Lecture 12: Dynamics and Control

ECE 481: Power Electronics

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Part II
Converter Dynamics and Control

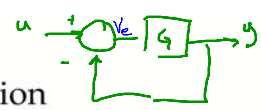
7. AC equivalent circuit modeling
8. Converter transfer functions
9. Controller design
10. Input filter design
11. AC and DC equivalent circuit modeling of the discontinuous conduction mode
12. Current programmed control

if time

Chapter 7. AC Equivalent Circuit Modeling

$V_e(s) = U(s) - Y(s)$

$Y(s) = G(s)\, V_e(s)$



7.1. Introduction

$Y(s) = G(s)[U(s) - Y(s)] \rightarrow \frac{Y(s)}{U(s)} = G_{yu} = \frac{G}{1+G}$ Negative feedback

Objective: maintain $v(t)$ equal to an accurate, constant value V.

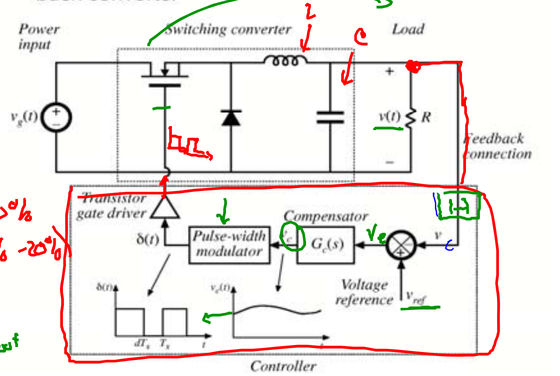
There are disturbances:

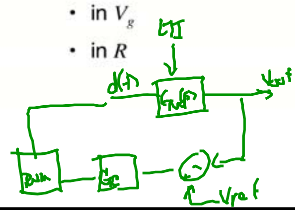
- in $v_g(t)$
- in R

There are uncertainties:

- in element values L ± 20% C (+80% -20%)
- in V_g
- in R

A simple dc-dc regulator system, employing a buck converter





Applications of control in power electronics

DC-DC converters

Regulate dc output voltage.

Control the duty cycle $d(t)$ such that $v(t)$ accurately follows a reference signal v_{ref}.

DC-AC inverters

Regulate an ac output voltage.

Control the duty cycle $d(t)$ such that $v(t)$ accurately follows a reference signal $v_{ref}(t)$.

AC-DC rectifiers

Regulate the dc output voltage.

Regulate the ac input current waveform.

Control the duty cycle $d(t)$ such that $i_g(t)$ accurately follows a reference signal i (t), and $v(t)$ accurately follows a reference signal v

Converter Modeling

Applications

- Aerospace worst-case analysis
- Commercial high-volume production: design for reliability and yield

High quality design

- Ensure that the converter works well under worst-case conditions
  - Steady state (losses, efficiency, voltage regulation)
  - Small-signal ac (controller stability and transient response)

Engineering methodology

- Simulate model during preliminary design (design verification)
- Construct laboratory prototype converter system and make it work under nominal conditions
- Develop a converter model. Refine model until it predicts behavior of nominal laboratory prototype
- Use model to predict behavior under worst-case conditions
- Improve design until worst-case behavior meets specifications (or until reliability and production yield are acceptable)

Objective of Part II

Develop tools for modeling, analysis, and design of converter control systems

Need dynamic models of converters:

How do ac variations in $v_g(t)$, R, or $d(t)$ affect the output voltage $v(t)$?

What are the small-signal transfer functions of the converter?

- Extend the steady-state converter models of Chapters 2 and 3, to include CCM converter dynamics (Chapter 7)
- Construct converter small-signal transfer functions (Chapter 8)
- Design converter control systems (Chapter 9)
- Design input EMI filters that do not disrupt control system operation (Chapter 10)
- Model converters operating in DCM (Chapter 11)
- Current-programmed control of converters (Chapter 12)

Modeling

- Representation of physical behavior by mathematical means
- Model dominant behavior of system, ignore other insignificant phenomena
- Simplified model yields physical insight, allowing engineer to design system to operate in specified manner
- Approximations neglect small but complicating phenomena
- After basic insight has been gained, model can be refined (if it is judged worthwhile to expend the engineering effort to do so), to account for some of the previously neglected phenomena

Neglecting the switching ripple

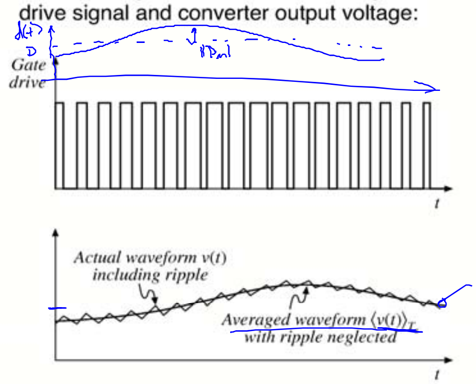
Suppose the duty cycle is modulated sinusoidally:

$$d(t) = D + D_m \cos \omega_m t$$

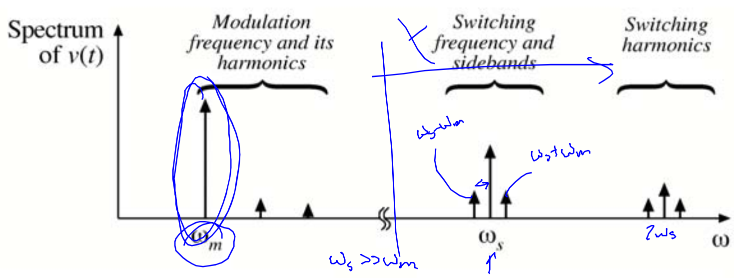
where D and D_m are constants, $\boxed{|D_m| \ll D}$, and the modulation frequency ω_m is much smaller than the converter switching frequency $\omega_s = 2\pi f_s$.

$\boxed{\omega_m \ll \omega_s}$

The resulting variations in transistor gate drive signal and converter output voltage:



Output voltage spectrum
with sinusoidal modulation of duty cycle



Contains frequency components at:

- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.

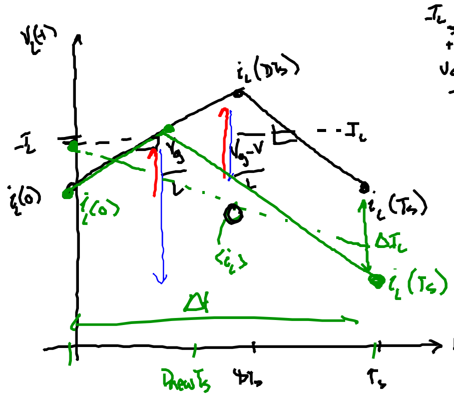
Objective of ac converter modeling

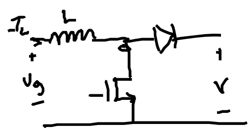
- Predict how low-frequency variations in duty cycle induce low-frequency variations in the converter voltages and currents
- Ignore the switching ripple
- Ignore complicated switching harmonics and sidebands

Approach:

- Remove switching harmonics by averaging all waveforms over one switching period

Transient Volt-Second Balance





for an inductor

$V_L = L \frac{di_L}{dt}$

$i_L(T_s) = \underbrace{i_L(0) + \frac{V_g}{L} DT_s}_{i_L(DT_s)} + \frac{V_g - V}{L} D' T_s$

$i_L(T_s) - i_L(0) = \Delta I_L = \frac{V_g}{L} D's + T_s \frac{V_g - V}{L} D'$

If we are in steady-state

$\Delta I_L = \cancel{0} = \frac{T_s}{L}\left(DV_g + D'V_g - D'V\right)$

$0 = (V_g - D'V)$

If we are not in steady-state

$\Delta I_L = \frac{V_g}{L} D_{new} T_s + \frac{V_g - V}{L} D'_{new} T_s$

$\Delta t = T_s$

$L \frac{\Delta I_L}{\Delta t} = \left(V_g - D'_{new} V\right) = \langle v_L(t) \rangle \big|_{T_s}$

$L \frac{d \langle i_L \rangle}{dt} = \langle v_L \rangle$

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Averaging to remove switching ripple

Average over one switching period to remove switching ripple:

$$\boxed{L \frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \langle v_L(t)\rangle_{T_s}}$$
$$\boxed{C \frac{d\langle v_C(t)\rangle_{T_s}}{dt} = \langle i_C(t)\rangle_{T_s}}$$

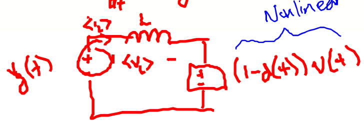
where

$$\langle x(t)\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} x(\tau)\,d\tau$$

e.g. Boost

$L\frac{d\langle i_L\rangle}{dt} = V_g - D'_{new} V$

Nonlinear

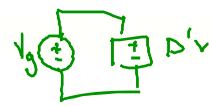


Note that, in steady-state,

$$\langle v_L(t)\rangle_{T_s} = 0$$
$$\langle i_C(t)\rangle_{T_s} = 0$$

by inductor volt-second balance and capacitor charge balance.

$\langle v_L\rangle = \emptyset = V_g - D'V$



Nonlinear averaged equations

The averaged voltages and currents are, in general, nonlinear functions of the converter duty cycle, voltages, and currents. Hence, the averaged equations

$$L \frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \langle v_L(t)\rangle_{T_s}$$
$$C \frac{d\langle v_C(t)\rangle_{T_s}}{dt} = \langle i_C(t)\rangle_{T_s}$$

constitute a system of nonlinear differential equations.

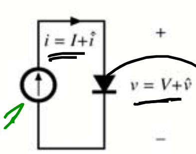
Hence, must linearize by constructing a small-signal converter model.

→ $\hat{i}$ is a small-signal value $\hat{i} = I_m \cos(\omega_m t)$

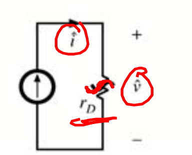
$I_m << I$

Small-signal modeling of the diode

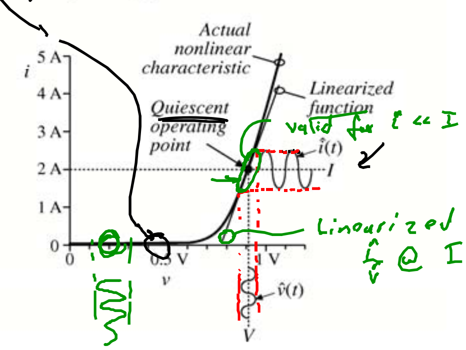
Nonlinear diode, driven by current source having a DC and small AC component



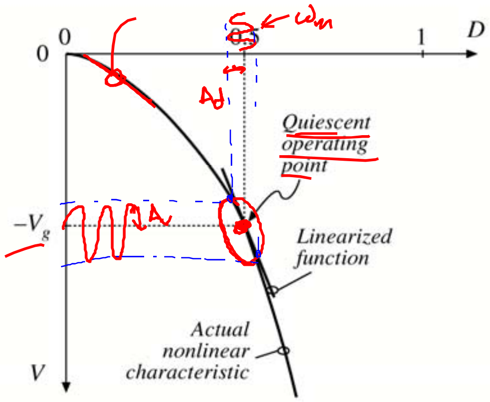
Small-signal AC model



Linearization of the diode *i-v* characteristic about a quiescent operating point



Buck-boost converter: nonlinear static control-to-output characteristic



$V = V_g\, D/(1-D)$ ← DC steady-state

$\hat{v} = G_{vd}(s)\, \hat{d}$

$\| G_{vd}(j\omega_m) \| = \dfrac{\Delta v}{\Delta d}$

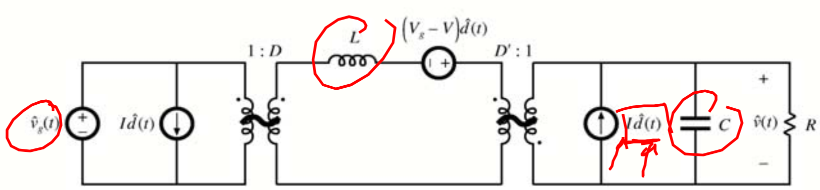
Example: linearization at the quiescent operating point

$D = 0.5$

Result of averaged small-signal ac modeling

hats mean small-signal

Small-signal ac equivalent circuit model



buck-boost example