Lecture 15: Averaged SSM and Linearization

ECE 481: Power Electronics

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7.5 The canonical circuit model

All PWM CCM dc-dc converters perform the same basic functions:

- Transformation of voltage and current levels, ideally with 100% efficiency
- Low-pass filtering of waveforms
- Control of waveforms by variation of duty cycle

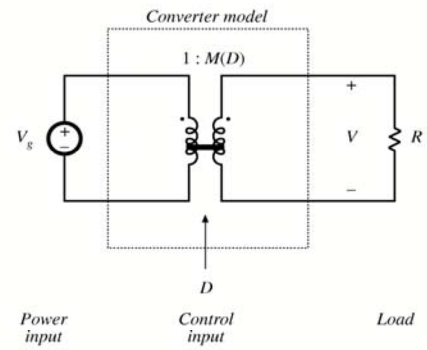
Hence, we expect their equivalent circuit models to be qualitatively similar.

Canonical model:

- A standard form of equivalent circuit model, which represents the above physical properties
- Plug in parameter values for a given specific converter

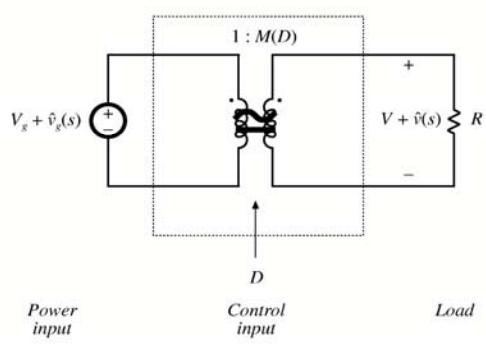
7.5.1. Development of the canonical circuit model

1. Transformation of dc voltage and current levels
- modeled as in Chapter 3 with ideal dc transformer
- effective turns ratio $M(D)$
- can refine dc model by addition of effective loss elements, as in Chapter 3



Steps in the development of the canonical circuit model

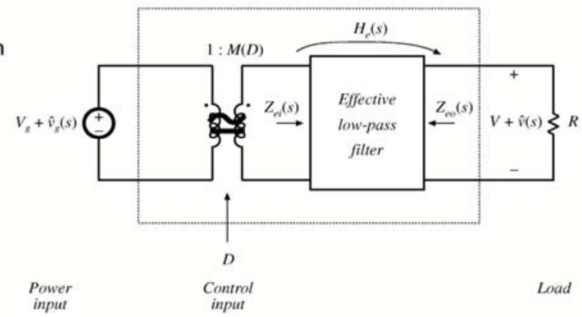
2. Ac variations in $v_g(t)$ induce ac variations in $v(t)$
- these variations are also transformed by the conversion ratio $M(D)$



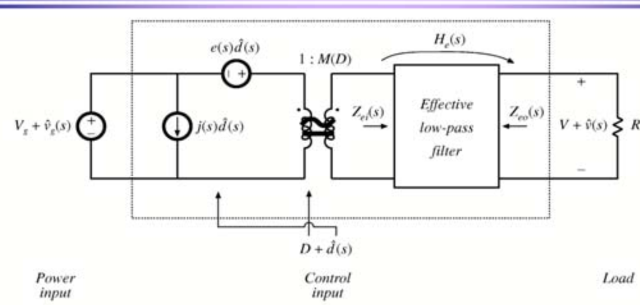
Steps in the development of the canonical circuit model

3. Converter must contain an effective low-pass filter characteristic

- necessary to filter switching ripple
- also filters ac variations
- effective filter elements may not coincide with actual element values, but can also depend on operating point



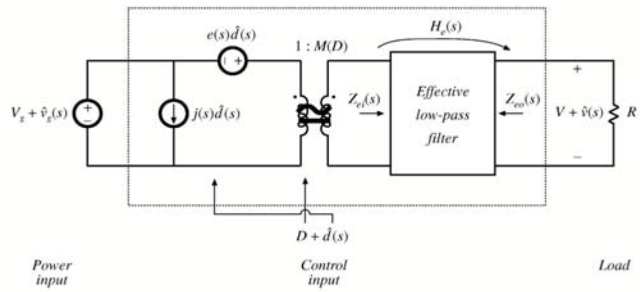
Steps in the development of the canonical circuit model



4. Control input variations also induce ac variations in converter waveforms

- Independent sources represent effects of variations in duty cycle
- Can push all sources to input side as shown. Sources may then become frequency-dependent

Transfer functions predicted by canonical model



Line-to-output transfer function: $G_{vg}(s) = \dfrac{\hat{v}(s)}{\hat{v}_g(s)} = M(D)\, H_e(s)$

Control-to-output transfer function: $G_{vd}(s) = \dfrac{\hat{v}(s)}{\hat{d}(s)} = e(s)\, M(D)\, H_e(s)$

7.5.3 Canonical circuit parameters for some common converters

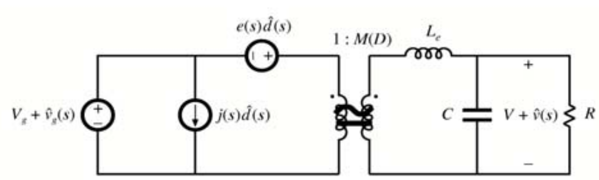


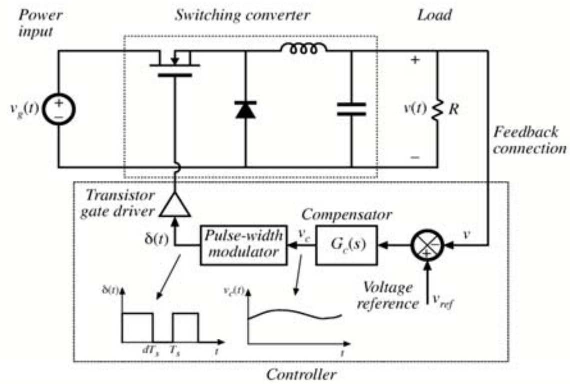
Table 7.1. Canonical model parameters for the ideal buck, boost, and buck-boost converters

Converter	$M(D)$	L_e	$e(s)$	$j(s)$
Buck	D	L	$\frac{V}{D^2}$	$\frac{V}{R}$
Boost	$\frac{1}{D'}$	$\frac{L}{D'^2}$	$V\left(1 - \frac{sL}{D'^2 R}\right)$	$\frac{V}{D'^2 R}$
Buck-boost	$-\frac{D}{D'}$	$\frac{L}{D'^2}$	$-\frac{V}{D^2}\left(1 - \frac{sDL}{D'^2 R}\right)$	$-\frac{V}{D'^2 R}$

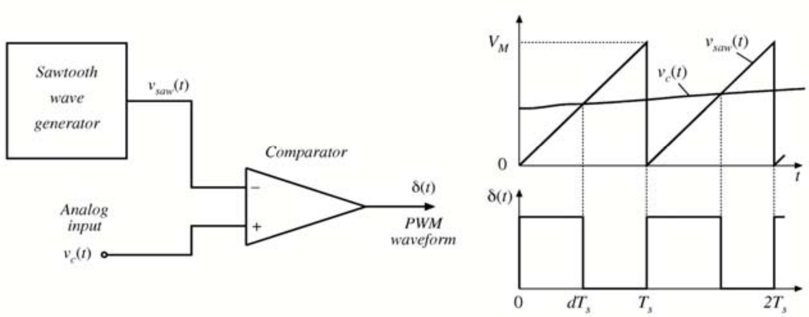
7.6 Modeling the pulse-width modulator

Pulse-width modulator converts voltage signal $v_c(t)$ into duty cycle signal $d(t)$.

What is the relationship between $v_c(t)$ and $d(t)$?



A simple pulse-width modulator

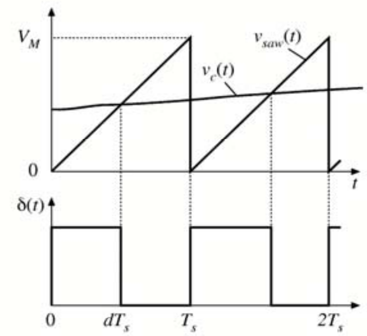


Equation of pulse-width modulator

For a linear sawtooth waveform:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

So $d(t)$ is a linear function of $v_c(t)$.



Perturbed equation of pulse-width modulator

PWM equation:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

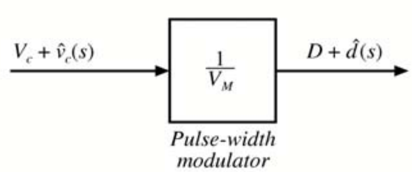
Perturb:

$$v_c(t) = V_c + \hat{v}_c(t)$$
$$d(t) = D + \hat{d}(t)$$

Result:

$$D + \hat{d}(t) = \frac{V_c + \hat{v}_c(t)}{V_M}$$

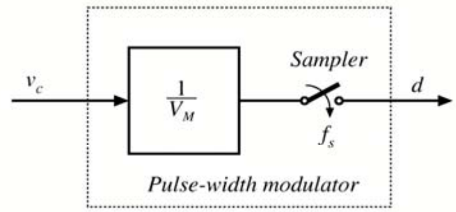
Block diagram:



Dc and ac relationships:

$$D = \frac{V_c}{V_M}$$
$$\hat{d}(t) = \frac{\hat{v}_c(t)}{V_M}$$

Sampling in the pulse-width modulator



The input voltage is a continuous function of time, but there can be only one discrete value of the duty cycle for each switching period.

Therefore, the pulse-width modulator samples the control waveform, with sampling rate equal to the switching frequency.

In practice, this limits the useful frequencies of ac variations to values much less than the switching frequency. Control system bandwidth must be sufficiently less than the Nyquist rate $f_s/2$. Models that do not account for sampling are accurate only at frequencies much less than $f_s/2$.

Chapter 8. Converter Transfer Functions

Converter Transfer Functions

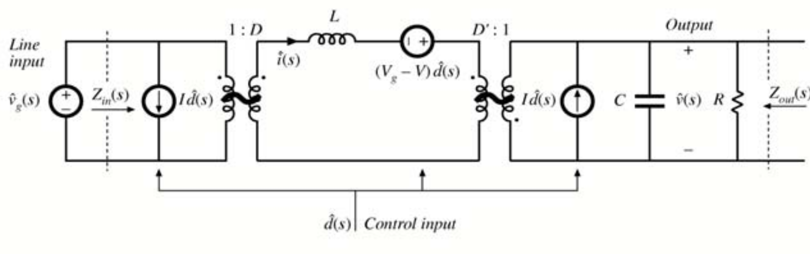
8.3. Graphical construction of converter transfer functions
- 8.3.1. Series impedances: addition of asymptotes
- 8.3.2. Parallel impedances: inverse addition of asymptotes
- 8.3.3. Another example
- 8.3.4. Voltage divider transfer functions: division of asymptotes

8.4. Measurement of ac transfer functions and impedances

8.5. Summary of key points

Buck-boost converter model

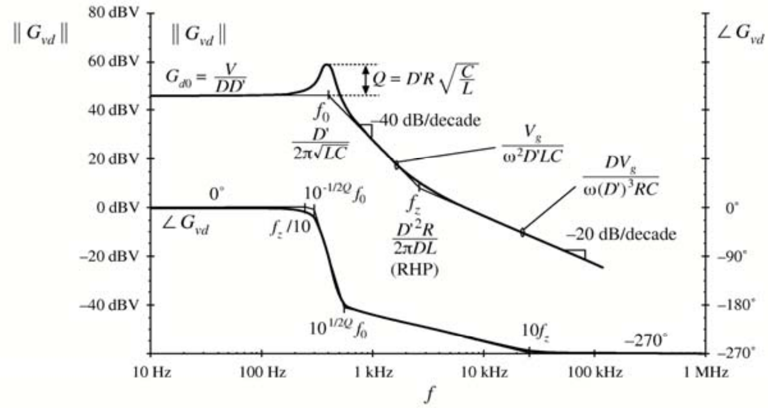
From Chapter 7



$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0} \qquad G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_g(s)=0}$$

Bode plot of control-to-output transfer function

with analytical expressions for important features



Design-oriented analysis

How to approach a real (and hence, complicated) system

Problems:

- Complicated derivations
- Long equations
- Algebra mistakes

Design objectives:

- Obtain physical insight which leads engineer to synthesis of a good design
- Obtain simple equations that can be inverted, so that element values can be chosen to obtain desired behavior. Equations that cannot be inverted are useless for design!

Design-oriented analysis is a structured approach to analysis, which attempts to avoid the above problems

Some elements of design-oriented analysis, discussed in this chapter

- Writing transfer functions in normalized form, to directly expose salient features
- Obtaining simple analytical expressions for asymptotes, corner frequencies, and other salient features, allows element values to be selected such that a given desired behavior is obtained
- Use of inverted poles and zeroes, to refer transfer function gains to the most important asymptote
- Analytical approximation of roots of high-order polynomials
- Graphical construction of Bode plots of transfer functions and polynomials, to
  - avoid algebra mistakes
  - approximate transfer functions
  - obtain insight into origins of salient features

8.1. Review of Bode plots

Decibels

$$\left\| G \right\|_{dB} = 20 \log_{10}\left(\left\| G \right\|\right)$$

Decibels of quantities having units (impedance example): normalize before taking log

$$\left\| Z \right\|_{dB} = 20 \log_{10}\left(\frac{\left\| Z \right\|}{R_{base}}\right)$$

Table 8.1. Expressing magnitudes in decibels

Actual magnitude	*Magnitude in dB*
1/2	– 6dB
1	0 dB
2	6 dB
5 = 10/2	20 dB – 6 dB = 14 dB
10	20dB
1000 = 10^3	3 · 20dB = 60 dB

5Ω is equivalent to 14dB with respect to a base impedance of R_{base} = 1Ω, also known as 14dBΩ.

60dBμA is a current 60dB greater than a base current of 1μA, or 1mA.

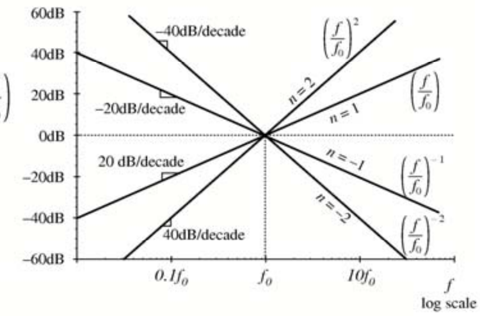
Bode plot of f^n

Bode plots are effectively log-log plots, which cause functions which vary as f^n to become linear plots. Given:

$$\| G \| = \left(\frac{f}{f_0} \right)^n$$

Magnitude in dB is

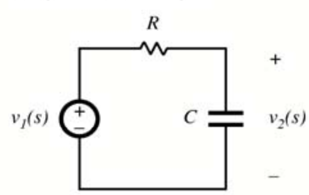
$$\| G \|_{\text{dB}} = 20 \log_{10} \left(\frac{f}{f_0} \right)^n = 20n \log_{10} \left(\frac{f}{f_0} \right)$$



- Slope is $20n$ dB/decade
- Magnitude is 1, or 0dB, at frequency $f = f_0$

8.1.1. Single pole response

Simple R-C example



Transfer function is

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R}$$

Express as rational fraction:

$$G(s) = \frac{1}{1 + sRC}$$

This coincides with the normalized form

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$

with $\omega_0 = \frac{1}{RC}$

$G(j\omega)$ and $\| G(j\omega) \|$

Let $s = j\omega$:

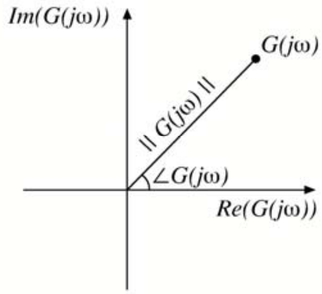
$$G(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_0}\right)} = \frac{1 - j\frac{\omega}{\omega_0}}{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

Magnitude is

$$\left\| G(j\omega) \right\| = \sqrt{\left[\mathrm{Re}\,(G(j\omega))\right]^2 + \left[\mathrm{Im}\,(G(j\omega))\right]^2}$$
$$= \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

Magnitude in dB:

$$\left\| G(j\omega) \right\|_{\mathrm{dB}} = -20 \log_{10}\left(\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right) \text{ dB}$$



Asymptotic behavior: low frequency

For small frequency,
$\omega << \omega_0$ and $f << f_0$:

$$\left(\frac{\omega}{\omega_0}\right) << 1$$

Then $\| G(j\omega) \|$ becomes

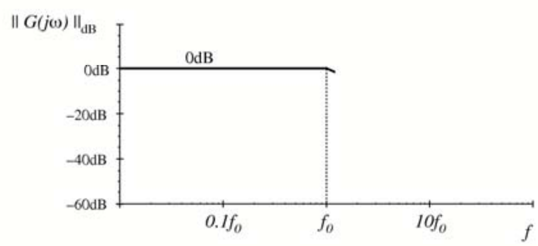
$$\left\| G(j\omega) \right\| \approx \frac{1}{\sqrt{1}} = 1$$

Or, in dB,

$$\left\| G(j\omega) \right\|_{\mathrm{dB}} \approx 0\mathrm{dB}$$

This is the low-frequency asymptote of $\| G(j\omega) \|$

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$



Asymptotic behavior: high frequency

For high frequency, $\omega >> \omega_0$ and $f >> f_0$:

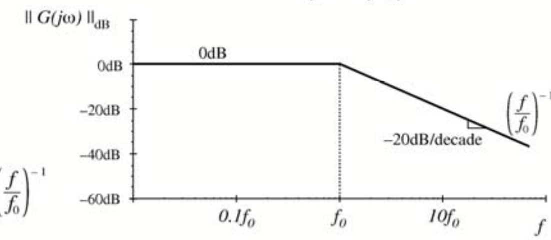
$$\left(\frac{\omega}{\omega_0}\right) >> 1$$

$$1 + \left(\frac{\omega}{\omega_0}\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^2$$

Then $\| G(j\omega) \|$ becomes

$$\left\| G(j\omega) \right\| \approx \frac{1}{\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}} = \left(\frac{f}{f_0}\right)^{-1}$$

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$



The high-frequency asymptote of $\| G(j\omega) \|$ varies as f^{-1}. Hence, $n = -1$, and a straight-line asymptote having a slope of -20dB/decade is obtained. The asymptote has a value of 1 at $f = f_0$.

Deviation of exact curve near $f = f_0$

Evaluate exact magnitude:

at $f = f_0$:

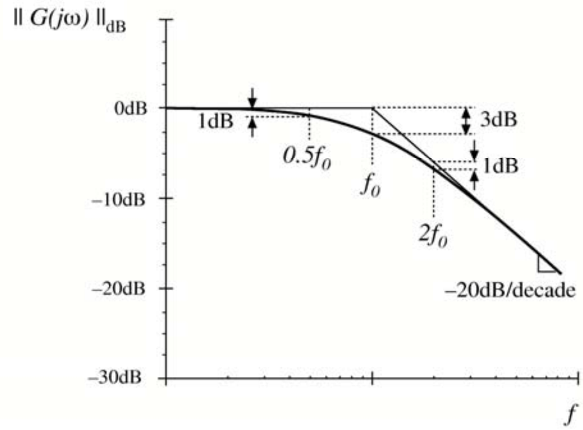
$$\left\| G(j\omega_0) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{\omega_0}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\left\| G(j\omega_0) \right\|_{dB} = -20 \log_{10}\left(\sqrt{1 + \left(\frac{\omega_0}{\omega_0}\right)^2}\right) \approx -3 \text{ dB}$$

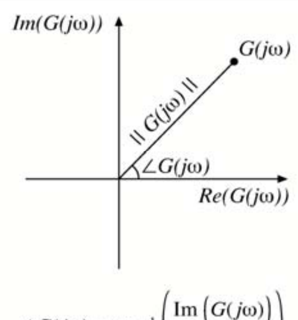
at $f = 0.5f_0$ *and* $2f_0$:

Similar arguments show that the exact curve lies 1dB below the asymptotes.

Summary: magnitude



Phase of $G(j\omega)$

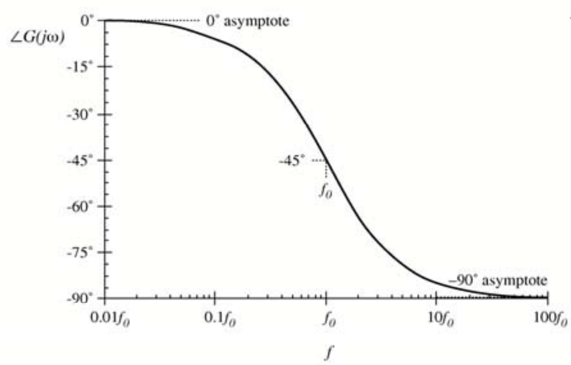


$$\angle G(j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left(G(j\omega)\right)}{\mathrm{Re}\left(G(j\omega)\right)}\right)$$

$$G(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_0}\right)} = \frac{1 - j\frac{\omega}{\omega_0}}{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Phase of $G(j\omega)$



$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

ω	$\angle G(j\omega)$
0	0°
ω_0	–45°
∞	–90°

Phase asymptotes

Low frequency: 0˚

High frequency: –90˚

Low- and high-frequency asymptotes do not intersect

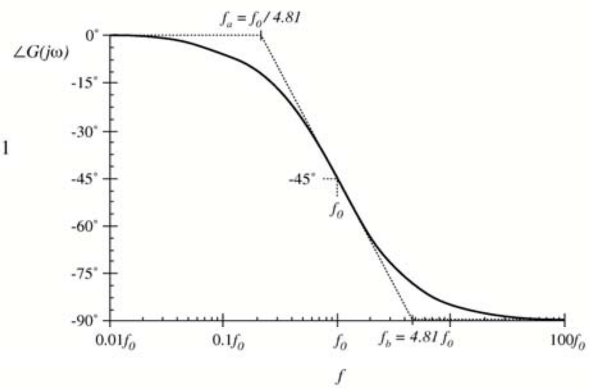
Hence, need a midfrequency asymptote

Try a midfrequency asymptote having slope identical to actual slope at the corner frequency f_0. One can show that the asymptotes then intersect at the break frequencies

$$f_a = f_0\, e^{-\pi/2} \approx f_0 / 4.81$$
$$f_b = f_0\, e^{\pi/2} \approx 4.81\, f_0$$

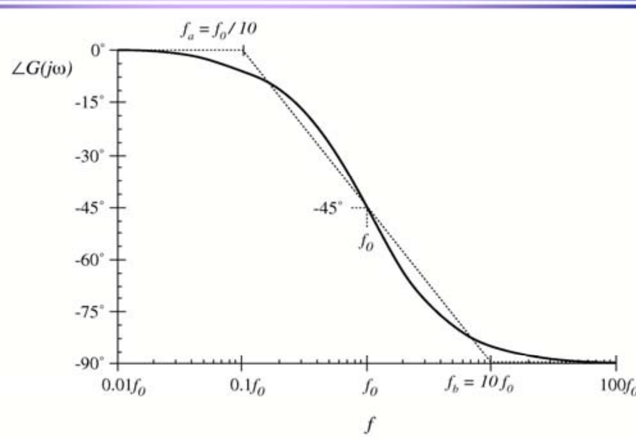
Phase asymptotes

$f_a = f_0\, e^{-\pi/2} \approx f_0 / 4.81$
$f_b = f_0\, e^{\pi/2} \approx 4.81\, f_0$

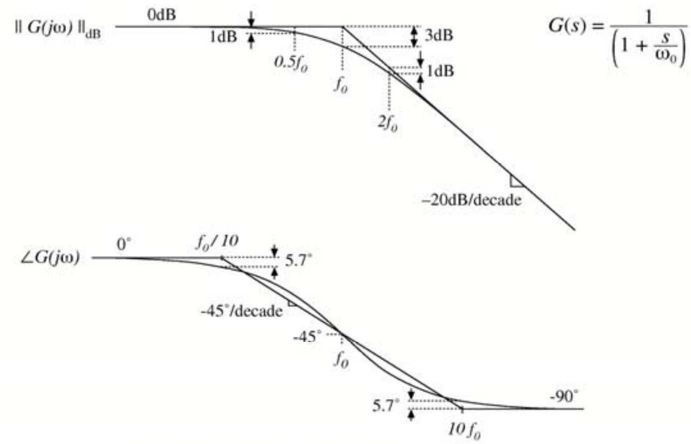


Phase asymptotes: a simpler choice

$f_a = f_0 / 10$
$f_b = 10\, f_0$



Summary: Bode plot of real pole



8.1.2. Single zero response

Normalized form:

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$

Magnitude:

$$\left\| G(j\omega) \right\| = \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

Use arguments similar to those used for the simple pole, to derive asymptotes:

0dB at low frequency, $\omega << \omega_0$

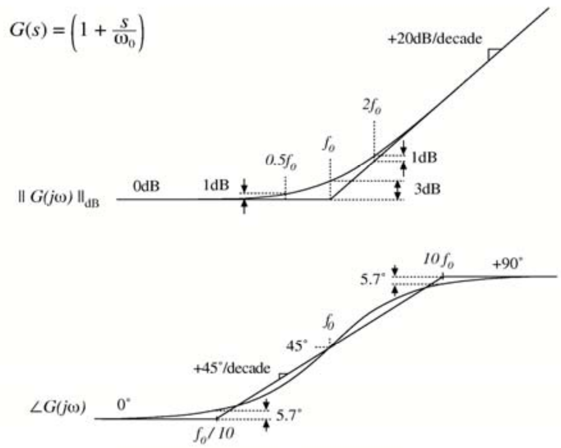
+20dB/decade slope at high frequency, $\omega >> \omega_0$

Phase:

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

—with the exception of a missing minus sign, same as simple pole

Summary: Bode plot, real zero



8.1.3. Right half-plane zero

Normalized form:

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

Magnitude:

$$\left\| G(j\omega) \right\| = \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

—same as conventional (left half-plane) zero. Hence, magnitude asymptotes are identical to those of LHP zero.

Phase:

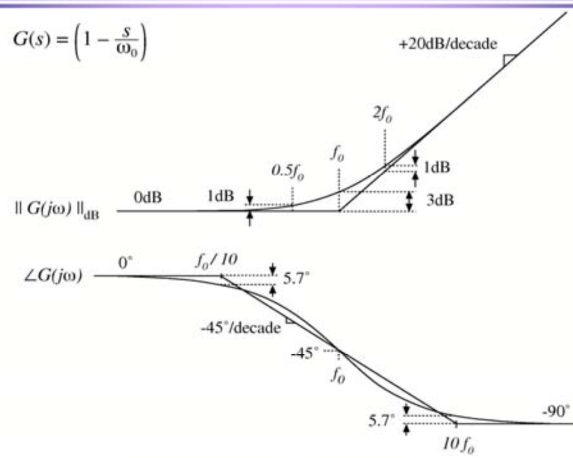
$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

—same as real pole.

The RHP zero exhibits the magnitude asymptotes of the LHP zero, and the phase asymptotes of the pole

Summary: Bode plot, RHP zero

$G(s) = \left(1 - \frac{s}{\omega_0}\right)$



8.1.4. Frequency inversion

Reversal of frequency axis. A useful form when describing mid- or high-frequency flat asymptotes. Normalized form, inverted pole:

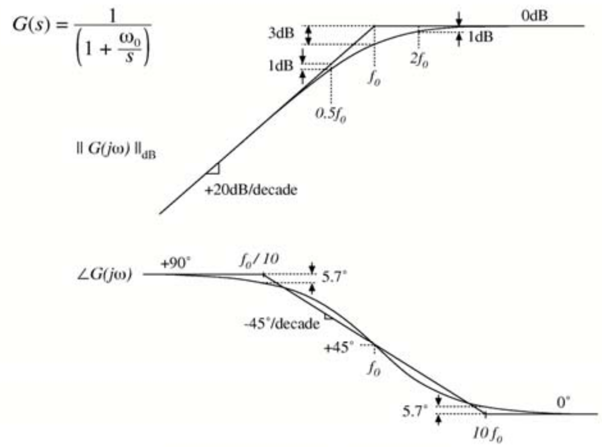
$$G(s) = \frac{1}{\left(1 + \frac{\omega_0}{s}\right)}$$

An algebraically equivalent form:

$$G(s) = \frac{\left(\frac{s}{\omega_0}\right)}{\left(1 + \frac{s}{\omega_0}\right)}$$

The inverted-pole format emphasizes the high-frequency gain.

Asymptotes, inverted pole



Inverted zero

Normalized form, inverted zero:

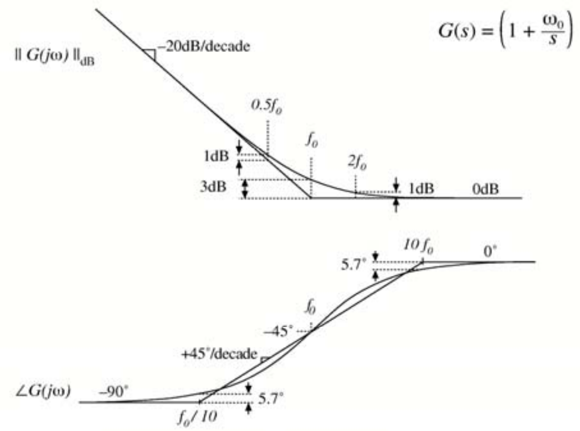
$$G(s) = \left(1 + \frac{\omega_0}{s}\right)$$

An algebraically equivalent form:

$$G(s) = \frac{\left(1 + \frac{s}{\omega_0}\right)}{\left(\frac{s}{\omega_0}\right)}$$

Again, the inverted-zero format emphasizes the high-frequency gain.

Asymptotes, inverted zero



8.1.5. Combinations

Suppose that we have constructed the Bode diagrams of two complex-values functions of frequency, $G_1(\omega)$ and $G_2(\omega)$. It is desired to construct the Bode diagram of the product, $G_3(\omega) = G_1(\omega)\, G_2(\omega)$.

Express the complex-valued functions in polar form:

$$G_1(\omega) = R_1(\omega)\, e^{j\theta_1(\omega)}$$
$$G_2(\omega) = R_2(\omega)\, e^{j\theta_2(\omega)}$$
$$G_3(\omega) = R_3(\omega)\, e^{j\theta_3(\omega)}$$

The product $G_3(\omega)$ can then be written

$$G_3(\omega) = G_1(\omega)\, G_2(\omega) = R_1(\omega)\, e^{j\theta_1(\omega)}\, R_2(\omega)\, e^{j\theta_2(\omega)}$$

$$G_3(\omega) = \Big(R_1(\omega)\, R_2(\omega)\Big)\, e^{j(\theta_1(\omega) + \theta_2(\omega))}$$

Combinations

$$G_3(\omega) = \left(R_1(\omega)\ R_2(\omega)\right)\ e^{j(\theta_1(\omega)+\theta_2(\omega))}$$

The composite phase is

$$\theta_3(\omega) = \theta_1(\omega) + \theta_2(\omega)$$

The composite magnitude is

$$R_3(\omega) = R_1(\omega)\ R_2(\omega)$$

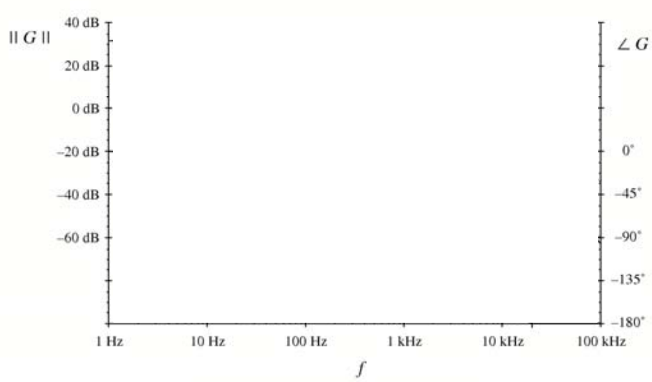
$$\left| R_3(\omega) \right|_{dB} = \left| R_1(\omega) \right|_{dB} + \left| R_2(\omega) \right|_{dB}$$

Composite phase is sum of individual phases.

Composite magnitude, when expressed in dB, is sum of individual magnitudes.

Example 1: $G(s) = \dfrac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$

with $G_0 = 40 \Rightarrow 32\ \text{dB}, f_1 = \omega_1/2\pi = 100\ \text{Hz}, f_2 = \omega_2/2\pi = 2\ \text{kHz}$



Example 1: $G(s) = \dfrac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$

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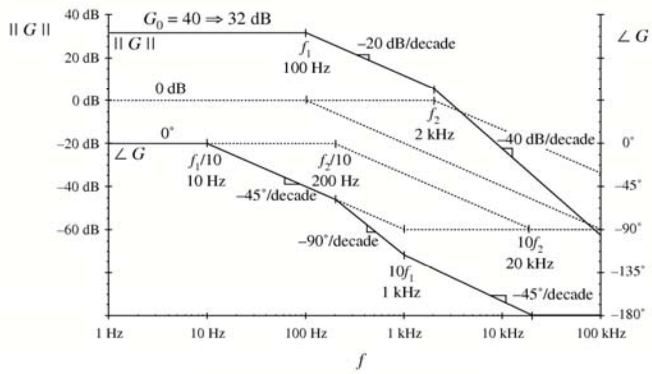

Example 2

Determine the transfer function *A(s)* corresponding to the following asymptotes: