Lecture 17: Converter Bode Plots

ECE 481: Power Electronics

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Announcements

- HW #7 due Tuesday, 11/5

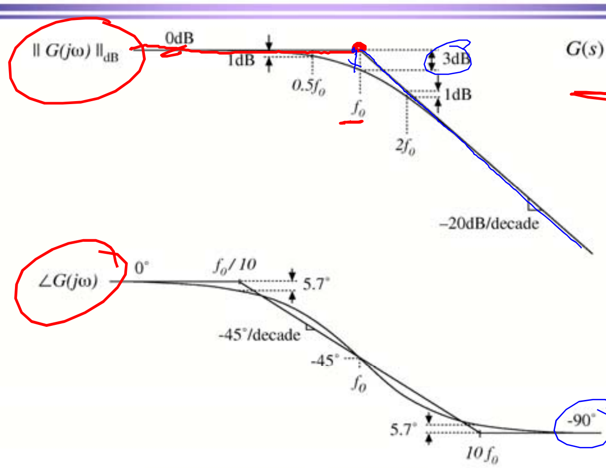
The 1st IEEE Workshop on Wide Bandgap Power Devices and Applications

TABLE I

PHYSICAL PROPERTIES AT 300 K OF SI, 4H-SIC, GAN AND DIAMOND AND RELATED FIGURES OF MERIT (JOHNSON, KEYES AND BALIGA).[1]

[unit]	Si	4H-SiC	GaN	Diamond
E_G [eV]	1.12 i	3.23 i	3.39 d	5.47 i
ϵ_r	11.7	9.66	8.9	5.7
E_B [MV/cm]	0.3	3	5	10
λ [W/cm.K]	1.3	3.7	1.3	22
v_s [10^7 cm/s]	1.0	2.0	2.2	1
μ_e [cm^2/V.s]	1400	900	1000	1000
μ_h [cm^2/V.s]	450	100	350	2000
JFM [10^{23} $\Omega.W/s^2$]	2.3	900	490	2530
KFM [10^7 W/K.s]	10	53	17	218
BFM [Si=1]	1	554	188	23068

Summary: Bode plot of real pole



$$G(s) = \frac{1}{\left(1+\frac{s}{\omega_0}\right)}$$

s = jw

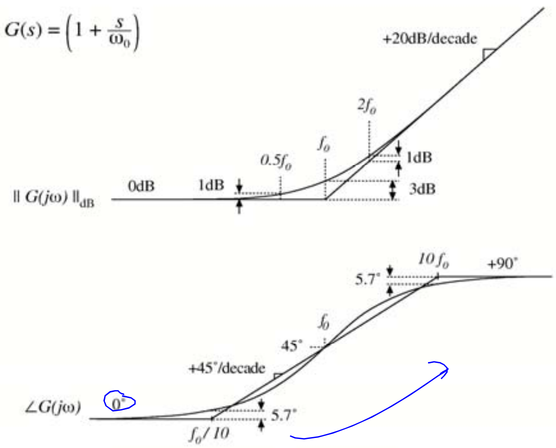
if w << w0

if w >> w0

$G(s) \approx \frac{1}{\frac{s}{\omega_0}} = \frac{\omega_0}{s}$

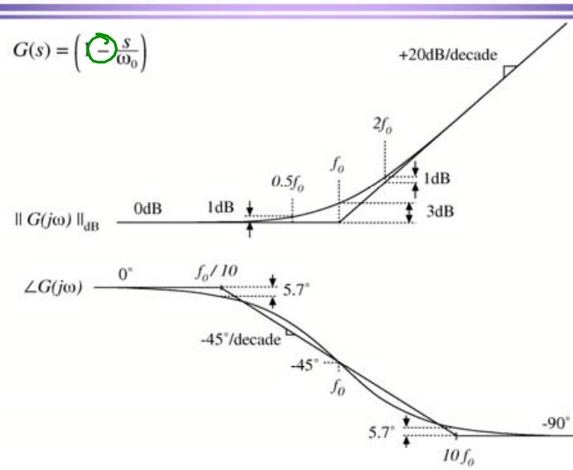
Summary: Bode plot, real zero

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$



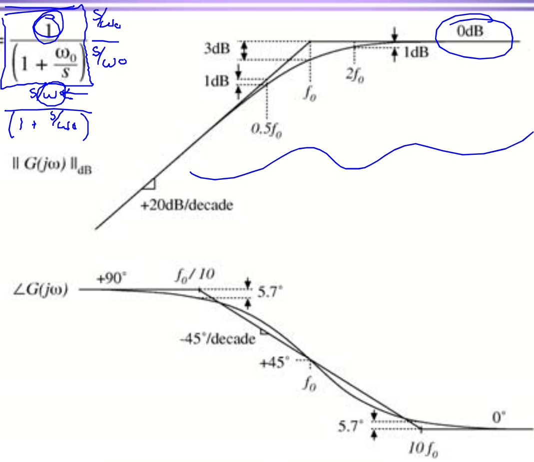
Summary: Bode plot, RHP zero

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$



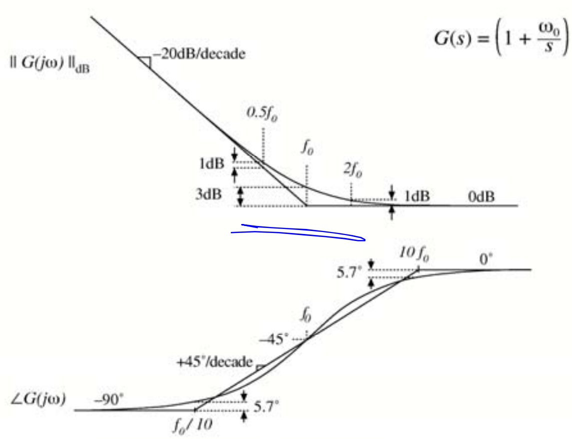
Asymptotes, inverted pole

$$G(s) = \frac{1}{\left(1 + \frac{\omega_0}{s}\right)} \cdot \frac{s/\omega_0}{s/\omega_0} = \frac{s/\omega_0}{\left(1 + s/\omega_0\right)}$$



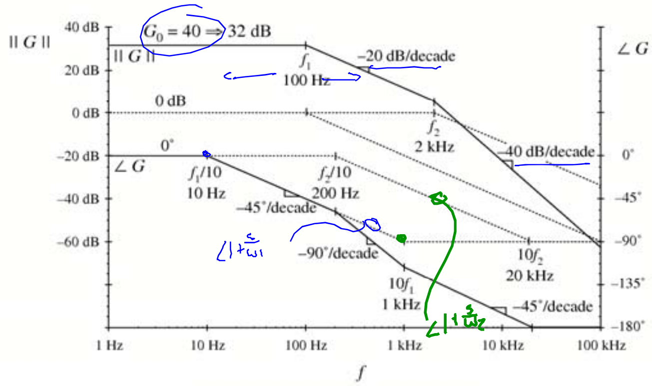
Asymptotes, inverted zero

$$G(s) = \left(1 + \frac{\omega_0}{s}\right)$$



Example 1: $G(s) = \dfrac{G_0}{\left(1 + \dfrac{s}{\omega_1}\right)\left(1 + \dfrac{s}{\omega_2}\right)}$

with $G_0 = 40 \Rightarrow 32$ dB, $f_1 = \omega_1/2\pi = 100$ Hz, $f_2 = \omega_2/2\pi = 2$ kHz



Example 2

$A(s) = A_0 \dfrac{\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$

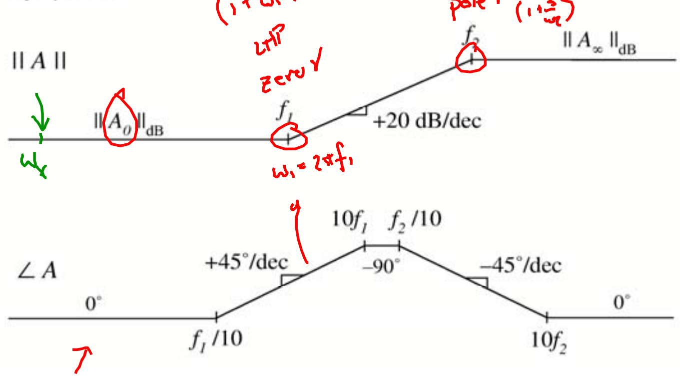
for $\omega_x \ll \omega_1, \omega_2$ $|A(s)| = A_0 \dfrac{\left(1 + \cancel{\frac{j\omega_x}{\omega_1}}\right)}{\left(1 + \cancel{\frac{j\omega_x}{\omega_2}}\right)}$ ≈ 0

$|A(j\omega_x)| = A_0$

Determine the transfer function $A(s)$ corresponding to the following asymptotes:

$\left(1 + \frac{s}{\omega_1}\right)$ LHP zero ✓

LHP pole ✓ $\frac{1}{\left(1 + \frac{s}{\omega_2}\right)}$



Example 2, continued

One solution:

$$A(s) = A_0 \frac{\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$$

Analytical expressions for asymptotes:

For $f < f_1$

$$\left| A_0 \frac{\left(1 + \frac{\cancel{s}}{\omega_1}\right)}{\left(1 + \frac{\cancel{s}}{\omega_2}\right)} \right|_{s = j\omega} = A_0 \frac{1}{1} = A_0$$

For $f_1 < f < f_2$

$$\left| A_0 \frac{\left(\cancel{1} + \frac{s}{\omega_1}\right)}{\left(1 + \frac{\cancel{s}}{\omega_2}\right)} \right|_{s = j\omega} = A_0 \frac{\left| \frac{s}{\omega_1} \right|_{s = j\omega}}{1} = A_0 \frac{\omega}{\omega_1} = A_0 \frac{f}{f_1}$$

Example 2, continued

For $f > f_2$

$$\left| A_0 \frac{\left(\cancel{1} + \frac{s}{\omega_1}\right)}{\left(\cancel{1} + \frac{s}{\omega_2}\right)} \right|_{s = j\omega} = A_0 \frac{\left| \frac{s}{\omega_1} \right|_{s = j\omega}}{\left| \frac{s}{\omega_2} \right|_{s = j\omega}} = A_0 \frac{\omega_2}{\omega_1} = A_0 \frac{f_2}{f_1}$$

So the high-frequency asymptote is

$$A_\infty = A_0 \frac{f_2}{f_1}$$

Another way to express $A(s)$: use inverted poles and zeroes, and express $A(s)$ directly in terms of A_∞

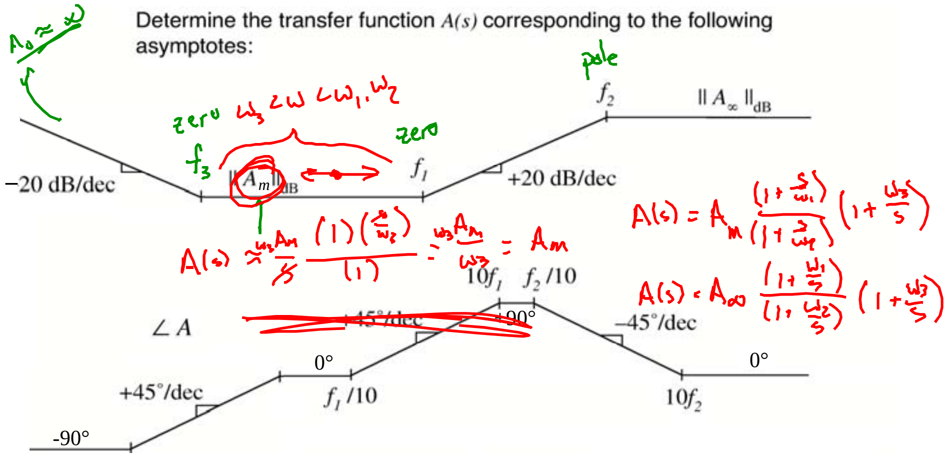
$$A(s) = A_\infty \frac{\left(1 + \frac{\omega_1}{s}\right)}{\left(1 + \frac{\omega_2}{s}\right)}$$

pole: $\frac{1}{1+\frac{s}{\omega}} \approx \frac{\omega}{s}$ for $\omega \to \emptyset$

$A(s) = \frac{\omega_3 A_m}{s} \frac{(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_3})}{(1+\frac{s}{\omega_2})}$

Example 2b

Determine the transfer function $A(s)$ corresponding to the following asymptotes:



$A(s) \approx \frac{\omega_3 A_m}{s} \frac{(1)(\frac{s}{\omega_3})}{(1)} = \frac{\omega_3 A_m}{\omega_3} = A_m$

$A(s) = A_m \frac{(1+\frac{s}{\omega_1})}{(1+\frac{s}{\omega_2})}(1+\frac{\omega_3}{s})$

$A(s) = A_\infty \frac{(1+\frac{\omega_1}{s})}{(1+\frac{\omega_2}{s})}(1+\frac{\omega_3}{s})$

8.1.6 Quadratic pole response: resonance

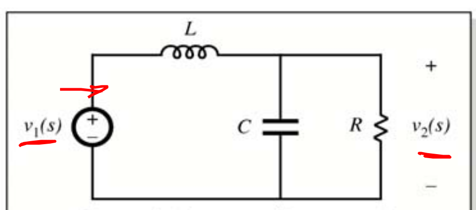
Example

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

Second-order denominator, of the form

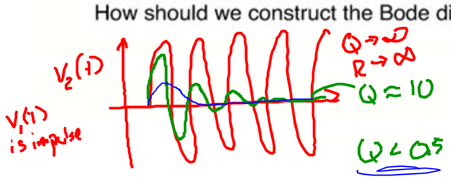
$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2}$$

with $a_1 = L/R$ and $a_2 = LC$



Two-pole low-pass filter example

How should we construct the Bode diagram?



$V_2(s) = V_1(s) \frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + sL}$

Approach 1: factor denominator

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2} = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

for real poles

$4a_2 \le a_1^2$

$4\frac{1}{\omega_0^2} \le \left(\frac{1}{Q\omega_0}\right)^2$

$Q \le 0.5$

We might factor the denominator using the quadratic formula, then construct Bode diagram as the combination of two real poles:

$$G(s) = \frac{1}{\left(1 - \frac{s}{s_1}\right)\left(1 - \frac{s}{s_2}\right)} \quad \text{with} \quad s_1 = -\frac{a_1}{2a_2}\left[1 - \sqrt{1 - \frac{4a_2}{a_1^2}}\right]$$

$$s_2 = -\frac{a_1}{2a_2}\left[1 + \sqrt{1 - \frac{4a_2}{a_1^2}}\right]$$

- If $4a_2 \le a_1^2$, then the roots s_1 and s_2 are real. We can construct Bode diagram as the combination of two real poles.
- If $4a_2 > a_1^2$, then the roots are complex. In Section 8.1.1, the assumption was made that ω_0 is real; hence, the results of that section cannot be applied and we need to do some additional work.

Approach 2: Define a standard normalized form for the quadratic case

$$G(s) = \frac{1}{1 + 2\zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{or} \quad \boxed{G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}}$$

- When the coefficients of s are real and positive, then the parameters ζ, ω_0, and Q are also real and positive
- The parameters ζ, ω_0, and Q are found by equating the coefficients of s
- The parameter ω_0 is the angular corner frequency, and we can define $f_0 = \omega_0/2\pi$
- The parameter ζ is called the *damping factor*. ζ controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $\zeta < 1$.
- In the alternative form, the parameter Q is called the *quality factor*. Q also controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $Q > 0.5$.

The Q-factor

In a second-order system, ζ and Q are related according to

$$Q = \frac{1}{2\zeta}$$

Q is a measure of the dissipation in the system. A more general definition of Q, for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that Q has a simple interpretation in the Bode diagrams of second-order transfer functions.

Analytical expressions for f_0 and Q

Two-pole low-pass filter example: we found that

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

Equate coefficients of like powers of s with the standard form

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Result:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}}$$

Magnitude asymptotes, quadratic form

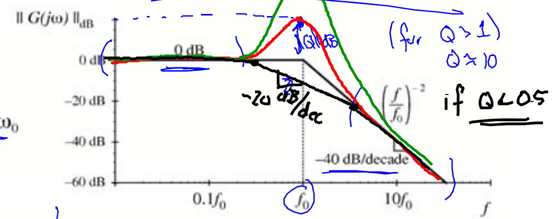
In the form $G(s) = \dfrac{1}{1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2}$

let $s = j\omega$ and find magnitude: $\left\| G(j\omega) \right\| = \dfrac{1}{\sqrt{\left(1 - \left(\dfrac{\omega}{\omega_0}\right)^2\right)^2 + \dfrac{1}{Q^2}\left(\dfrac{\omega}{\omega_0}\right)^2}}$

Asymptotes are

$\| G \| \to 1$ for $\omega \ll \omega_0$

$\| G \| \to \left(\dfrac{f}{f_0}\right)^{-2}$ for $\omega \gg \omega_0$



@ $\omega = \omega_0$ $\quad \| G(j\omega_0) \| = \dfrac{1}{\sqrt{(1-1)^2 + \frac{1}{Q^2}(1^2)}} = \dfrac{1}{\sqrt{1/Q^2}} = Q$

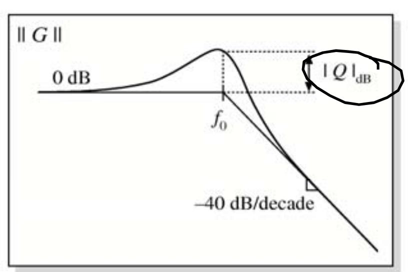
Deviation of exact curve from magnitude asymptotes

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$

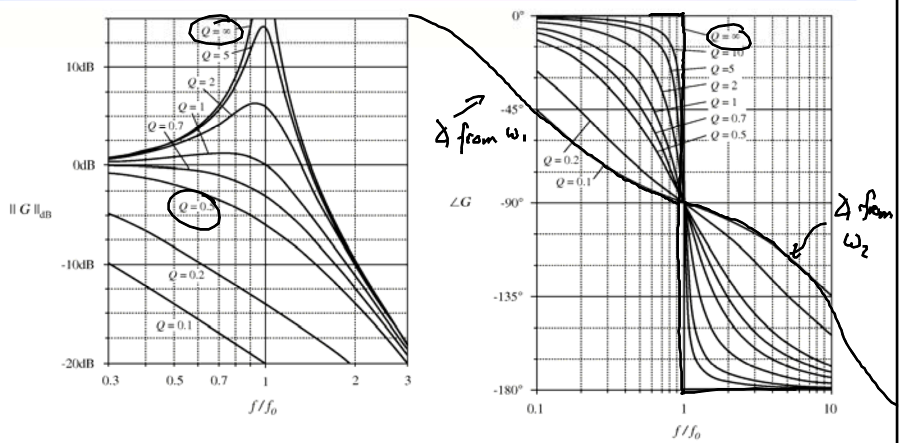
At $\omega = \omega_0$, the exact magnitude is

$\left\| G(j\omega_0) \right\| = Q$ or, in dB: $\left\| G(j\omega_0) \right\|_{dB} = | Q |_{dB}$

The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $| Q |_{dB}$



Two-pole response: exact curves



8.1.7. The low-Q approximation

Given a second-order denominator polynomial, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \qquad \text{or} \qquad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

When the roots are real, i.e., when $Q < 0.5$, then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

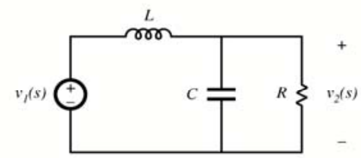
This is a particularly desirable approach when $Q << 0.5$, i.e., when the corner frequencies ω_1 and ω_2 are well separated.

An example

A problem with this procedure is the complexity of the quadratic formula used to find the corner frequencies.

R-L-C network example:

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$



Use quadratic formula to factor denominator. Corner frequencies are:

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4\,LC}}{2\,LC}$$

Factoring the denominator

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4\,LC}}{2\,LC}$$

This complicated expression yields little insight into how the corner frequencies ω_1 and ω_2 depend on R, L, and C.

When the corner frequencies are well separated in value, it can be shown that they are given by the much simpler (approximate) expressions

$$\omega_1 \approx \frac{R}{L}, \qquad \omega_2 \approx \frac{1}{RC}$$

ω_1 is then independent of C, and ω_2 is independent of L.

These simpler expressions can be derived via the Low-Q Approximation.

Derivation of the Low-Q Approximation

Given

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Use quadratic formula to express corner frequencies ω_1 and ω_2 in terms of Q and ω_0 as:

$$\omega_1 = \frac{\omega_0}{Q}\,\frac{1 - \sqrt{1 - 4Q^2}}{2} \qquad \omega_2 = \frac{\omega_0}{Q}\,\frac{1 + \sqrt{1 - 4Q^2}}{2}$$

Corner frequency ω_2

$$\omega_2 = \frac{\omega_0}{Q}\,\frac{1 + \sqrt{1 - 4Q^2}}{2}$$

can be written in the form

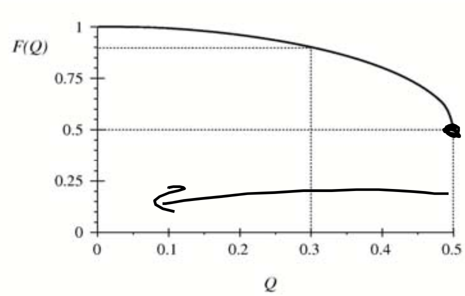
$$\omega_2 = \frac{\omega_0}{Q}\,F(Q)$$

where

$$F(Q) = \frac{1}{2}\left(1 + \sqrt{1 - 4Q^2}\right)$$

For small Q, $F(Q)$ tends to 1.
We then obtain

$$\omega_2 \approx \frac{\omega_0}{Q} \quad \text{for } Q << \frac{1}{2}$$



For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.

Corner frequency ω_1

$$\omega_1 = \frac{\omega_0}{Q}\,\frac{1-\sqrt{1-4Q^2}}{2}$$

can be written in the form

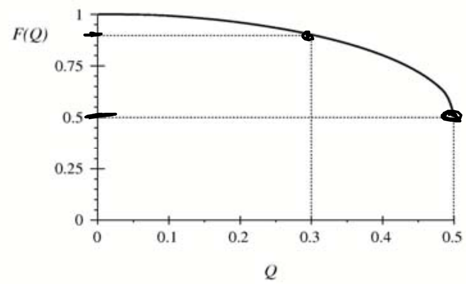
$$\omega_1 = \frac{Q\,\omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2}\left(1+\sqrt{1-4Q^2}\right)$$

For small Q, $F(Q)$ tends to 1.
We then obtain

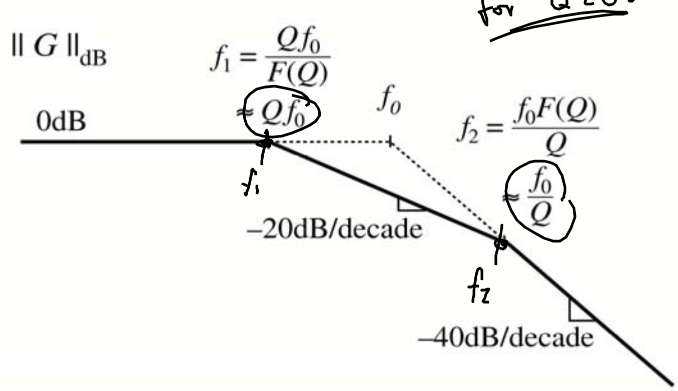
$$\omega_1 \approx Q\,\omega_0 \qquad \text{for } Q \ll \frac{1}{2}$$



For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.

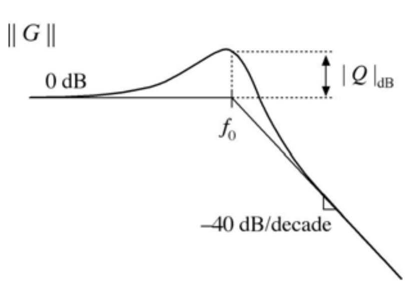
The Low-Q Approximation

for $Q < 0.5$

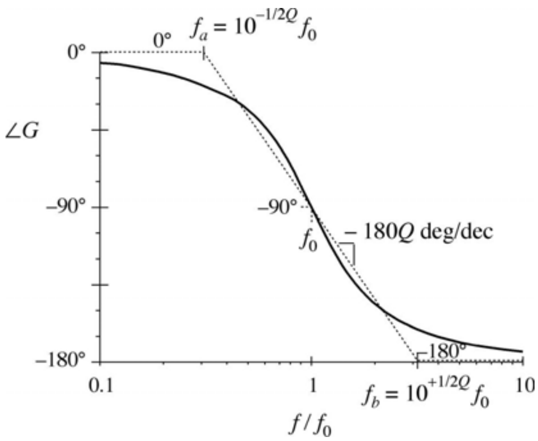


Summary: Asymptotes for Complex Poles

Magnitude



Phase



R-*L*-*C* Example

For the previous example:

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}}$$

Use of the Low-Q Approximation leads to

$$\omega_1 \approx Q\,\omega_0 = R\sqrt{\frac{C}{L}}\,\frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$\omega_2 \approx \frac{\omega_0}{Q} = \frac{1}{\sqrt{LC}}\,\frac{1}{R\sqrt{\frac{C}{L}}} = \frac{1}{RC}$$

8.1.8. Approximate Roots of an Arbitrary-Degree Polynomial

Generalize the low-Q approximation to obtain approximate factorization of the n^{th}-order polynomial

$$P(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n$$

It is desired to factor this polynomial in the form

$$P(s) = \left(1 + \tau_1 s\right)\left(1 + \tau_2 s\right) \cdots \left(1 + \tau_n s\right)$$

When the roots are real and well separated in value, then approximate analytical expressions for the time constants τ_1, τ_2, ... τ_n can be found, that typically are simple functions of the circuit element values.

Objective: find a general method for deriving such expressions. Include the case of complex root pairs.

Result

when roots are real and well separated

If the following inequalities are satisfied

$$\left|a_1\right| >> \left|\frac{a_2}{a_1}\right| >> \left|\frac{a_3}{a_2}\right| >> \cdots >> \left|\frac{a_n}{a_{n-1}}\right|$$

Then the polynomial $P(s)$ has the following approximate factorization

$$P(s) \approx \left(1 + a_1 s\right)\left(1 + \frac{a_2}{a_1} s\right)\left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

- If the a_n coefficients are simple analytical functions of the element values L, C, etc., then the roots are similar simple analytical functions of L, C, etc.
- Numerical values are used to justify the approximation, but analytical expressions for the roots are obtained

When two roots are not well separated

then leave their terms in quadratic form

Suppose inequality k is not satisfied:

$$\left| a_1 \right| \gg \left| \frac{a_2}{a_1} \right| \gg \cdots \gg \underbrace{\left| \frac{a_k}{a_{k-1}} \right| \not\gg \left| \frac{a_{k+1}}{a_k} \right|}_{\text{not satisfied}} \gg \cdots \gg \left| \frac{a_n}{a_{n-1}} \right|$$

Then leave the terms corresponding to roots k and $(k + 1)$ in quadratic form, as follows:

$$P(s) \approx \left(1 + a_1 s\right) \left(1 + \frac{a_2}{a_1} s\right) \cdots \left(1 + \frac{a_k}{a_{k-1}} s + \frac{a_{k+1}}{a_{k-1}} s^2\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is accurate provided

$$\left| a_1 \right| \gg \left| \frac{a_2}{a_1} \right| \gg \cdots \gg \left| \frac{a_k}{a_{k-1}} \right| \gg \left| \frac{a_{k-2}\, a_{k+1}}{a_{k-1}^2} \right| \gg \left| \frac{a_{k+2}}{a_{k+1}} \right| \gg \cdots \gg \left| \frac{a_n}{a_{n-1}} \right|$$

When the first inequality is violated

A special case for quadratic roots

When inequality 1 is not satisfied:

$$\underbrace{\left| a_1 \right| \not\gg \left| \frac{a_2}{a_1} \right|}_{\text{not satisfied}} \gg \left| \frac{a_3}{a_2} \right| \gg \cdots \gg \left| \frac{a_n}{a_{n-1}} \right|$$

Then leave the first two roots in quadratic form, as follows:

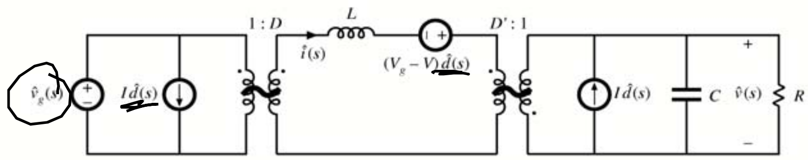
$$P(s) \approx \left(1 + a_1 s + a_2 s^2\right) \left(1 + \frac{a_3}{a_2} s\right) \cdots \left(1 + \frac{a_n}{a_{n-1}} s\right)$$

This approximation is justified provided

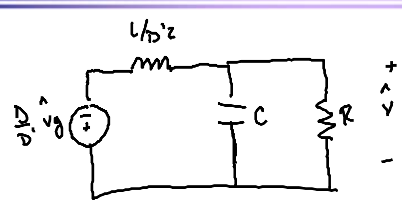
$$\left| \frac{a_2^2}{a_3} \right| \gg \left| a_1 \right| \gg \left| \frac{a_3}{a_2} \right| \gg \left| \frac{a_4}{a_3} \right| \gg \cdots \gg \left| \frac{a_n}{a_{n-1}} \right|$$

8.2.1. Example: transfer functions of the buck-boost converter

Small-signal ac model of the buck-boost converter, derived in Chapter 7:



From Lecture 15



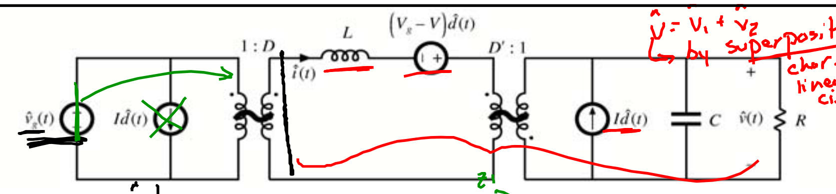
$$\hat v = \frac{\frac{1}{sC}\parallel R}{\frac{1}{sC}\parallel R + sL/D'^2}\,\frac{-D}{D'}\hat v_g$$

$$G_{vg} = \frac{\hat v}{\hat v_g}\bigg|_{\hat d=0} = \frac{-D}{D'}\left(\frac{\frac{1}{sC+\frac{1}{R}}}{\frac{1}{sC+\frac{1}{R}} + s\frac{L}{D'^2}}\right) = \frac{-D}{D'}\left(\frac{1}{1+s^2\frac{LC}{D'^2}+s\frac{L}{D'^2R}}\right)$$

$$\Downarrow$$

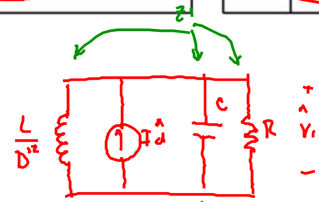
$$\frac{G_{vg0}}{\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1}$$

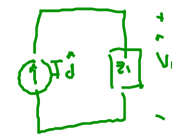
$\hat{v} = \hat{v}_1 + \hat{v}_2$ ↳ by superposition char. of linear circuits



$$G_{vd} = \left.\frac{\hat{v}}{\hat{d}}\right|_{\hat{v}_g = 0}$$

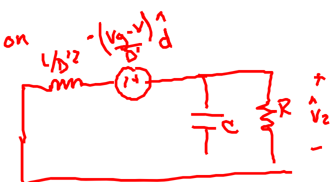
(1) $I\hat{d}$ on





$$\hat{v}_1 = I\hat{d}\left(\frac{sL}{D'^2} \,\|\, \frac{1}{sC} \,\|\, R\right)$$

(2) $(V_g - V)\hat{d}$ on



$$\hat{v}_2 = \frac{V_g - V}{D'}\hat{d}\,\frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + sL/D'^2}$$

$$\hat{v} = \hat{v}_1 + \hat{v}_2 = I\hat{d}\underbrace{\left(\frac{1}{sC} \,\|\, \frac{sL}{D'^2} \,\|\, R\right)}_{} + \frac{-(V_g - V)}{D'}\hat{d}\,\underbrace{\frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + sL/D'^2}}_{}$$

$$\left(\frac{sL}{D'^2}\right)\frac{1}{\left(\frac{sL}{D'^2}\right)\left(sC + \frac{D'^2}{sL} + \frac{1}{R}\right)} = \left(\frac{sL}{D'^2}\right)\frac{1}{s^2\frac{LC}{D'^2} + 1 + \frac{sL}{RD'^2}}$$

$$\frac{1}{s^2\frac{LC}{D'^2} + s\frac{L}{D'^2 R} + 1}$$

$$\frac{\hat{v}}{\hat{d}} = \frac{I\frac{sL}{D'^2} - \frac{V_g - V}{D'}}{s^2\frac{LC}{D'^2} + \frac{sL}{RD'^2} + 1} = G_{vd}$$

Derivation of transfer functions

Divide numerator and denominator by R. Result: the line-to-output transfer function is

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0} = \left(-\frac{D}{D'}\right) \frac{1}{1 + s\frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}}$$

which is of the following standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Salient features of the line-to-output transfer function

Equate standard form to derived transfer function, to determine expressions for the salient features:

$$G_{g0} = -\frac{D}{D'}$$

$$\frac{1}{\omega_0^2} = \frac{LC}{D'^2} \qquad \omega_0 = \frac{D'}{\sqrt{LC}}$$

$$\frac{1}{Q\omega_0} = \frac{L}{D'^2 R} \qquad Q = D'R\sqrt{\frac{C}{L}}$$

Control-to-output transfer function

Express in normalized form:

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_g(s)=0} = \left(-\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{\left(1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}\right)}$$

This is of the following standard form:

$$G_{vd}(s) = G_{d0}\frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

Salient features of control-to-output transfer function

$$G_{d0} = -\frac{V_g - V}{D'} = -\frac{V_g}{D'^2} = \frac{V}{DD'}$$

$$\omega_z = \frac{V_g - V}{L\,I} = \frac{D'\,R}{D\,L} \quad \text{(RHP)}$$

$$\omega_0 = \frac{D'}{\sqrt{LC}}$$

$$Q = D'R\sqrt{\frac{C}{L}}$$

— Simplified using the dc relations: $\begin{cases} V = -\frac{D}{D'}V_g \\ I = -\frac{V}{D'\,R} \end{cases}$

Plug in numerical values

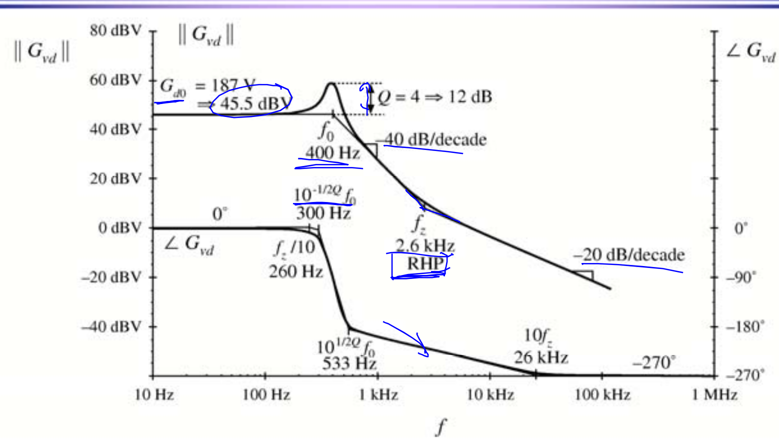
Suppose we are given the following numerical values:

$$D = 0.6$$
$$R = 10\Omega$$
$$V_g = 30\text{V}$$
$$L = 160\mu\text{H}$$
$$C = 160\mu\text{F}$$

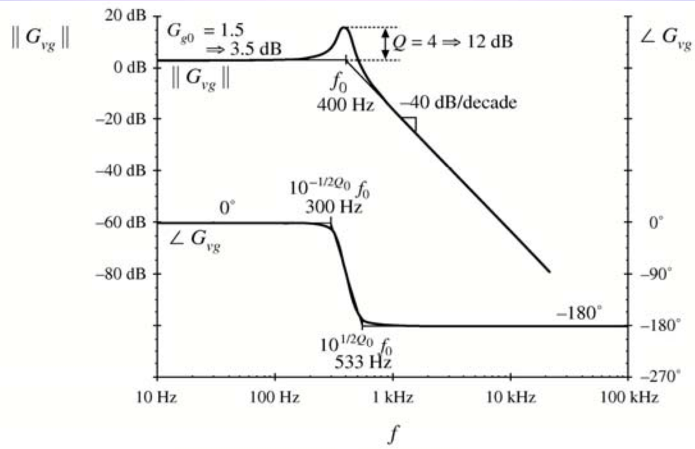
Then the salient features have the following numerical values:

$$\left| G_{g0} \right| = \frac{D}{D'} = 1.5 \Rightarrow 3.5 \text{ dB}$$
$$\left| G_{d0} \right| = \frac{\left| V \right|}{DD'} = 187.5 \text{ V} \Rightarrow 45.5 \text{ dBV}$$
$$f_0 = \frac{\omega_0}{2\pi} = \frac{D'}{2\pi\sqrt{LC}} = 400 \text{ Hz}$$
$$Q = D'R\sqrt{\frac{C}{L}} = 4 \Rightarrow 12 \text{ dB}$$
$$f_z = \frac{\omega_z}{2\pi} = \frac{D'^2 R}{2\pi DL} = 2.65 \text{ kHz}$$

Bode plot: control-to-output transfer function

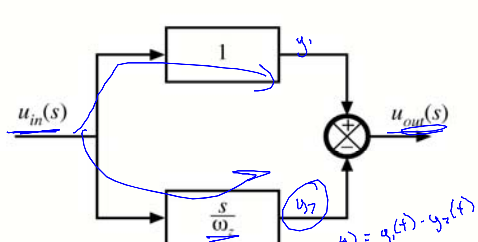


Bode plot: line-to-output transfer function



8.2.3. Physical origins of the right half-plane zero

$$G(s) = \left(1 - \frac{s}{\omega_z}\right)$$



- *phase reversal at high frequency*
- *transient response: output initially tends in wrong direction*