Lecture 18: Z_{out}, Closed Loop Converters

ECE 481: Power Electronics

Prof. Daniel Costinett
Department of Electrical Engineering and Computer Science
University of Tennessee Knoxville
Fall 2013

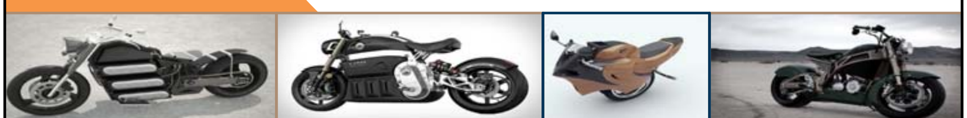
Announcements

- No office hours this afternoon
- HW#7 due, solutions and HW#8 this afternoon

ECE 482: Power Electronic Circuits

Course reformatted for Spring 2014

- Design-oriented introduction to the analysis, modeling, and testing of power electronics
  - Fabrication of the multiple switched-mode power converters
    - Analog and digital control systems
      - Realize a functioning, sub-kW electric vehicle
        - Compete to achieve best performance of EV drive train



ECE482 Schedule

ECE 599

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
9am	ECE 482-001 29437 Class 9:05 am-9:55 am MINKAO 406 ?		ECE 482-001 29437 Class 9:05 am-9:55 am MINKAO 406 9-11		~~ECE 482-001~~ ~~29437 Class~~ ~~9:05 am-9:55 am~~ ~~MINKAO 406~~	

ECE 433 @ 9:00 ?

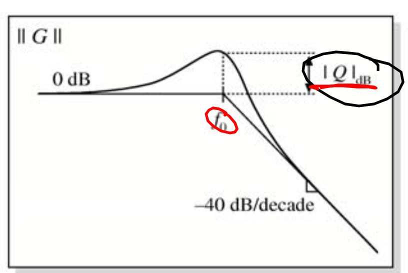
Deviation of exact curve from magnitude asymptotes

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$

At $\omega = \omega_0$, the exact magnitude is

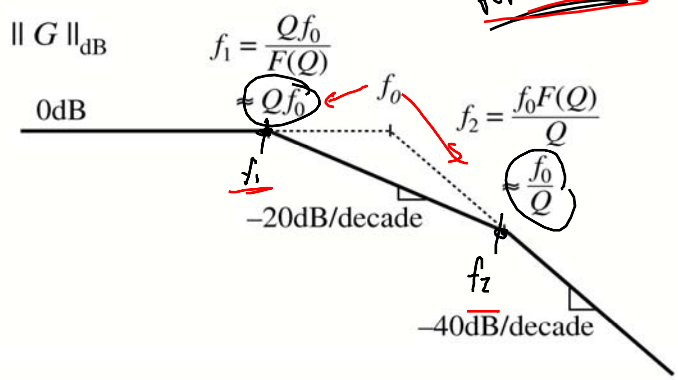
$$\left\| G(j\omega_0) \right\| = Q \qquad \text{or, in dB:} \qquad \left\| G(j\omega_0) \right\|_{dB} = \| Q \|_{dB}$$

The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $\| Q \|_{dB}$



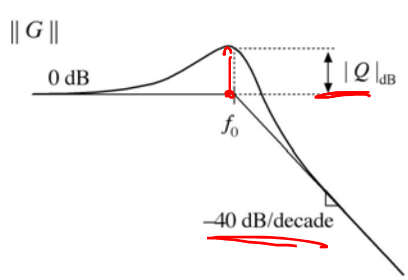
The Low-Q Approximation

for $Q < 0.5$

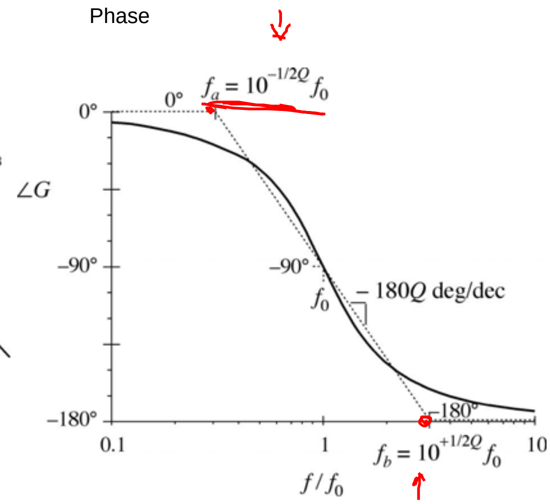


Summary: Asymptotes for Complex Poles

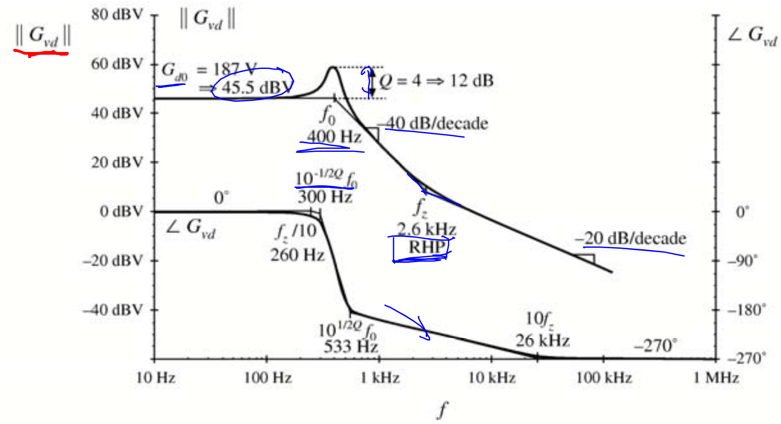
Magnitude



Phase

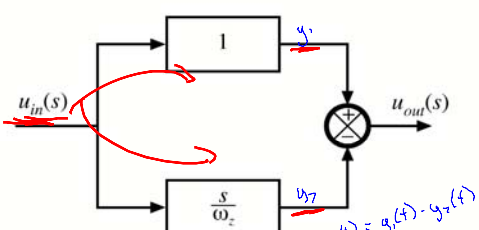


Bode plot: control-to-output transfer function

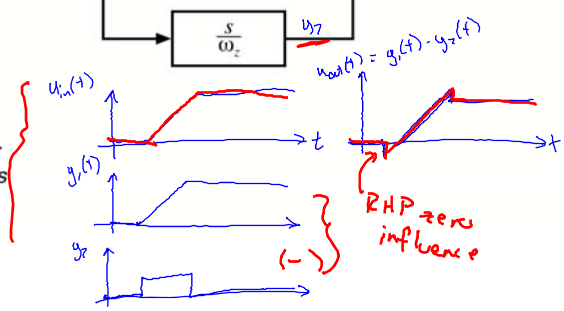


8.2.3. Physical origins of the right half-plane zero

$$G(s) = \left(1 - \frac{s}{\omega_z}\right)$$



- *phase reversal at high frequency*
- *transient response: output initially tends in wrong direction*



Two converters whose CCM control-to-output transfer functions exhibit RHP zeroes

if $d\uparrow$, $(1-d)\downarrow$

$\langle i_d \rangle \downarrow$

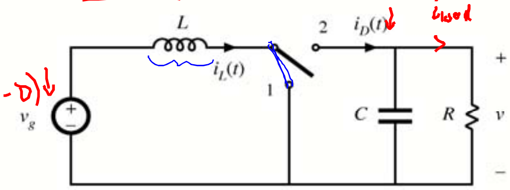
$V\downarrow$

$$\underbrace{\langle i_D \rangle_{T_s}}_{} = d' \langle i_L \rangle_{T_s} = (1-d) \langle i_L \rangle_{T_s}$$

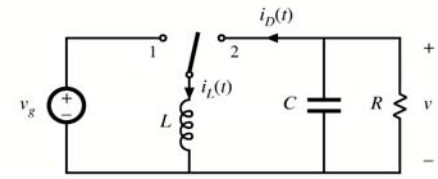
Boost

$V = \frac{V_g}{1-D}$ $D\uparrow$, $(1-D)\downarrow$

$V\uparrow$



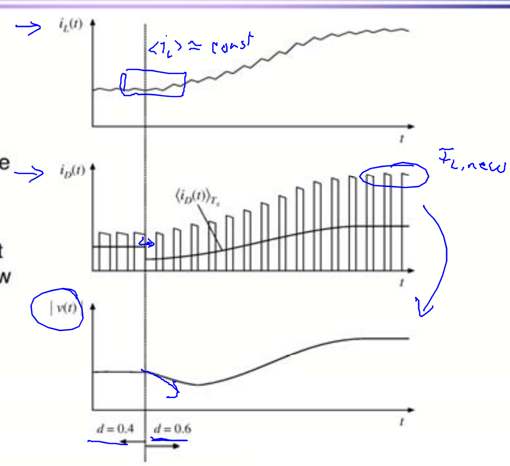
Buck-boost



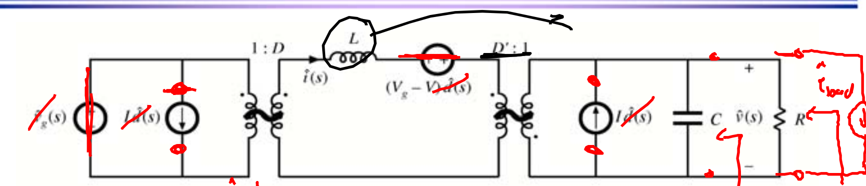
Waveforms, step increase in duty cycle

$$\langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s}$$

- Increasing $d(t)$ causes the average diode current to initially decrease
- As inductor current increases to its new equilibrium value, average diode current eventually increases



Buck-Boost Output Impedance



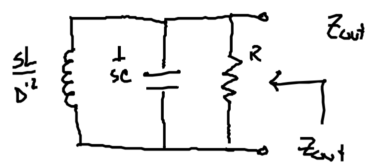
$D = 0.6$
$R = 10\Omega$
$V_g = 30\text{V}$
$L = 160\mu\text{H}$
$C = 160\mu\text{F}$

$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz

$G_{vd} = \frac{\hat{v}}{\hat{d}}\Big|$ All other small-signal inputs set to zero

$G_{vg} = \frac{\hat{v}}{\hat{v}_g}\Big|$

$Z_{out} = \frac{\hat{v}}{\hat{i}_{load}}\Big|_{\hat{v}_g = 0,\ \hat{d} = 0}$



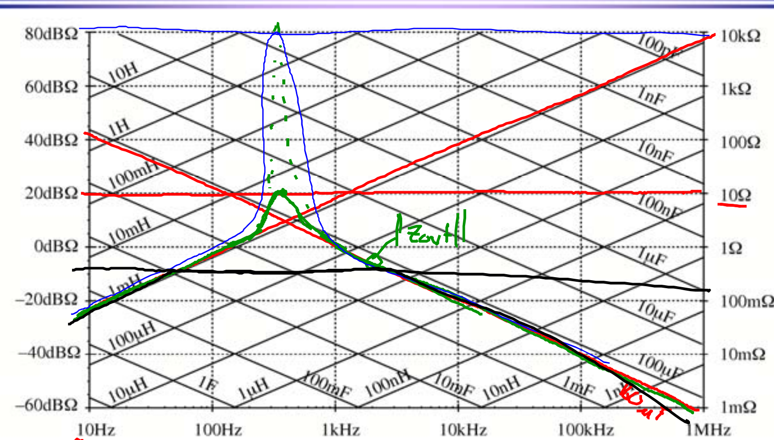
$Z_{out} = R \,\|\, \frac{1}{sC} \,\|\, \frac{sL}{D'^2}$

$D = 0.6$
$R = 10\Omega$
$V_g = 30\text{V}$
$L = 160\mu\text{H}$
$C = 160\mu\text{F}$

$X_1 \| X_2 \approx \begin{cases} X_1 & \text{if } X_1 < X_2 \\ X_2 & \text{if } X_2 < X_1 \end{cases}$

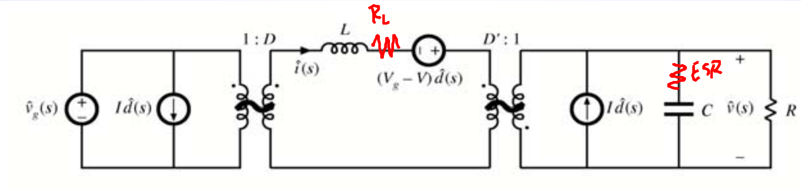
$X_1 + X_2 \approx \begin{cases} X_1 & \text{if } X_1 > X_2 \\ X_2 & \text{if } X_2 > X_1 \end{cases}$

Impedance graph paper



$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz

Nonideal Output Impedance



$D = 0.6$
$R = 10\Omega$
$V_g = 30\text{V}$
$L = 160\mu\text{H}$
$C = 160\mu\text{F}$

$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz
$R_L = 316\text{m}\Omega = -10\text{dB}\Omega$
ESR = $10\text{m}\Omega = -40\text{dB}\Omega$

$D = 0.6$
$R = 10\Omega$
$V_g = 30\text{V}$
$L = 160\mu\text{H}$
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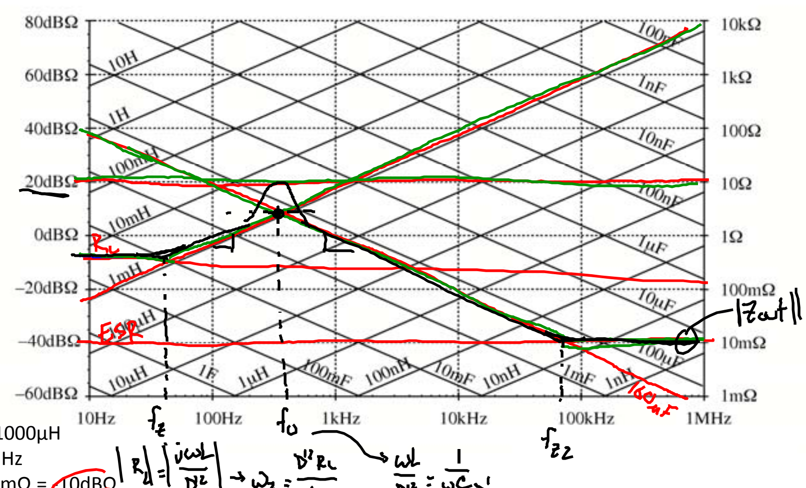
Nonideal Output Impedance



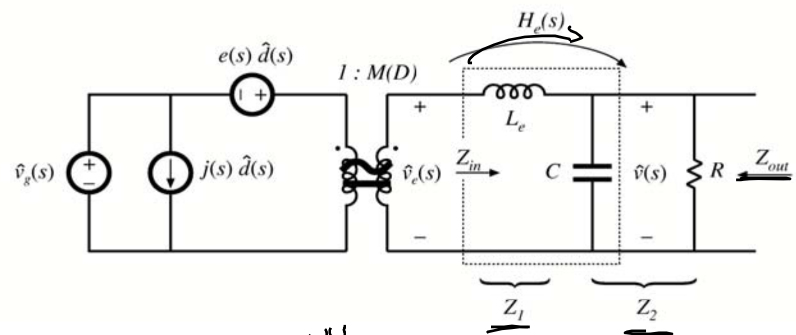
$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz
$R_L = 316\text{m}\Omega = 10\text{dB}\Omega$
ESR = $10\text{m}\Omega = -40\text{dB}\Omega$

$|R_L| = \left|\frac{j\omega L}{D'^2}\right| \rightarrow \omega_z = \frac{D'^2 R_L}{L}$

$\frac{\omega L}{D'^2} = \frac{1}{\omega C}$

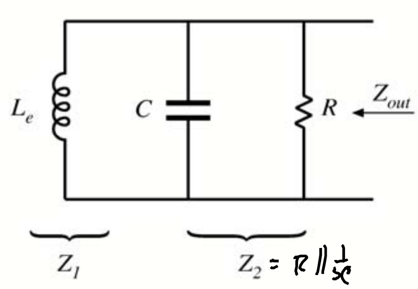
$\omega_0 = \frac{D'}{\sqrt{LC}}$

Transfer functions predicted by canonical model



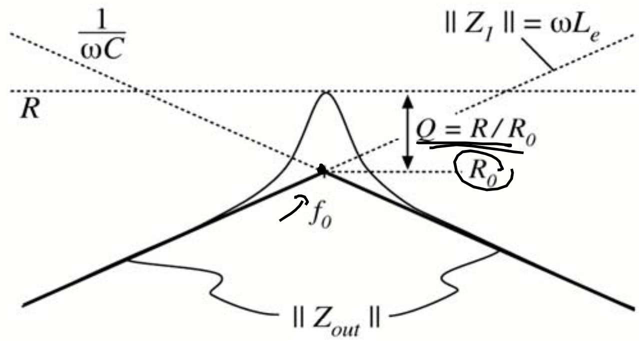
$$H_e = \frac{R \parallel \frac{1}{sC}}{R \parallel \frac{1}{sC} + sL_e}$$

Output impedance Z_{out}: set sources to zero

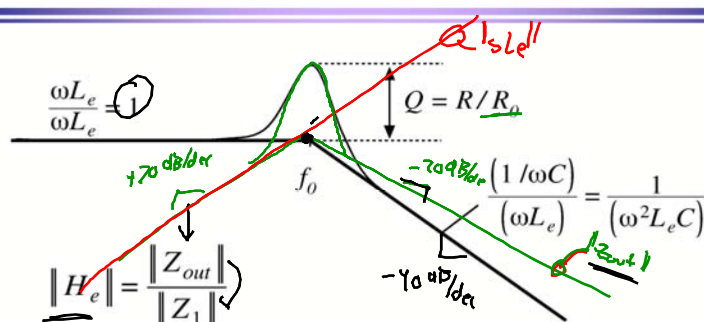


$$Z_{out} = Z_1 \parallel Z_2$$

Graphical construction of output impedance

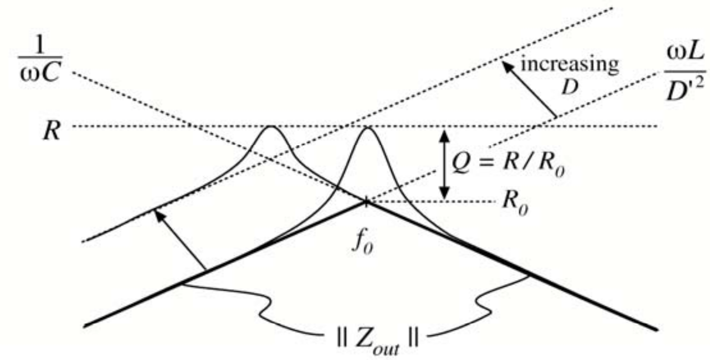


Graphical construction of filter effective transfer function



$$\boxed{\log\left(\frac{x_1}{x_2}\right) = \log(x_1) - \log(x_2)}$$

Boost and buck-boost converters: $L_e = L / D'^2$



Chapter 9. Controller Design

9.1. Introduction

9.2. Effect of negative feedback on the network transfer functions

- 9.2.1. Feedback reduces the transfer function from disturbances to the output
- 9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$ and the closed-loop transfer functions

Controller design

9.4. Stability
- 9.4.1. The phase margin test
- 9.4.2. The relation between phase margin and closed-loop damping factor
- 9.4.3. Transient response vs. damping factor

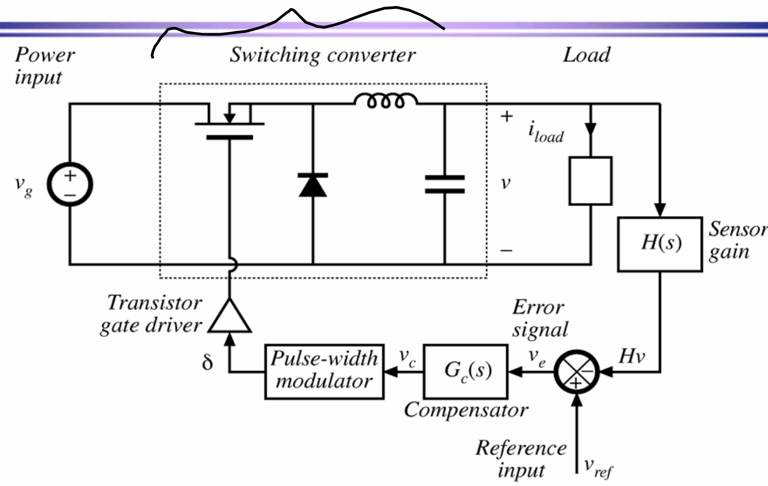
9.5. Regulator design
- 9.5.1. Lead (PD) compensator
- 9.5.2. Lag (PI) compensator
- 9.5.3. Combined (PID) compensator
- 9.5.4. Design example

Controller design

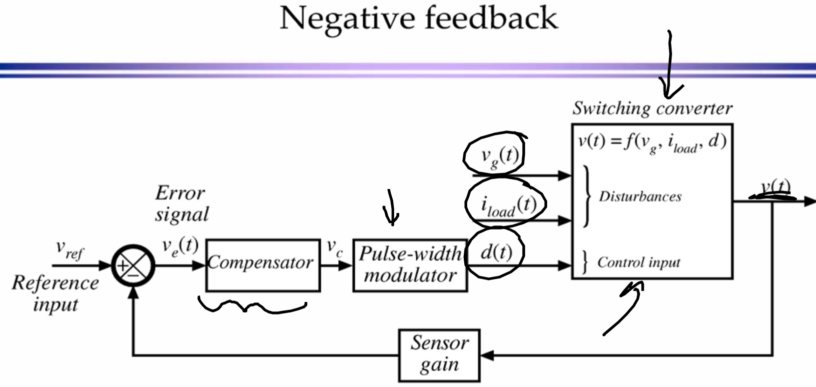
9.6. Measurement of loop gains
- 9.6.1. Voltage injection
- 9.6.2. Current injection
- 9.6.3. Measurement of unstable systems

9.7. Summary of key points

Negative feedback: a switching regulator system



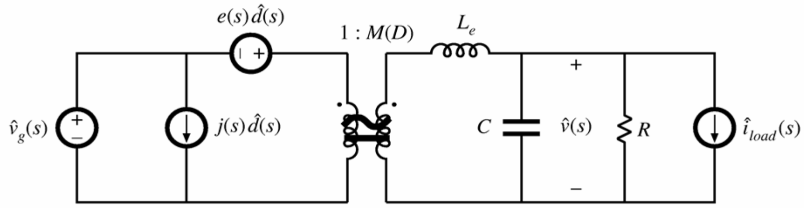
Negative feedback



H(s)
often just a Gain
may have some filtering

9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter



Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s) - Z_{out}(s)\,\hat{i}_{load}(s)$$

where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}} \qquad Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

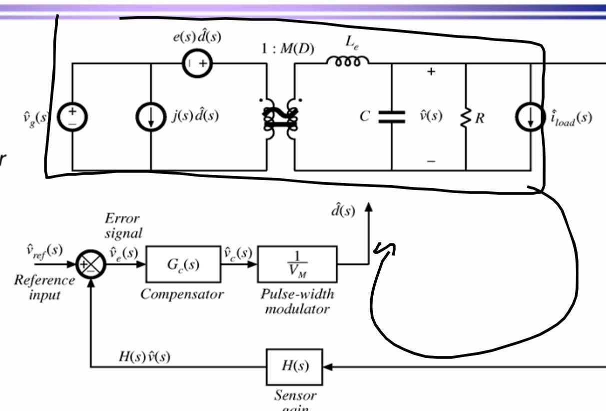
Voltage regulator system small-signal model

- *Use small-signal converter model*
- *Perturb and linearize remainder of feedback loop:*

$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$

$v_e(t) = V_e + \hat{v}_e(t)$

etc.



(1) Very large DC gain gives $\hat{v} \approx \frac{1}{H} V_{ref}$

Regulator system small-signal block diagram

e.g. $\left.\frac{\hat{v}}{\hat{v}_{ref}}\right|_{\hat{v}_g=0,\ \hat{i}_{load}=0} =$

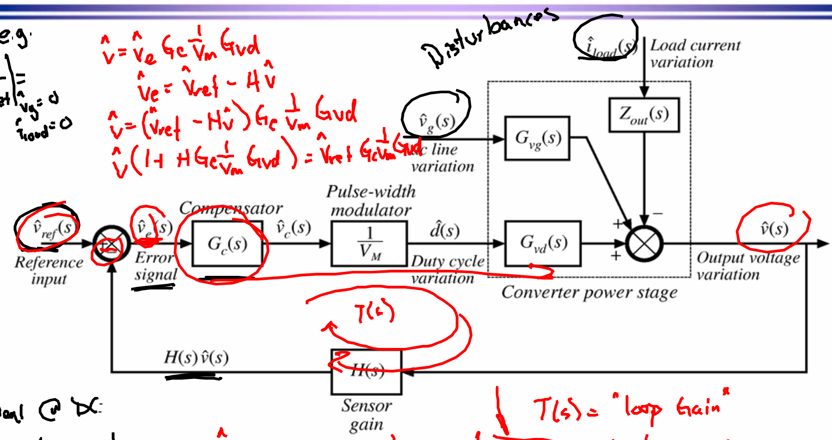
$\hat{v} = \hat{v}_e G_c \frac{1}{V_m} G_{vd}$

$\hat{v}_e = \hat{v}_{ref} - H\hat{v}$

$\hat{v} = (\hat{v}_{ref} - H\hat{v}) G_c \frac{1}{V_m} G_{vd}$

$\hat{v}\left(1 + H G_c \frac{1}{V_m} G_{vd}\right) = \hat{v}_{ref} G_c \frac{1}{V_m} G_{vd}$

Disturbances



Ideal @ DC:

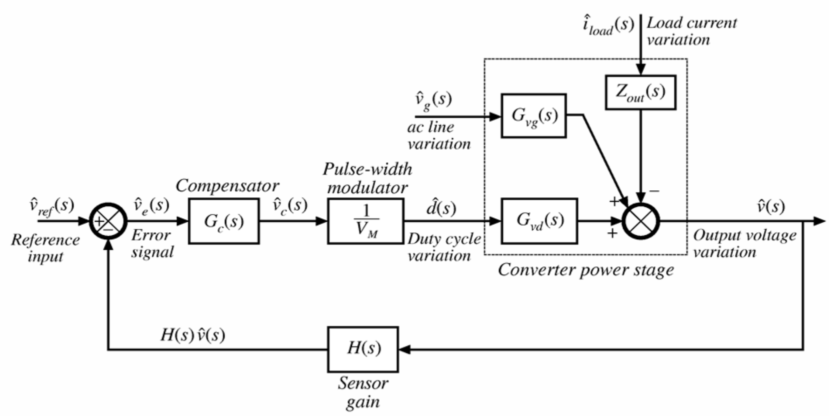
$v_e = \emptyset \quad V = \frac{1}{H} V_{ref}$

$\frac{V}{V_{ref}} = \frac{1}{H}$

$T(s) =$ "loop Gain" $= G_c \frac{1}{V_m} G_{vd} H$

$$\left.\frac{\hat{v}}{\hat{v}_{ref}}\right|_{\hat{v}_g=0,\ \hat{i}_{load}=0} = \frac{G_c \frac{1}{V_m} G_{vd}}{1 + H G_c \frac{1}{V_m} G_{vd}} = \boxed{\frac{1}{H}\frac{T}{1+T}}$$

Closed Loop Reference-to-Output



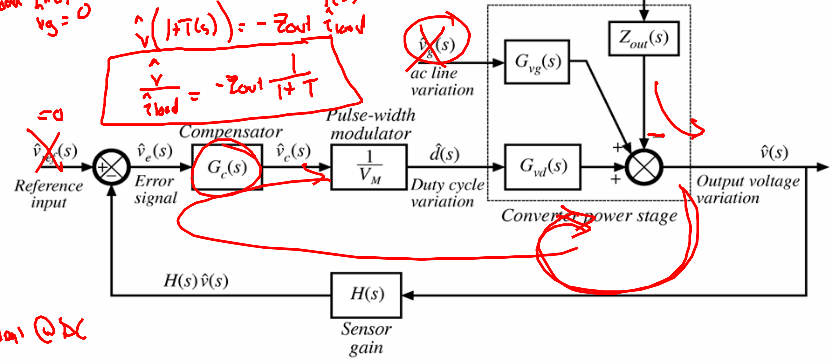
Closed Loop Disturbance-to-Output

$\frac{\hat{v}}{\hat{i}_{load}}$ $\hat{v}_{ref} = 0$, $\hat{v}_g = 0$

$\hat{v} = -Z_{out}\,\hat{i}_{load} + \hat{v}\,\underbrace{H G_c \frac{1}{V_m} G_{vd}}_{T(s)}$

$\hat{v}\left(1+T(s)\right) = -Z_{out}\,\hat{i}_{load}$

$$\boxed{\frac{\hat{v}}{\hat{i}_{load}} = -Z_{out}\frac{1}{1+T}}$$



Ideal @ DC

$\frac{\partial V}{\partial I_{load}} = \phi$

(if T is very large, disturbances have little effect on output)

Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref}\frac{G_c G_{vd}/V_M}{1+HG_cG_{vd}/V_M} + \hat{v}_g\frac{G_{vg}}{1+HG_cG_{vd}/V_M} - \hat{i}_{load}\frac{Z_{out}}{1+HG_cG_{vd}/V_M}$$

which is of the form

$$\boxed{\hat{v} = \hat{v}_{ref}\frac{1}{H}\cancel{\frac{T}{1+T}}^{\to 1} + \hat{v}_g\cancel{\frac{G_{vg}}{1+T}}^{\to \phi} - \hat{i}_{load}\cancel{\frac{Z_{out}}{1+T}}^{\to \phi}}$$

@ f such that T is "very large"

with $T(s) = H(s)\,G_c(s)\,G_{vd}(s)/V_M$ = "*loop gain*"

$\hat{v} \approx \hat{v}_{ref}\frac{1}{H}$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}}$$

With addition of negative feedback, the line-to-output transfer function becomes:

$$\left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{i}_{load} = 0}} = \frac{G_{vg}(s)}{1 + T(s)}$$

Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the line-to-output transfer function becomes small.

Closed-loop output impedance

Original (open-loop) output impedance:

$$Z_{out}(s) = - \left. \frac{\hat{v}(s)}{\hat{i}_{load}(s)} \right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

With addition of negative feedback, the output impedance becomes:

$$\left. \frac{\hat{v}(s)}{- \hat{i}_{load}(s)} \right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{v}_g = 0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

Feedback reduces the output impedance by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the output impedance is greatly reduced in magnitude.

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

If the loop gain is large in magnitude, i.e., $\| T \| >> 1$, then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

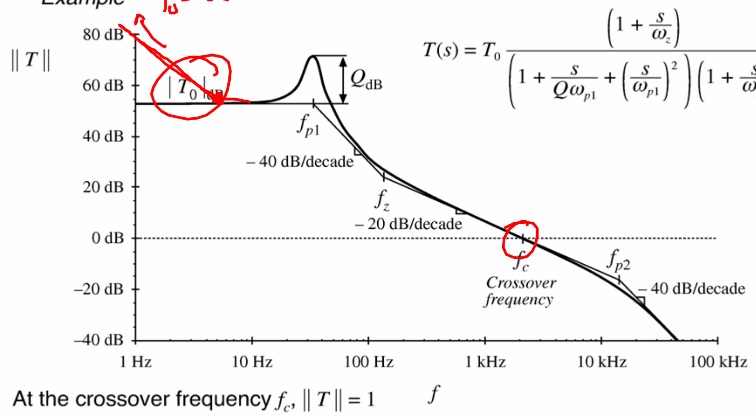
$$\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}$$

9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$

Example

$T_0 \to \infty$

$$T(s) = T_0 \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_{p1}} + \left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



At the crossover frequency f_c, $\| T \| = 1$

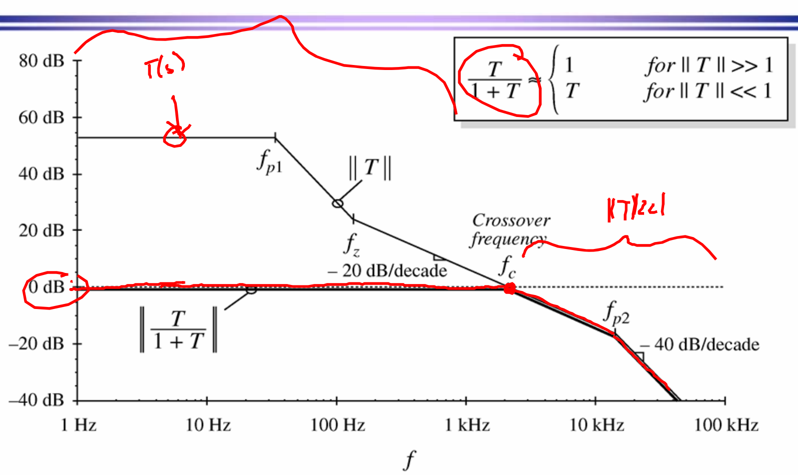
Approximating $1/(1+T)$ and $T/(1+T)$

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } \| T \| \gg 1 \\ T & \text{for } \| T \| \ll 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & \text{for } \| T \| \gg 1 \\ 1 & \text{for } \| T \| \ll 1 \end{cases}$$

Example: construction of $T/(1+T)$

|T|>>1



Example: analytical expressions for approximate reference to output transfer function

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \approx \frac{1}{H(s)}$$

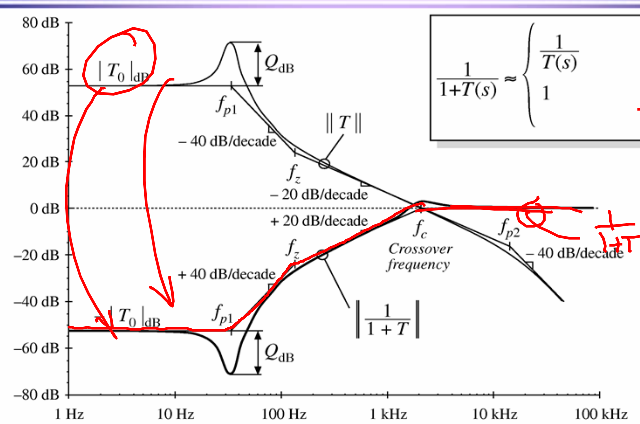
This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T/(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s)G_{vd}(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

Same example: construction of 1/(1+T)



$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| \gg 1 \\ 1 & \text{for } \| T \| \ll 1 \end{cases}$$