Lecture 15: Converter Transfer Functions

ECE 481: Power Electronics

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Announcements

- No class Thursday; enjoy Engineer's Day
- Tuesday: Graphical construction of converter transfer functions with Zhuxian
- HW #5 due today
- HW #6 due Tuesday, 10/29; posted this afternoon

7.5 The canonical circuit model

All PWM CCM dc-dc converters perform the same basic functions:

- Transformation of voltage and current levels, ideally with 100% efficiency
- Low-pass filtering of waveforms
- Control of waveforms by variation of duty cycle

Hence, we expect their equivalent circuit models to be qualitatively similar.

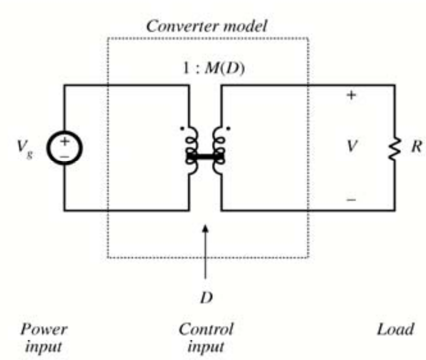
Canonical model:

- A standard form of equivalent circuit model, which represents the above physical properties
- Plug in parameter values for a given specific converter

7.5.1. Development of the canonical circuit model

1. Transformation of dc voltage and current levels

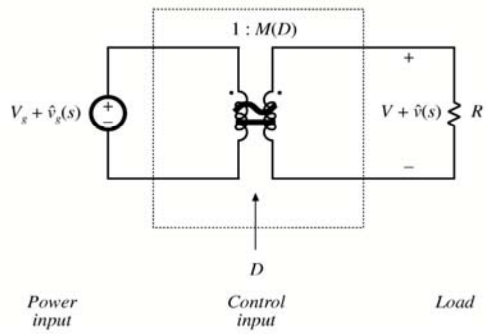
- modeled as in Chapter 3 with ideal dc transformer
- effective turns ratio $M(D)$
- can refine dc model by addition of effective loss elements, as in Chapter 3



Steps in the development of the canonical circuit model

2. Ac variations in $v_g(t)$ induce ac variations in $v(t)$

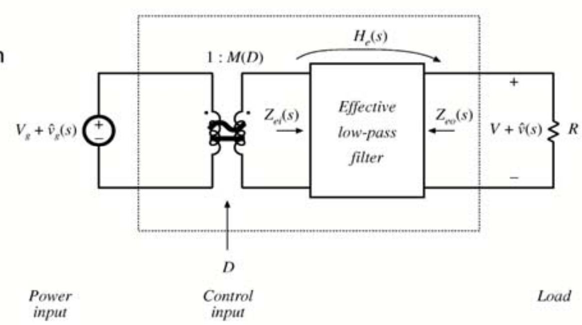
- these variations are also transformed by the conversion ratio $M(D)$



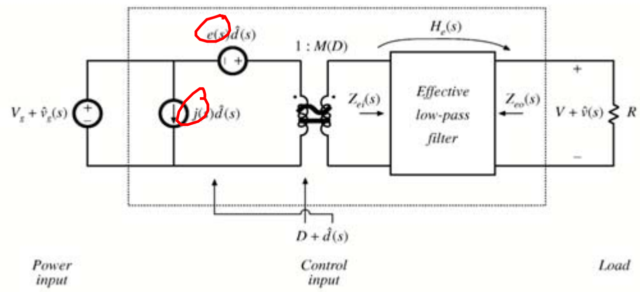
Steps in the development of the canonical circuit model

3. Converter must contain an effective low-pass filter characteristic

- necessary to filter switching ripple
- also filters ac variations
- effective filter elements may not coincide with actual element values, but can also depend on operating point



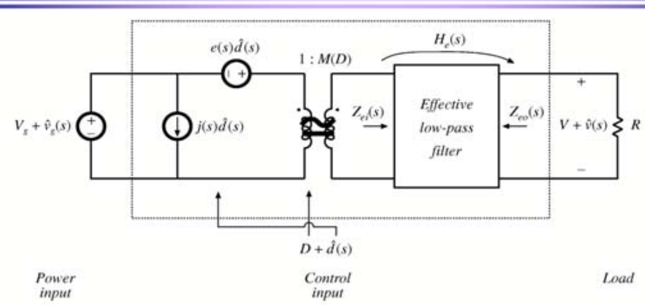
Steps in the development of the canonical circuit model



4. Control input variations also induce ac variations in converter waveforms

- Independent sources represent effects of variations in duty cycle
- Can push all sources to input side as shown. Sources may then become frequency-dependent

Transfer functions predicted by canonical model



Line-to-output transfer function: $G_{vg}(s) = \dfrac{\hat{v}(s)}{\hat{v}_g(s)} = M(D)\, H_e(s)$

Control-to-output transfer function: $G_{vd}(s) = \dfrac{\hat{v}(s)}{\hat{d}(s)} = e(s)\, M(D)\, H_e(s)$

7.5.3 Canonical circuit parameters for some common converters

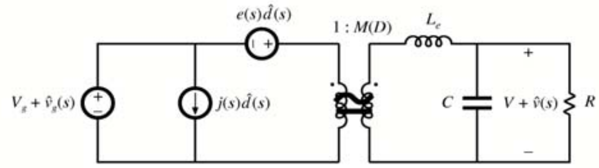
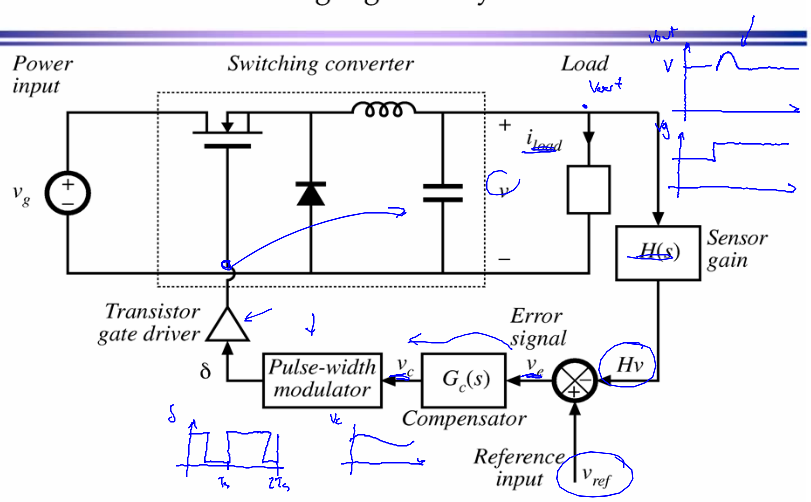


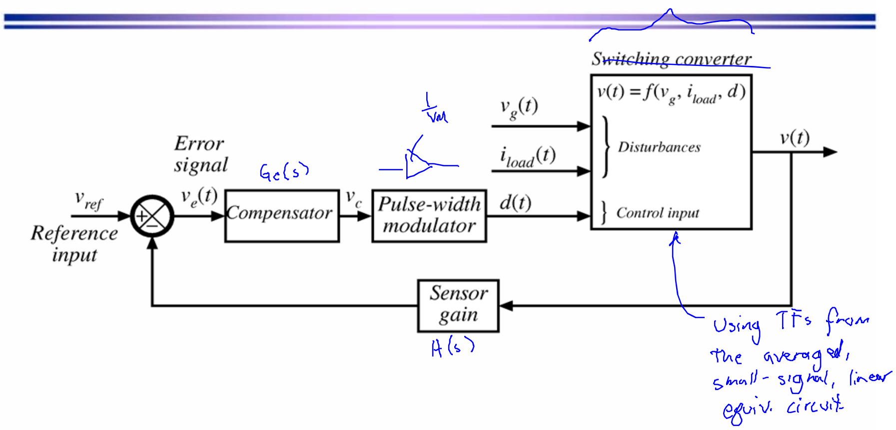
Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	$M(D)$	L_e	$e(s)$	$j(s)$
Buck	D	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$	$\frac{L}{D'^2}$	$V\left(1 - \frac{sL}{D'^2 R}\right)$	$\frac{V}{D'^2 R}$
Buck-boost	$-\frac{D}{D'}$	$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{sDL}{D'^2 R}\right)$	$-\frac{V}{D'^2 R}$

Negative feedback: a switching regulator system



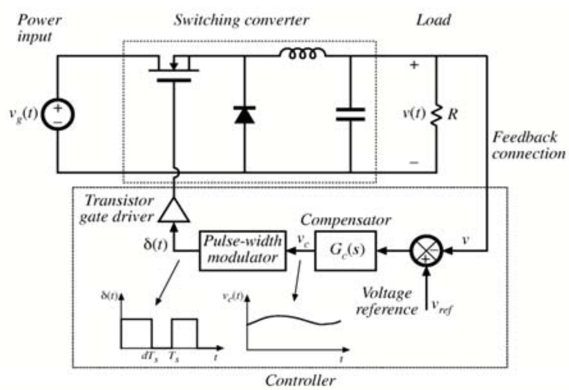
Negative feedback



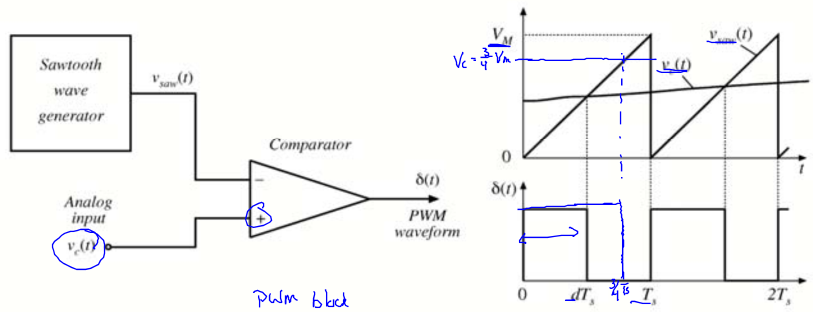
7.6 Modeling the pulse-width modulator

Pulse-width modulator converts voltage signal $v_c(t)$ into duty cycle signal $d(t)$.

What is the relationship between $v_c(t)$ and $d(t)$?



A simple pulse-width modulator



PWM block

$$d = \frac{\langle v_c \rangle}{V_M}$$

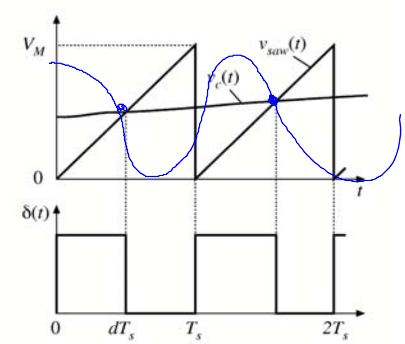
$$G_{pwm} = \frac{d}{\langle v_c \rangle} = \frac{1}{V_M}$$

Equation of pulse-width modulator

For a linear sawtooth waveform:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

So $d(t)$ is a linear function of $v_c(t)$.



Perturbed equation of pulse-width modulator

PWM equation:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

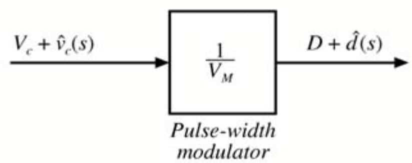
Perturb:

$$v_c(t) = V_c + \hat{v}_c(t)$$
$$d(t) = D + \hat{d}(t)$$

Result:

$$D + \hat{d}(t) = \frac{V_c + \hat{v}_c(t)}{V_M}$$

Block diagram:

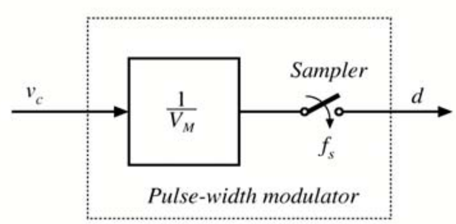


Dc and ac relationships:

$$D = \frac{V_c}{V_M}$$
$$\hat{d}(t) = \frac{\hat{v}_c(t)}{V_M}$$

Sampling in the pulse-width modulator

The input voltage is a continuous function of time, but there can be only one discrete value of the duty cycle for each switching period.



Therefore, the pulse-width modulator samples the control waveform, with sampling rate equal to the switching frequency.

In practice, this limits the useful frequencies of ac variations to values much less than the switching frequency. Control system bandwidth must be sufficiently less than the Nyquist rate $f_s/2$. Models that do not account for sampling are accurate only at frequencies much less than $f_s/2$.

Chapter 8. Converter Transfer Functions

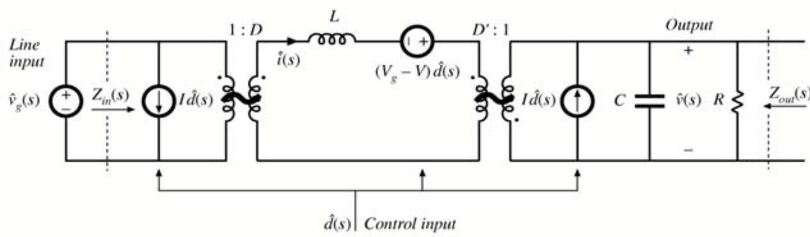
Converter Transfer Functions

8.2. Analysis of converter transfer functions

8.2.1. Example: transfer functions of the buck-boost converter
8.2.2. Transfer functions of some basic CCM converters
8.2.3. Physical origins of the right half-plane zero in converters

Buck-boost converter model

From Chapter 7

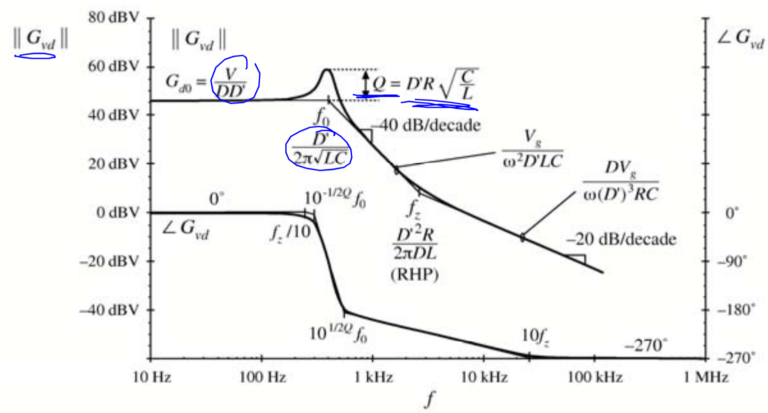


$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\hat{d}(s) = 0} \qquad G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s) = 0}$$

All other independent sources off

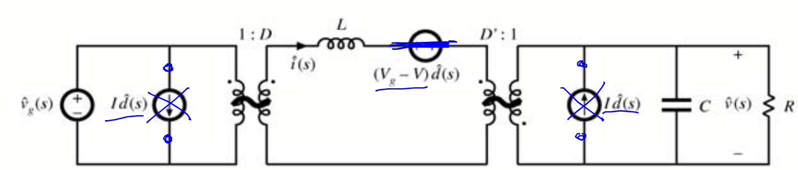
Bode plot of control-to-output transfer function

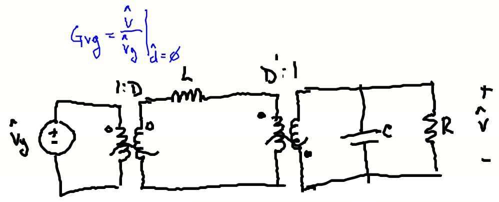
with analytical expressions for important features



8.2.1. Example: transfer functions of the buck-boost converter

Small-signal ac model of the buck-boost converter, derived in Chapter 7:

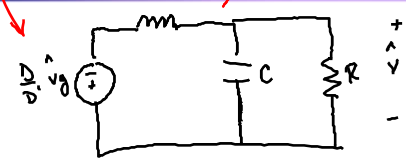




Z_2 → L/D'^2

$v = V_{in}\frac{Z_1}{Z_1+Z_2}$

V_{in} Z_1



$$\hat{v} = \frac{\frac{1}{sC}\|R}{\frac{1}{sC}\|R + sL/D'^2}\,\frac{-D}{D'}\hat{v}_g$$

$$G_{vg} = \frac{\hat{v}}{\hat{v}_g}\bigg|_{\hat{d}=0} = \frac{-D}{D'}\left(\frac{\frac{1}{sC+\frac{1}{R}}}{\frac{1}{sC+\frac{1}{R}} + s\frac{L}{D'^2}}\right) = \frac{-D}{D'}\left(\frac{1}{1+s^2\frac{LC}{D'^2}+s\frac{L}{D'^2R}}\right)$$

$$\Downarrow$$

$$\frac{G_{vg0}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

Definition of transfer functions

The converter contains two inputs, $\hat{d}(s)$ and $\hat{v}_g(s)$ and one output, $\hat{v}(s)$

Hence, the ac output voltage variations can be expressed as the superposition of terms arising from the two inputs:

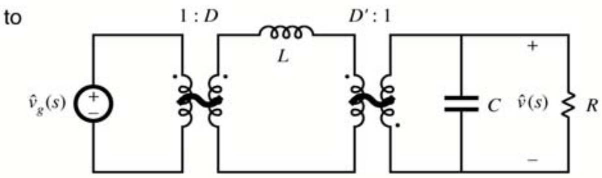
$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s)$$

The control-to-output and line-to-output transfer functions can be defined as

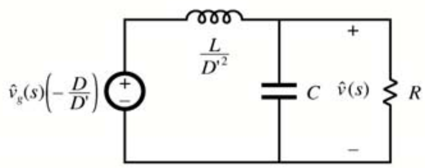
$$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)}\bigg|_{\hat{v}_g(s)=0} \quad \text{and} \quad G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)}\bigg|_{\hat{d}(s)=0}$$

Derivation of line-to-output transfer function $G_{vg}(s)$

Set $\hat{d}$ sources to zero:



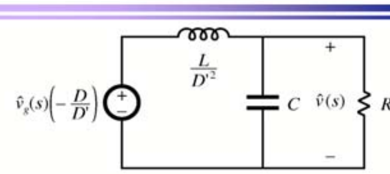
Push elements through transformers to output side:



Derivation of transfer functions

Use voltage divider formula to solve for transfer function:

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0} = -\frac{D}{D'}\frac{\left(R \parallel \frac{1}{sC}\right)}{\frac{sL}{D'^2} + \left(R \parallel \frac{1}{sC}\right)}$$



Expand parallel combination and express as a rational fraction:

$$G_{vg}(s) = \left(-\frac{D}{D'}\right)\frac{\left(\frac{R}{1+sRC}\right)}{\frac{sL}{D'^2} + \left(\frac{R}{1+sRC}\right)}$$

$$= \left(-\frac{D}{D'}\right)\frac{R}{R + \frac{sL}{D'^2} + \frac{s^2RLC}{D'^2}}$$

We aren't done yet! Need to write in normalized form, where the coefficient of s^0 is 1, and then identify salient features

Derivation of transfer functions

Divide numerator and denominator by R. Result: the line-to-output transfer function is

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0} = \left(-\frac{D}{D'}\right)\frac{1}{1 + s\dfrac{L}{D'^2 R} + s^2 \dfrac{LC}{D'^2}}$$

which is of the following standard form:

$$G_{vg}(s) = G_{g0}\frac{1}{1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2}$$

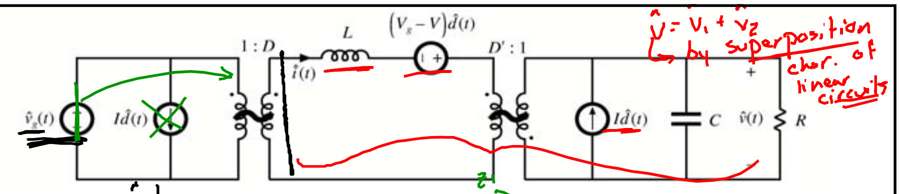
Salient features of the line-to-output transfer function

Equate standard form to derived transfer function, to determine expressions for the salient features:

$$G_{g0} = -\frac{D}{D'}$$

$$\frac{1}{\omega_0^2} = \frac{LC}{D'^2} \qquad \omega_0 = \frac{D'}{\sqrt{LC}}$$

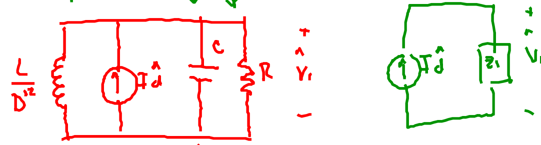
$$\frac{1}{Q\omega_0} = \frac{L}{D'^2 R} \qquad Q = D'R\sqrt{\frac{C}{L}}$$



$\hat v = \hat v_1 + \hat v_2$ by superposition char. of linear circuits

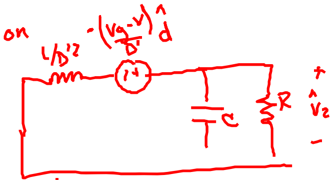
$$G_{vd} = \frac{\hat v}{\hat d}\bigg|_{\hat v_g = 0}$$

(1) $I\hat d$ on



$$\hat v_1 = I\hat d\left(\frac{sL}{D'^2} \,\|\, \frac{1}{sC} \,\|\, R\right)$$

(2) $(V_g - V)\hat d$ on



$$\hat v_2 = \frac{V_g - V}{D'}\hat d\,\frac{R\|\frac{1}{sC}}{R\|\frac{1}{sC} + sL/D'^2}$$

$$\hat v = \hat v_1 + \hat v_2 = I\hat d\left(\frac{1}{sC}\,\|\,\frac{sL}{D'^2}\,\|\,R\right) + \frac{-(V_g - V)}{D'}\hat d\,\frac{R\|\frac{1}{sC}}{R\|\frac{1}{sC} + sL/D'^2}$$

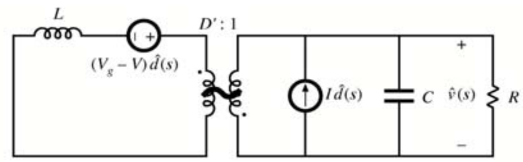
$$\left(\frac{sL}{D'^2}\right)\frac{1}{\left(\frac{sL}{D'^2}\right)\left(sC + \frac{D'^2}{sL} + \frac{1}{R}\right)} = \left(\frac{sL}{D'^2}\right)\frac{1}{s^2\frac{LC}{D'^2} + 1 + \frac{sL}{RD'^2}}$$

$$\frac{1}{s^2\frac{LC}{D'^2} + s\frac{L}{D'^2R} + 1}$$

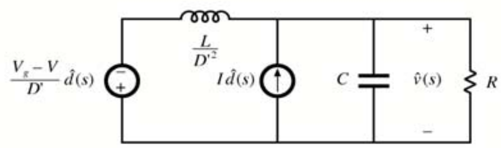
$$\frac{\hat v}{\hat d} = \frac{I\frac{sL}{D'^2} - \frac{V_g - V}{D'}}{s^2\frac{LC}{D'^2} + \frac{sL}{RD'^2} + 1} = G_{vd}$$

Derivation of control-to-output transfer function $G_{vd}(s)$

In small-signal model, set $\hat{v}_g$ source to zero:



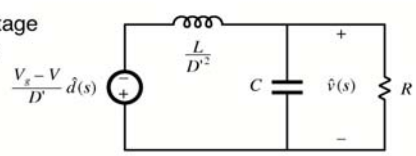
Push all elements to output side of transformer:



There are two $\hat{d}$ sources. One way to solve the model is to use superposition, expressing the output $\hat{v}$ as a sum of terms arising from the two sources.

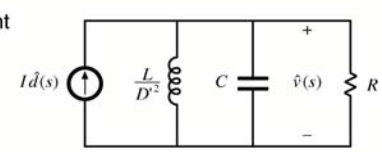
Superposition

With the voltage source only:



$$\frac{\hat{v}(s)}{\hat{d}(s)} = \left(-\frac{V_g - V}{D'}\right)\frac{\left(R \parallel \frac{1}{sC}\right)}{\frac{sL}{D'^2} + \left(R \parallel \frac{1}{sC}\right)}$$

With the current source alone:



$$\frac{\hat{v}(s)}{\hat{d}(s)} = I\left(\frac{sL}{D'^2} \parallel R \parallel \frac{1}{sC}\right)$$

Total:

$$G_{vd}(s) = \left(-\frac{V_g - V}{D'}\right)\frac{\left(R \parallel \frac{1}{sC}\right)}{\frac{sL}{D'^2} + \left(R \parallel \frac{1}{sC}\right)} + I\left(\frac{sL}{D'^2} \parallel R \parallel \frac{1}{sC}\right)$$

Control-to-output transfer function

Express in normalized form:

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_g(s)=0} = \left(-\frac{V_g - V}{D'^2}\right) \frac{\left(1 - s\frac{LI}{V_g - V}\right)}{\left(1 + s\frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}\right)}$$

This is of the following standard form:

$$G_{vd}(s) = G_{d0} \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

8.2.2. Transfer functions of some basic CCM converters

Table 8.2. Salient features of the small-signal CCM transfer functions of some basic dc-dc converters

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\frac{V}{D}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
boost	$\frac{1}{D'}$	$\frac{V}{D'}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2 R}{L}$
buck-boost	$-\frac{D}{D'}$	$\frac{V}{D D'^2}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2 R}{D L}$

where the transfer functions are written in the standard forms

$$G_{vd}(s) = G_{d0} \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

control-to-output

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

line-to-output