Lecture 13: Midterm, Dynamics and Control (cont.)

ECE 481: Power Electronics

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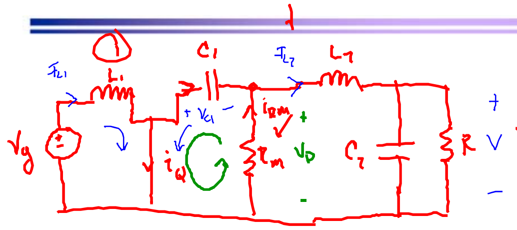
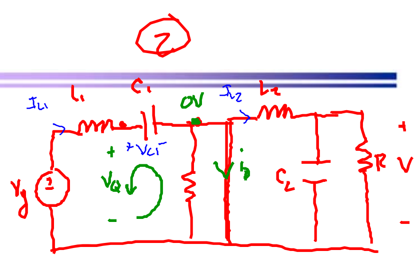




In subinterval 1 (MOSFET on)

$i_Q = I_{L1} - I_{L2} + i_{RM} = I_{L1} - I_{L2} + \frac{V_{c1}}{R_M}$

$V_D = -V_{c1}$

$V_{L1} = V_g$

$i_{c1} = I_{L2} - \frac{V_{c1}}{R_M}$

$V_{L2} = -V_{c1} - V$

$i_{c2} = I_{L2} - \frac{V}{R}$

Sub. 2 (Diode on)

$V_Q = V_{c1}$

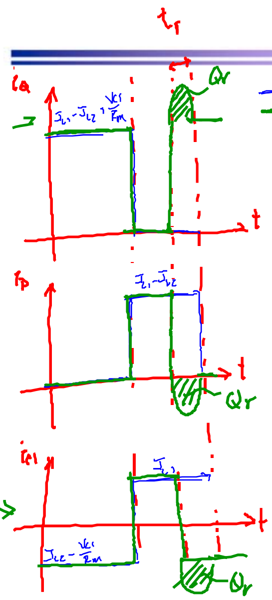
$i_D = -I_{L2} + I_{L1}$

$V_{L1} = V_g - V_{c1}$

$i_{c1} = I_{L1}$

$V_{L2} = 0 - V$

$i_{c2} = I_{L2} - \frac{V}{R}$



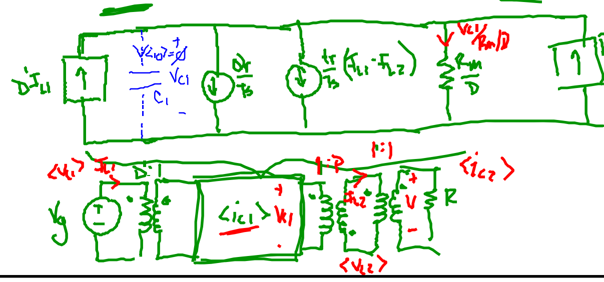
— w/out RR
— w/ RR

$$\langle v_{L1}\rangle = V_g - D'V_{C1} = 0$$
$$\langle v_{L2}\rangle = -DV_{C1} - V = 0$$
$$\langle i_{C2}\rangle = I_{L2} - \frac{V}{R} = 0$$

"Normal" eq. circs

$$\langle i_{C1}\rangle = \boxed{D\left(I_{L2} - \frac{V_{C1}}{R_m}\right) + D'I_{L1}} - \frac{1}{T_s}\left[t_r\left(I_{L1} - I_{L2}\right) + Q_r\right]$$

$$= DI_{L2} - \underbrace{\frac{D}{R_m}V_{C1}}_{}^{\frac{V_{C1}}{(R_m/D)}} + D'I_{L1} - \frac{t_r}{T_s}\left(I_{L1} - I_{L2}\right) - \frac{Q_r}{T_s} = 0$$



Chapter 7. AC Equivalent Circuit Modeling

7.1 Introduction

7.2 The basic AC modeling approach

7.3 State-space averaging

7.4 Circuit averaging and averaged switch modeling

7.5 The canonical circuit model

7.6 Modeling the pulse-width modulator

7.7 Summary of key points

DC

AC, small-signal variation

$$i_L(t) = I_L + \hat{i}_L$$

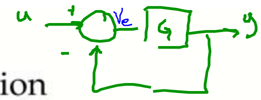
$|\hat{i}| \ll I_L$ (small ripple)

$\omega_m \ll \omega_s$ — switching freq

freq of AC

$V_e(s) = U(s) - Y(s)$

$Y(s) = G(s) V_e(s)$



7.1. Introduction

$Y(s) = G(s)[U(s) - Y(s)] \Rightarrow \frac{Y(s)}{U(s)} = \boxed{G_{yu} = \frac{G}{1-G}}$ negative feedback

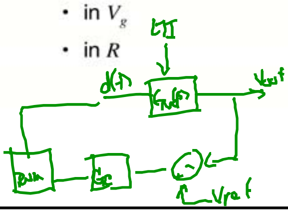
Objective: maintain $v(t)$ equal to an accurate, constant value V.

There are disturbances:

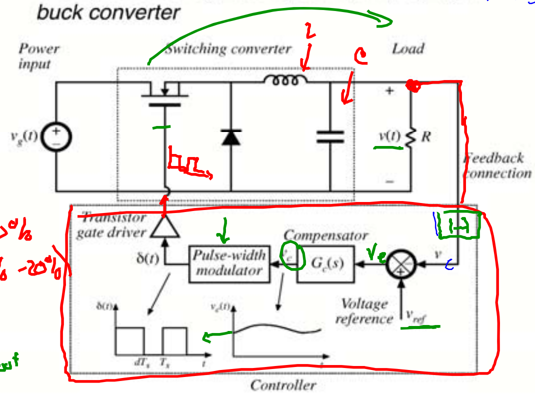
- in $v_g(t)$
- in R

There are uncertainties:

- in element values $L \pm 20\%$ $C(+80\% \; -20\%)$
- in V_g
- in R



A simple dc-dc regulator system, employing a buck converter



Neglecting the switching ripple

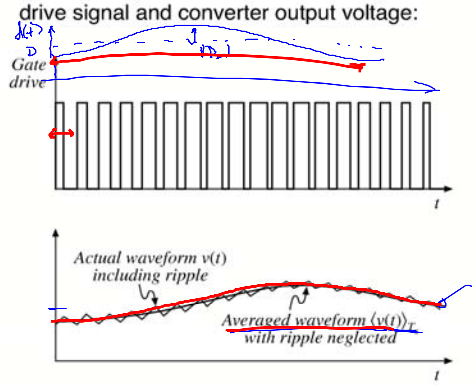
Suppose the duty cycle is modulated sinusoidally:

$$d(t) = D + D_m \cos \omega_m t$$

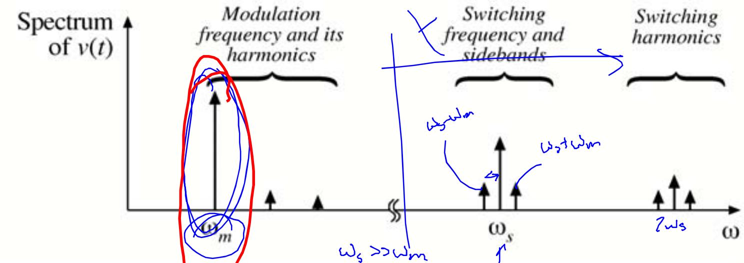
where D and D_m are constants, $\boxed{|D_m| \ll D}$, and the modulation frequency ω_m is much smaller than the converter switching frequency $\omega_s = 2\pi f_s$.

$\boxed{\omega_m \ll \omega_s}$

The resulting variations in transistor gate drive signal and converter output voltage:



Output voltage spectrum
with sinusoidal modulation of duty cycle



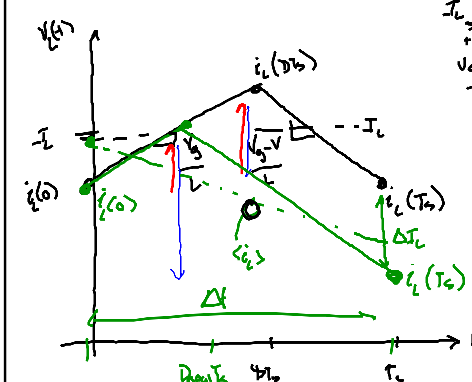
Contains frequency components at:

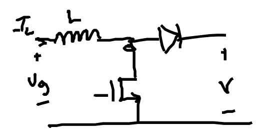
- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.

Transient Volt-Second Balance





for an inductor

$V_L = L \frac{di_L}{dt}$

$$i_L(T_s) = \underbrace{i_L(0) + \frac{V_g}{L} DT_s}_{i_L(DT_s)} + \frac{V_g - V}{L} D' T_s$$

$$i_L(T_s) - i_L(0) = \Delta I_L = \frac{V_g}{L} DT_s + T_s \frac{V_g - V}{L} D'$$

If we are in steady state

$$\Delta I_L = \emptyset = \frac{T_s}{L}\left(DV_g + D'V_g - D'V\right)$$

$$\emptyset = (V_g - D'V)$$

If we are not in steady-state

$$\Delta I_L = \frac{V_g}{L} D_{new} T_s + \frac{V_g - V}{L} D'_{new} T_s$$

$$L \frac{d\langle i_L \rangle}{dt} = \langle v_L \rangle$$

$\Delta t = T_s$

$$L \frac{\Delta I_L}{\Delta t} = \left(V_g - D'_{new} V\right) = \langle v_L(t) \rangle \big|_{T_s}$$

Averaging to remove switching ripple

Average over one switching period to remove switching ripple:

$$\boxed{L \frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \langle v_L(t)\rangle_{T_s}}$$

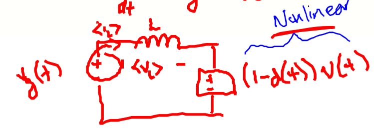
$$\boxed{C \frac{d\langle v_C(t)\rangle_{T_s}}{dt} = \langle i_C(t)\rangle_{T_s}}$$

where

$$\langle x(t)\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} x(\tau)\,d\tau$$

e.g. Boost

$L\frac{d\langle i_L\rangle}{dt} = V_g - D'_{new}V$

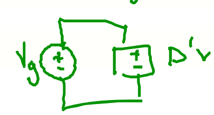


Note that, in steady-state,

$$\langle v_L(t)\rangle_{T_s} = 0$$
$$\langle i_C(t)\rangle_{T_s} = 0$$

by inductor volt-second balance and capacitor charge balance.

$\langle v_L \rangle = 0 = V_g - D'V$

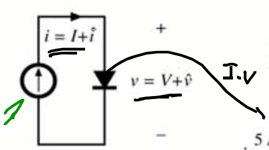


→ $\hat{i}$ is a small-signal value $\hat{i} = I_m \cos(\omega_m t)$

$I_m \ll I$

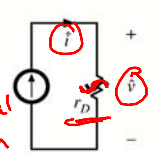
Small-signal modeling of the diode

Nonlinear diode, driven by current source having a DC and small AC component



DC from Quiescent

Small-signal AC model

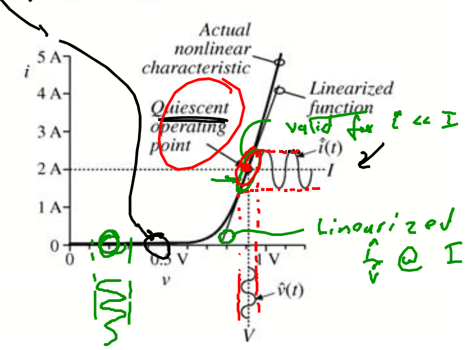


Small-signal equation

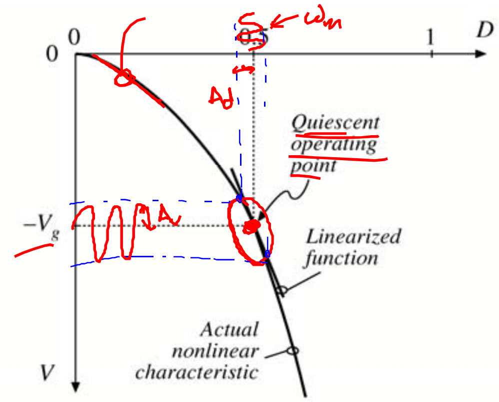
→ $\hat{i} = \frac{1}{r_D}(\hat{v})$

↳ so, linearized

Linearization of the diode *i-v* characteristic about a quiescent operating point



Buck-boost converter: nonlinear static control-to-output characteristic



$V = V_g\, D/(1 - D)$ ← DC steady-state

$\hat{v} = G_{vd}(s)\, \hat{d}$

$\|G_{vd}(j\omega_m)\| = \dfrac{\Delta v}{\Delta d}$

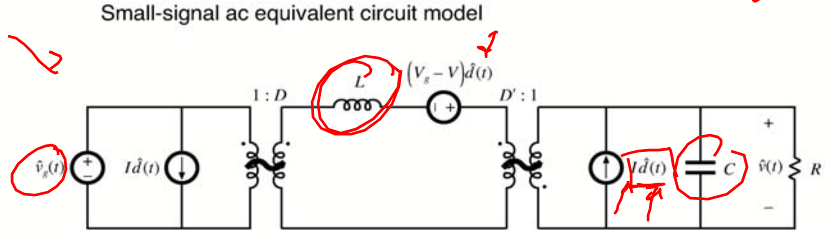
Example: linearization at the quiescent operating point

$D = 0.5$

Result of averaged small-signal ac modeling

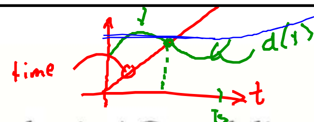
hats mean small-signal

Small-signal ac equivalent circuit model



$G_{vd}(s) = \dfrac{v(s)}{d(s)}$

buck-boost example



$\omega_m << \omega_s$

7.2. The basic AC modeling approach

$0 < t < d(t)T_s$ $\qquad$ $d(t)T_s < t < T_s$

Buck-boost converter example

(1)

$v_L(t) = v_g(t)$

$i_c(t) = -\frac{v(t)}{R}$

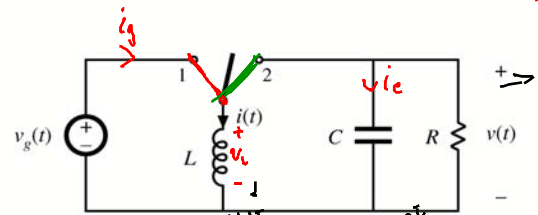
$i_g(t) = i_L(t)$

(2)

$i_g(t) = \emptyset$

$v_L(t) = v(t)$

$i_c(t) = -\frac{v(t)}{R} - i_L(t)$



$$\langle v_L(t)\rangle = L\frac{d\langle i_L(t)\rangle}{dt} = \frac{1}{T_s}\int_0^{d(t)T_s} v_g(t)\,dt + \frac{1}{T_s}\int_{d(t)T_s}^{T_s} v(t)\,dt$$

$$\langle v_L(t)\rangle = L\frac{d\langle i_L(t)\rangle}{dt} \approx d(t)v_g(t) + d'(t)v(t)$$ → Large signal averaged over T_s nonlinear

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$$\langle v_L(t)\rangle = L\frac{d\langle i_L\rangle}{dt} = d(t)v_g(t) + d'(t)v(t)$$

$d(t) = D + \hat{d}$

$v_g(t) = V_g + \hat{v}_g$

$v(t) = V + \hat{v}$