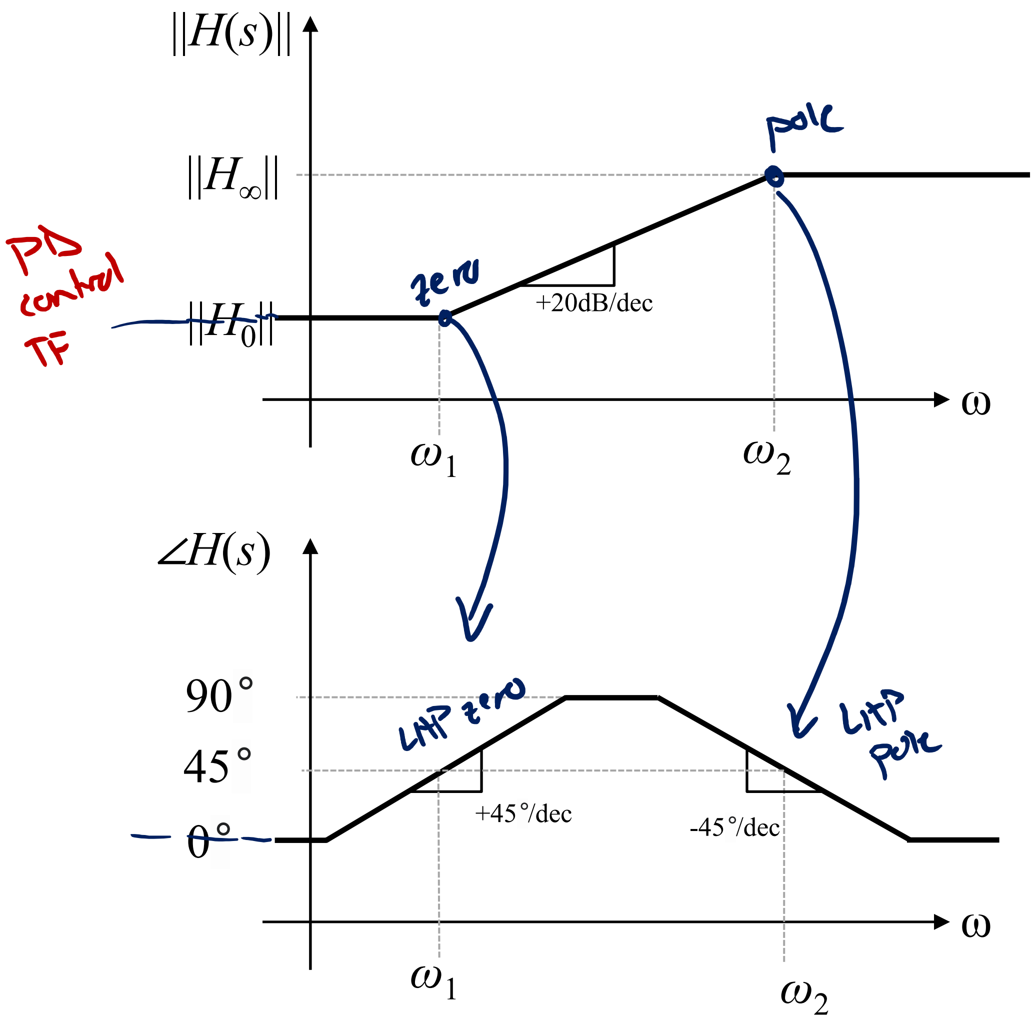
$H_0 = 40$

$f_1 = 100Hz$

$f_2 = 2kHz$



Example 2



$$H(s) = H_0 \frac{\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$$

what is H_∞?

High frequency gain as $s \to j\omega$, $\omega \to \infty$

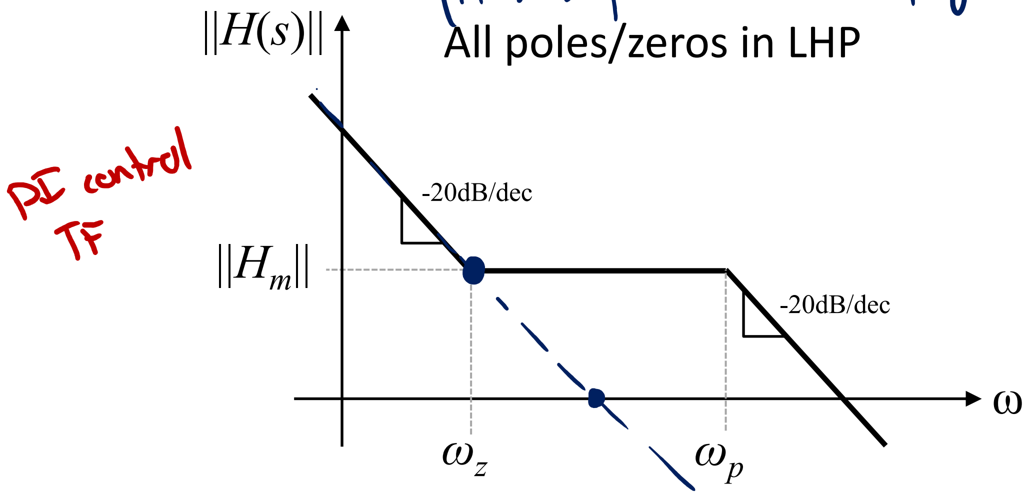
$$\|H_\infty\| = \left\| \lim_{\omega \to \infty} \left| H_0 \frac{1 + \frac{j\omega}{\omega_1}}{1 + \frac{j\omega}{\omega_2}} \right| \right\|$$

$$\boxed{H_\infty = H_0 \frac{\omega_2}{\omega_1}}$$

Example 3

|H(s)| is positive at low frequencies

All poles/zeros in LHP

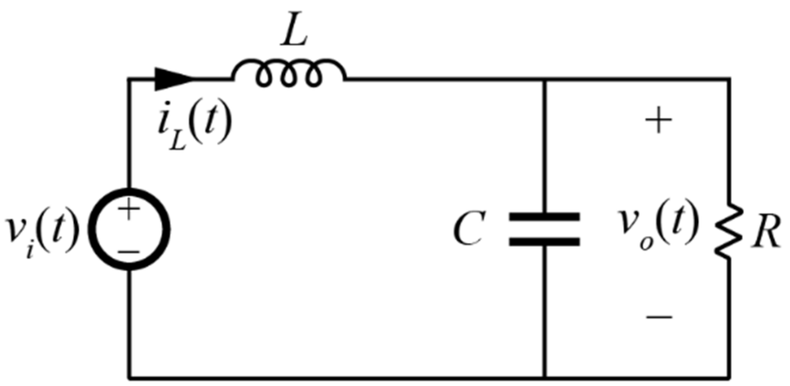


$$H(s) = H_m \omega_z \cdot \frac{1}{s} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

@ ω_z $\left|\frac{A}{s}\right| = |H_m|$

$$\left|\frac{A}{j\omega_z}\right| = |H_m| \rightarrow A = H_m \omega_z$$

Complex Poles



$$H(s) = \frac{V_o(s)}{V_I(s)} = \frac{R \parallel \frac{1}{sC}}{sL + R \parallel \frac{1}{sC}} = \frac{\frac{R}{1+RsC}}{sL + \frac{R}{1+RsC}}$$

$$H(s) = \frac{1}{s^2LC + s\frac{L}{R} + 1}$$

Standard form for 2nd order polynomial

$$\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1$$

find ω_0 & Q by matching coefficients

$$\frac{1}{\omega_0^2} = LC \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{Q\omega_0} = \frac{L}{R} \rightarrow Q = \frac{R}{\sqrt{L/C}} = \frac{R}{R_0}$$

Also define $R_0 = \sqrt{L/C}$, "characteristic impedance"

Standard Form for Complex Poles

$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1} \qquad H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\frac{s}{\omega_0} + 1}$$

- ω_0, Q, and ζ are real for real circuits
  - Found by coefficient matching to solved transfer function
- ω_0 is the angular/resonant frequency
  - Q is the "quality factor"
    - Roots are complex when $Q > \frac{1}{2}$
    - If $Q < \frac{1}{2}$, the polynomial can be factored into two real poles
  - ζ is the "damping factor"
    - Roots are complex when $\zeta > 1$
    - If $\zeta > 1$, the polynomial can be factored into two real poles

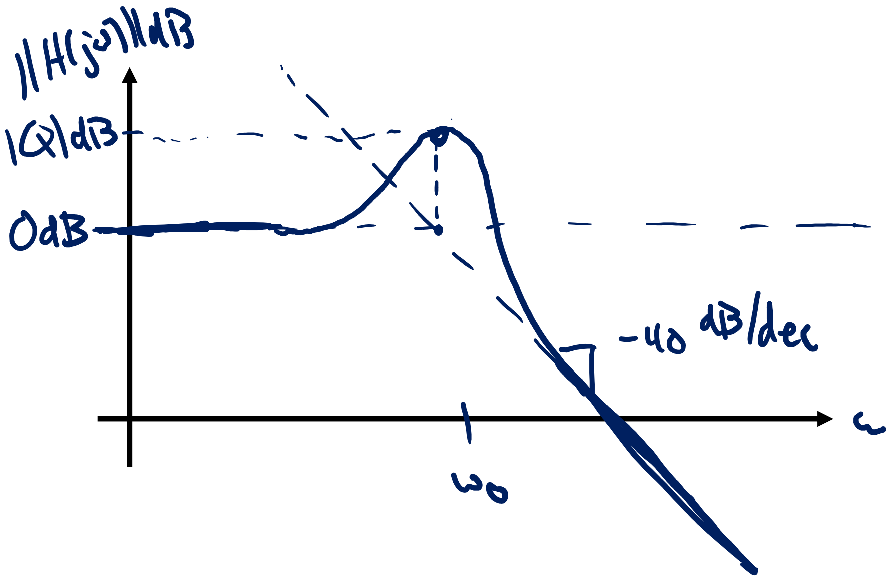
$$\zeta = \frac{1}{2Q}, \qquad Q = \frac{1}{2\zeta}$$

Magnitude Asymptotes

when $Q > 1/2$

$$H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

$$\|H(j\omega)\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{\omega}{Q\omega_0}\right)^2}}$$

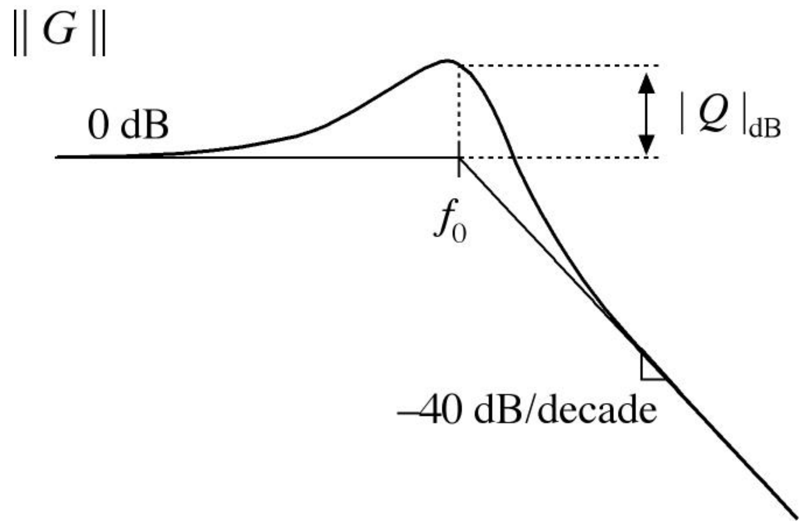


$$\|H(j\omega)\|_{dB} \approx \begin{cases} 0\text{dB}, & \omega \ll \omega_0 \\ 20\log(Q), & \omega = \omega_0 \\ 40\log(\omega_0) - 40\log(\omega), & \omega \gg \omega_0 \end{cases}$$

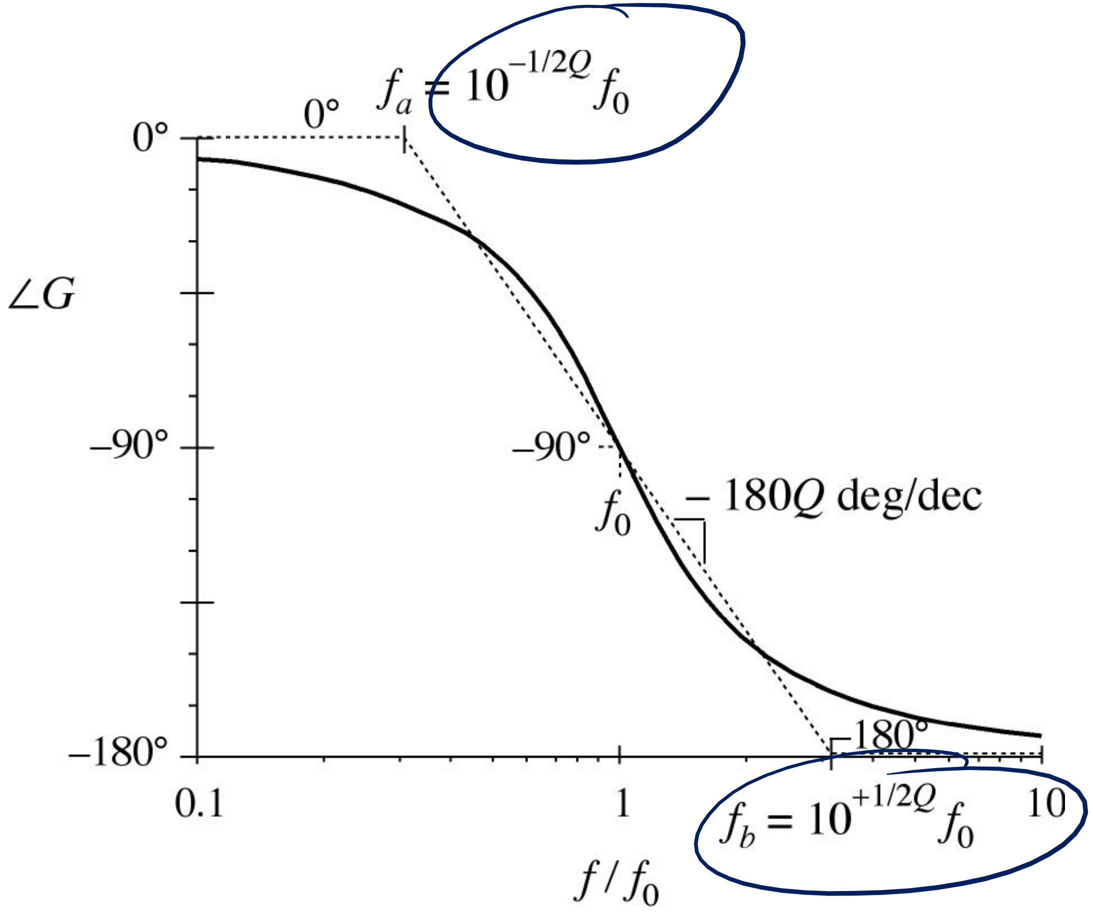
$$|H(j\omega)| = \frac{1}{(\omega/\omega_0)^2}$$

Asymptotes for Complex Poles, Q>0.5

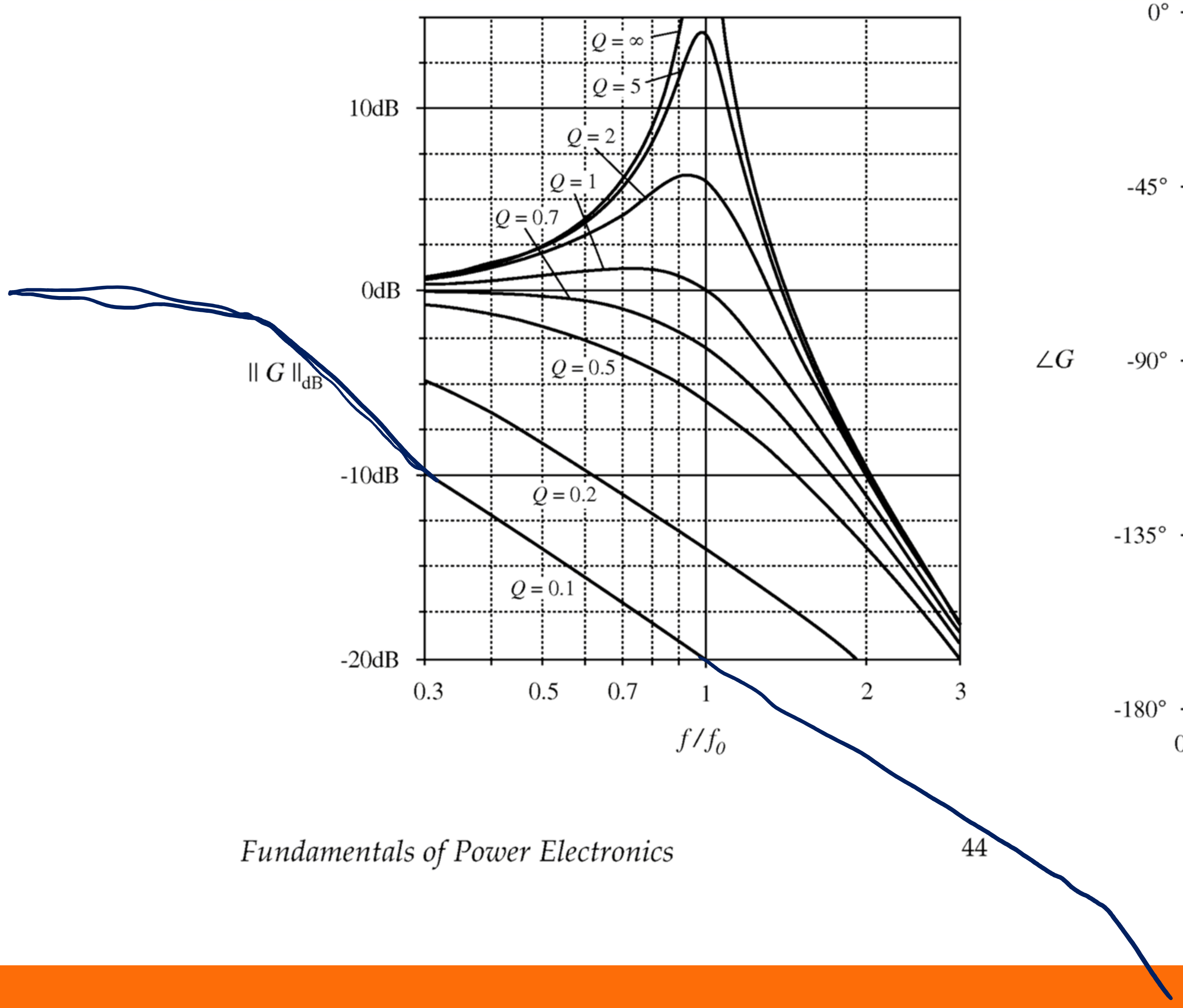
Magnitude

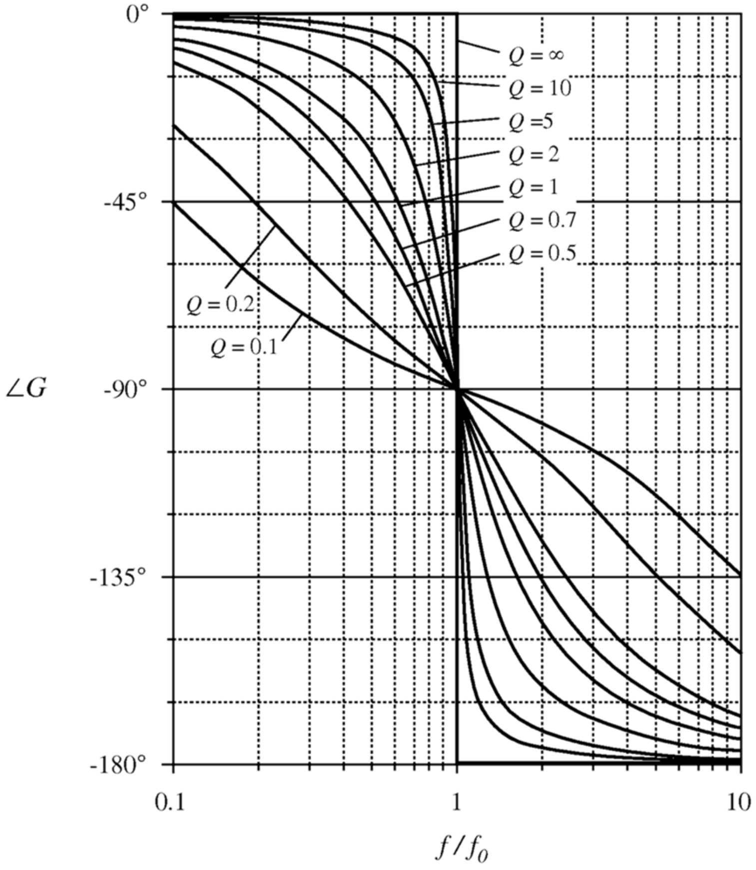


Phase



Curves for Varying Q

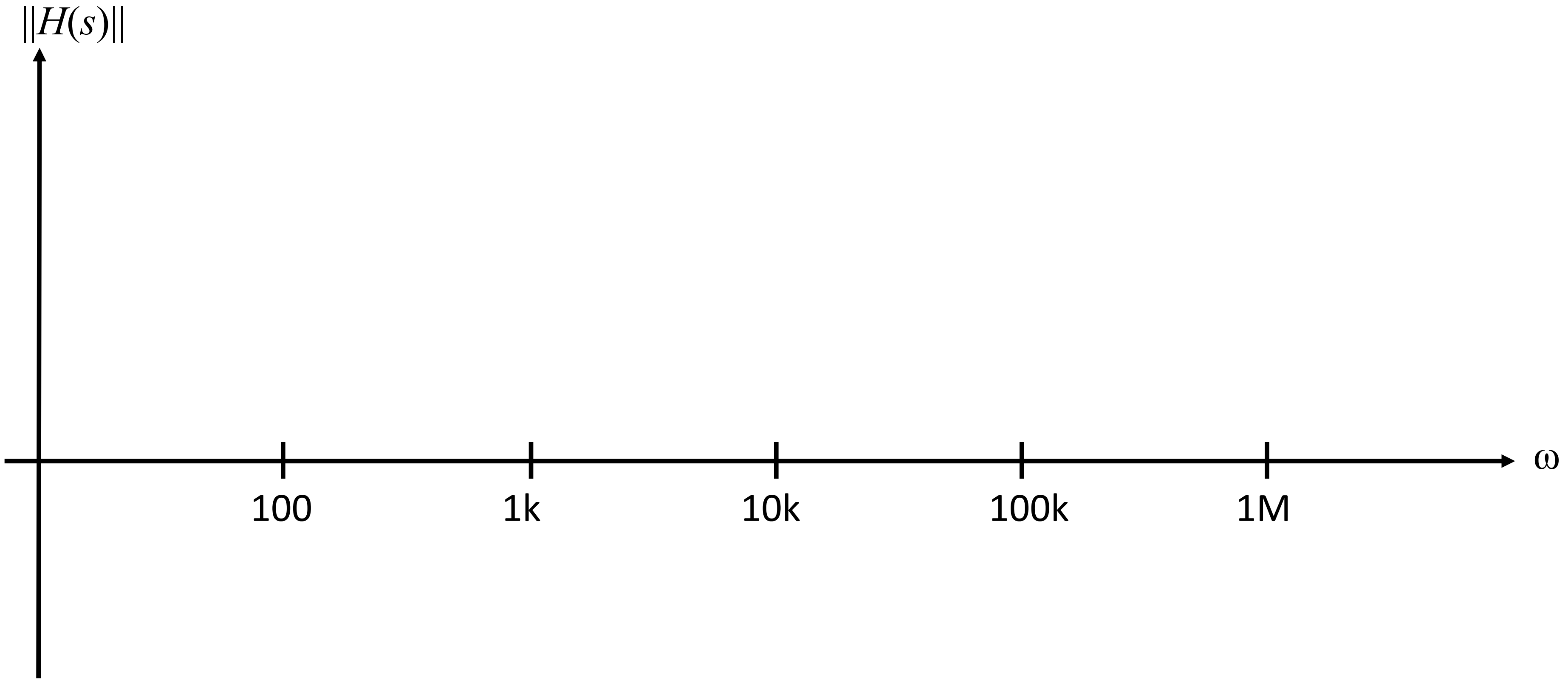




Example

A	w_{z1}	w_{z2}	ω_0	Q	ω_p
1000	100	10k	1k	10	100k

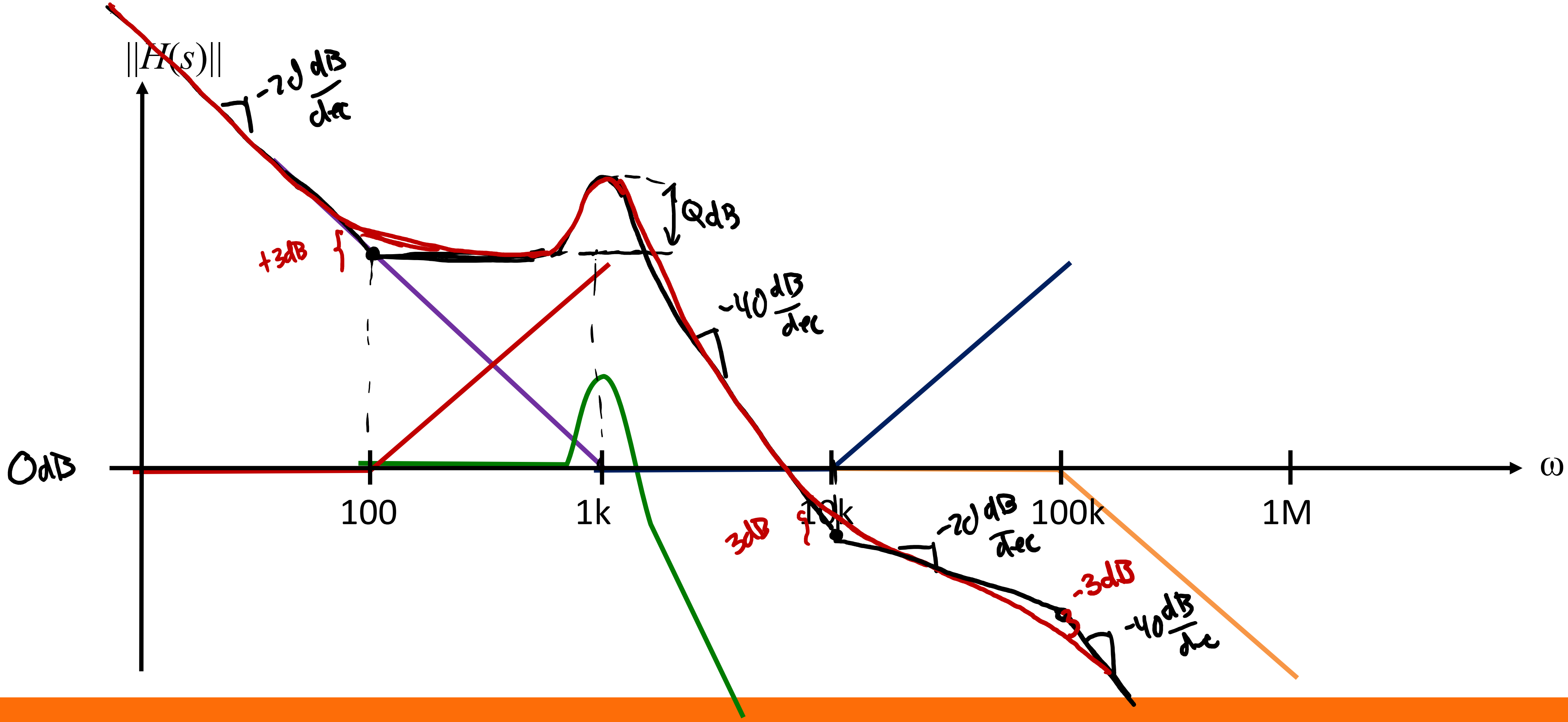
$$H(s) = \frac{A}{s}\frac{\left(1+\frac{s}{w_{z1}}\right)\left(1+\frac{s}{w_{z2}}\right)}{\left(\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1\right)\left(1+\frac{s}{\omega_p}\right)}$$



Example

A	w_{z1}	w_{z2}	ω_0	Q	ω_p
1000	100	10k	1k	10	100k

$$H(s) = \frac{A}{s}\frac{\left(1+\frac{s}{w_{z1}}\right)\left(1+\frac{s}{w_{z2}}\right)}{\left(\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1\right)\left(1+\frac{s}{\omega_p}\right)}$$



MATLAB

```
A = 1000;
wz1 = 100;
wz2 = 10e3;
w0 = 1e3;
Q = 10;
wp = 100e3;

s = tf('s');

H = A*(1+s/wz1)*(1+s/wz2)/...
       (s*(1+s/wp)*((s/w0)^2+s/Q/w0+1));

bode(H)
```