Lecture 18: Z_{out}, Closed Loop Converters

ECE 481: Power Electronics

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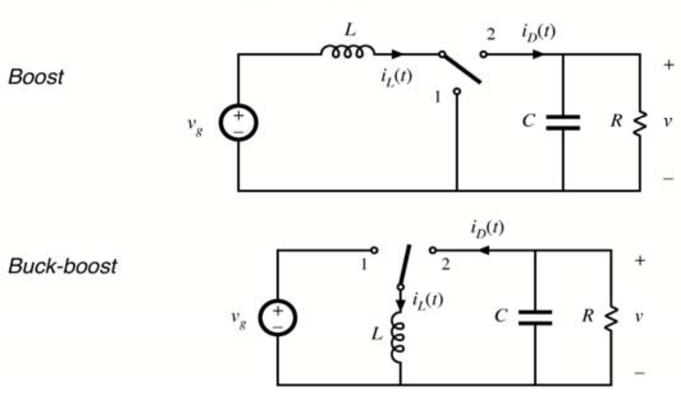
Fall 2013

Two converters whose CCM control-to-output transfer functions exhibit RHP zeroes

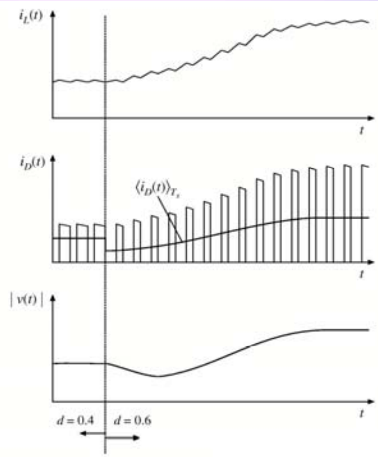
$$\langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s}$$



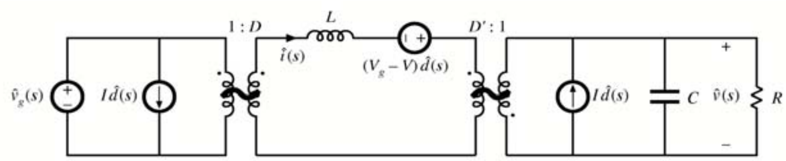
Waveforms, step increase in duty cycle

$$\langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s}$$

- Increasing $d(t)$ causes the average diode current to initially decrease
- As inductor current increases to its new equilibrium value, average diode current eventually increases



Buck-Boost Output Impedance



$D = 0.6$
$R = 10\Omega$
$V_g = 30\text{V}$
$L = 160\mu\text{H}$
$C = 160\mu\text{F}$

$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz

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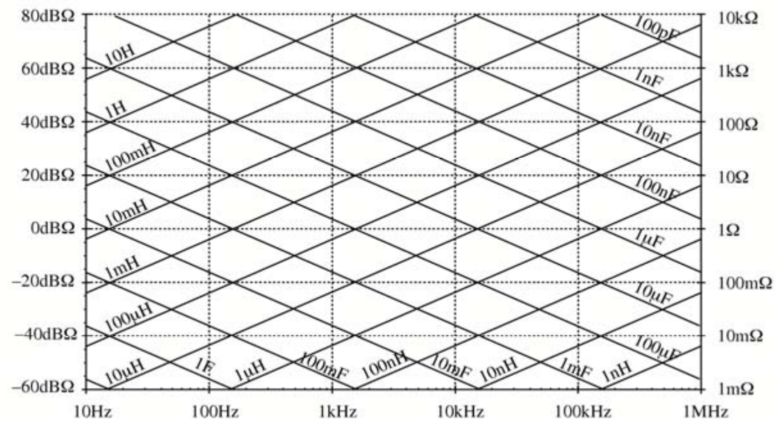
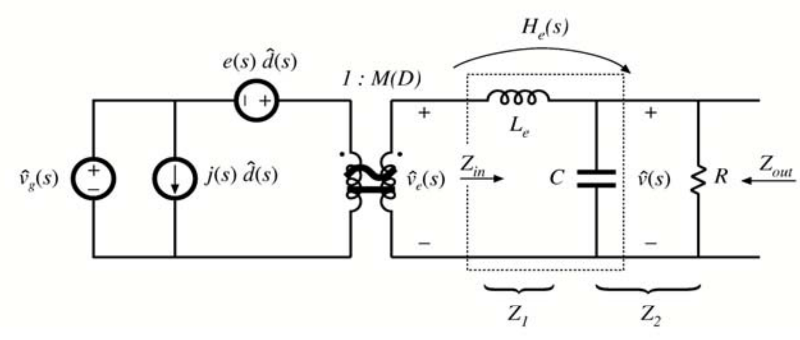
$L/D'^2 = 1000\mu\text{H}$
$f_o = 400$ Hz

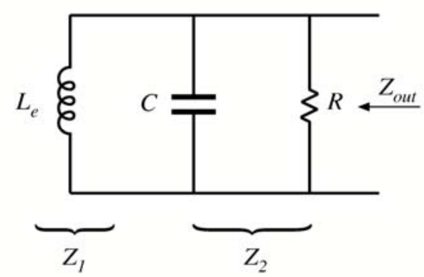
Impedance graph paper



Transfer functions predicted by canonical model

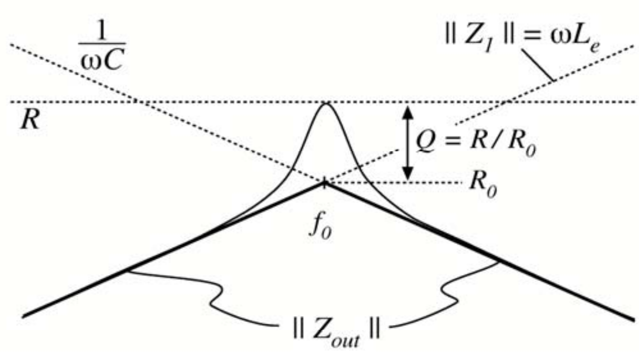


Output impedance Z_{out}: set sources to zero

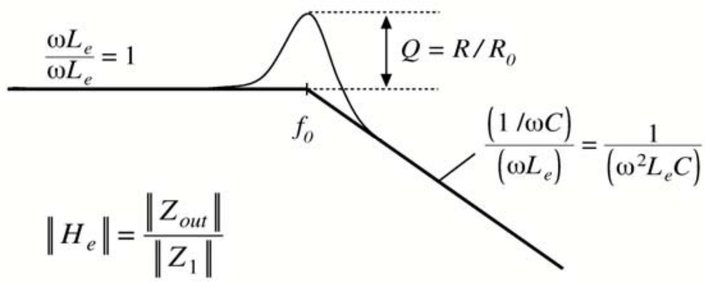


$$Z_{out} = Z_1 \parallel Z_2$$

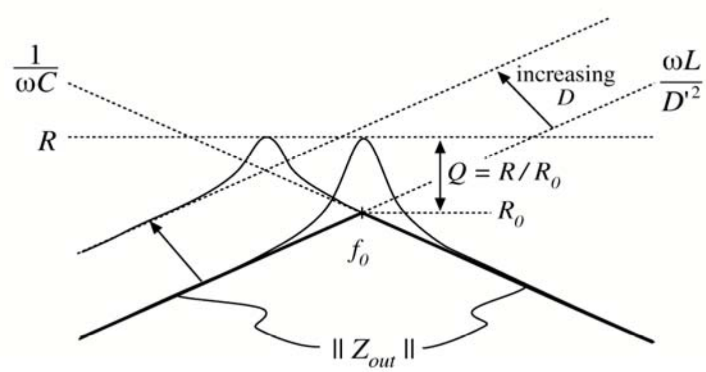
Graphical construction of output impedance



Graphical construction of filter effective transfer function



Boost and buck-boost converters: $L_e = L / D'^2$



Chapter 9. Controller Design

9.1. Introduction

9.2. Effect of negative feedback on the network transfer functions

9.2.1. Feedback reduces the transfer function from disturbances to the output

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$ and the closed-loop transfer functions

Controller design

9.4. Stability

9.4.1. The phase margin test

9.4.2. The relation between phase margin and closed-loop damping factor

9.4.3. Transient response vs. damping factor

9.5. Regulator design

9.5.1. Lead (PD) compensator

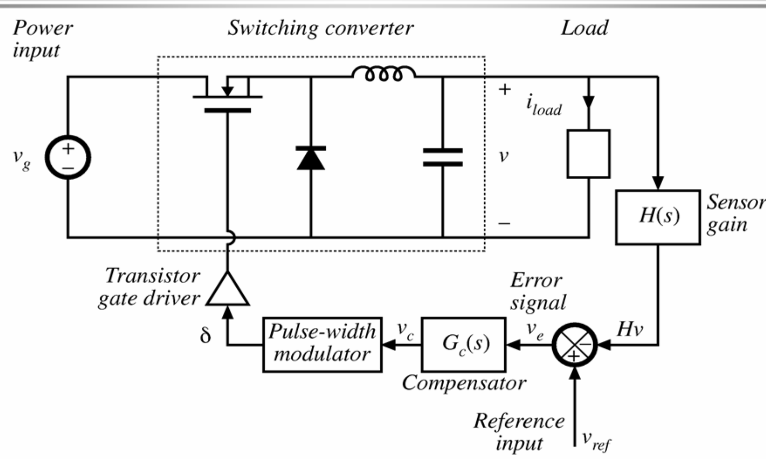
9.5.2. Lag (PI) compensator

9.5.3. Combined (PID) compensator

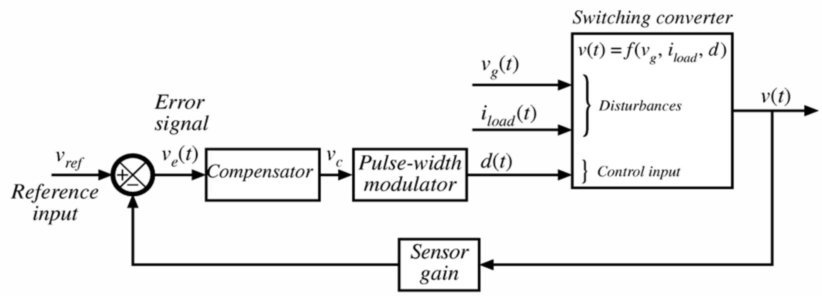
9.5.4. Design example

Controller design

Negative feedback:
a switching regulator system

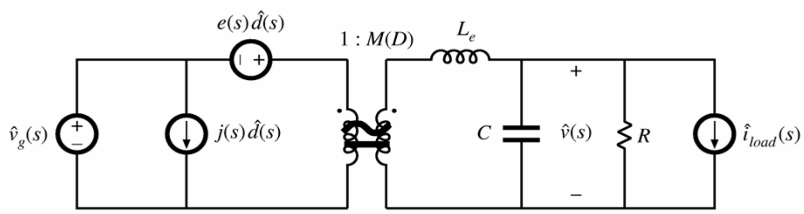


Negative feedback



9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter



Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s) - Z_{out}(s)\,\hat{i}_{load}(s)$$

where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}} \qquad Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

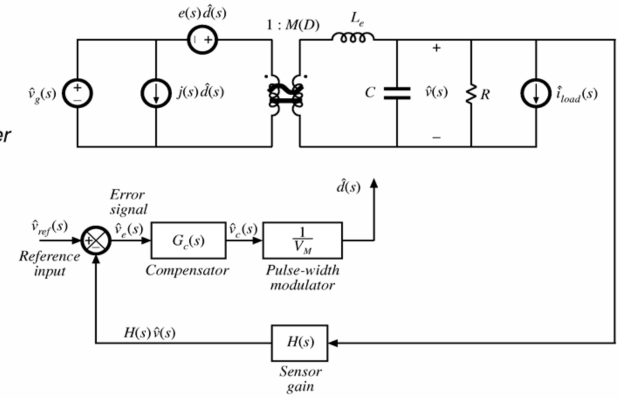
Voltage regulator system small-signal model

- *Use small-signal converter model*
- *Perturb and linearize remainder of feedback loop:*

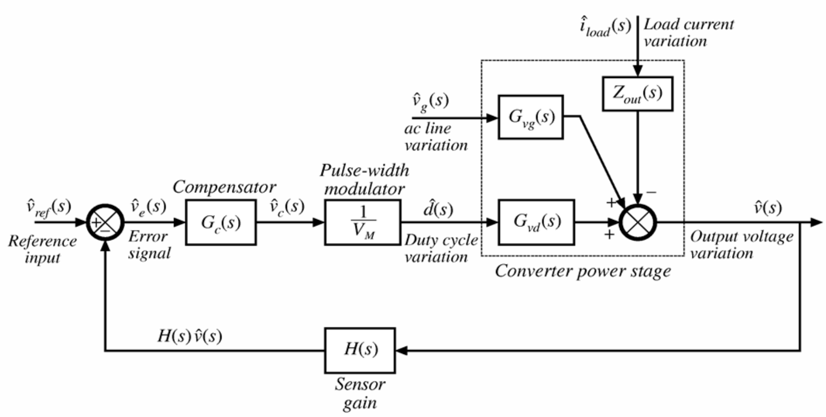
$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$

$v_e(t) = V_e + \hat{v}_e(t)$

etc.



Regulator system small-signal block diagram



Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

which is of the form

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} - \hat{i}_{load} \frac{Z_{out}}{1+T}$$

$$with \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M \quad = \text{"loop gain"}$$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

$$G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{d}=0 \\ \hat{i}_{load}=0}}$$

With addition of negative feedback, the line-to-output transfer function becomes:

$$\left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\substack{\hat{v}_{ref}=0 \\ \hat{i}_{load}=0}} = \frac{G_{vg}(s)}{1 + T(s)}$$

Feedback reduces the line-to-output transfer function by a factor of

$$\frac{1}{1 + T(s)}$$

If $T(s)$ is large in magnitude, then the line-to-output transfer function becomes small.

Closed-loop output impedance

Original (open-loop) output impedance:

$$Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d}=0\\ \hat{v}_g=0}}$$

With addition of negative feedback, the output impedance becomes:

$$\left.\frac{\hat{v}(s)}{-\hat{i}_{load}(s)}\right|_{\substack{\hat{v}_{ref}=0\\ \hat{v}_g=0}} = \frac{Z_{out}(s)}{1+T(s)}$$

Feedback reduces the output impedance by a factor of

$$\frac{1}{1+T(s)}$$

If $T(s)$ is large in magnitude, then the output impedance is greatly reduced in magnitude.

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g=0\\ \hat{i}_{load}=0}} = \frac{1}{H(s)}\,\frac{T(s)}{1+T(s)}$$

If the loop gain is large in magnitude, i.e., $\| T \| >> 1$, then $(1+T) \approx T$ and $T/(1+T) \approx T/T = 1$. The transfer function then becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}$$

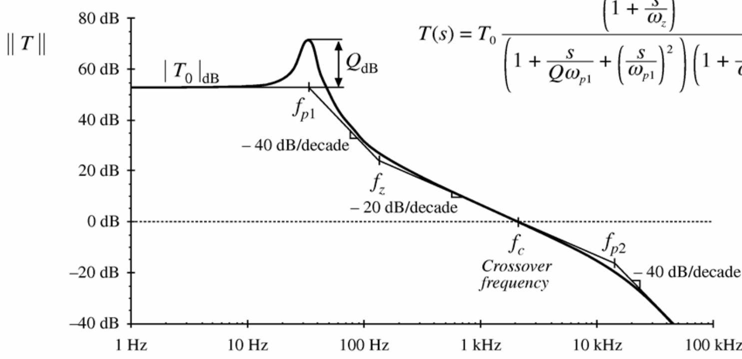
which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

$$\frac{V}{V_{ref}} = \frac{1}{H(0)}\,\frac{T(0)}{1+T(0)} \approx \frac{1}{H(0)}$$

9.3. Construction of the important quantities 1/(1+T) and T/(1+T)

Example



$$T(s) = T_0 \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_{p1}} + \left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

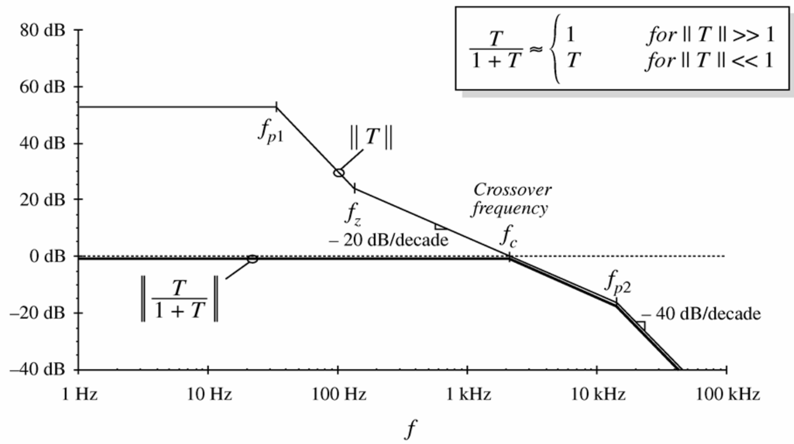
At the crossover frequency f_c, $\| T \| = 1$

Approximating 1/(1+T) and T/(1+T)

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } \| T \| >> 1 \\ T & \text{for } \| T \| << 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Example: construction of $T/(1+T)$



Example: analytical expressions for approximate reference to output transfer function

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{1}{H(s)}$$

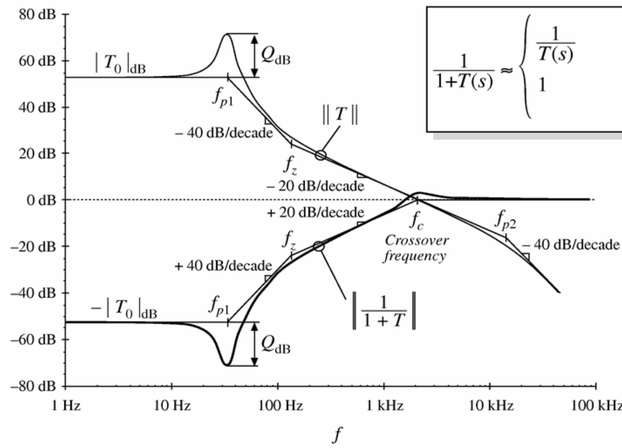
This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T/(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s) G_{vd}(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

Same example: construction of 1/(1+T)



$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Interpretation: how the loop rejects disturbances

Below the crossover frequency: $f < f_c$ and $\| T \| > 1$

Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\| T \|$

Above the crossover frequency: $f > f_c$ and $\| T \| < 1$

Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Terminology: open-loop vs. closed-loop

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$$G_{vd}(s) \qquad G_{vg}(s) \qquad Z_{out}(s)$$

Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \qquad \frac{G_{vg}(s)}{1+T(s)} \qquad \frac{Z_{out}(s)}{1+T(s)}$$

The loop gain:

$$T(s)$$

Determination of stability directly from $T(s)$

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

  Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

  A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

9.4.1. The phase margin test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

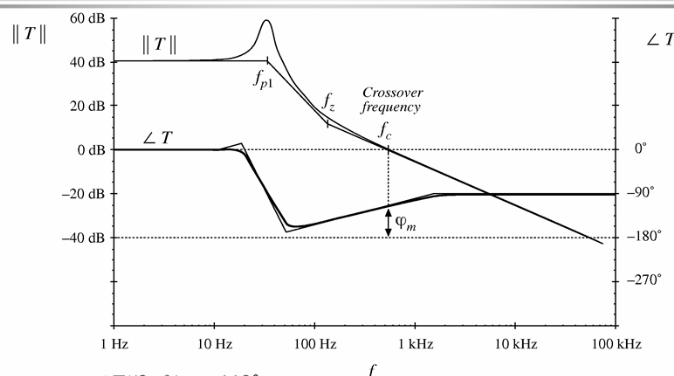
The phase margin φ_m is determined from the phase of $T(s)$ at f_c, as follows:

$$\varphi_m = 180^\circ + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

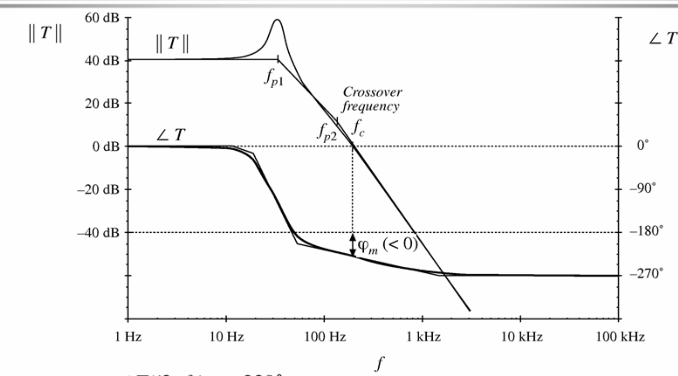
Example: a loop gain leading to a stable closed-loop system



$\angle T(j2\pi f_c) = -112^\circ$

$\varphi_m = 180^\circ - 112^\circ = +68^\circ$

Example: a loop gain leading to an unstable closed-loop system



$\angle T(j2\pi f_c) = -\ 230^\circ$

$\varphi_m = 180^\circ - 230^\circ = -\ 50^\circ$

9.4.2. The relation between phase margin and closed-loop damping factor

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

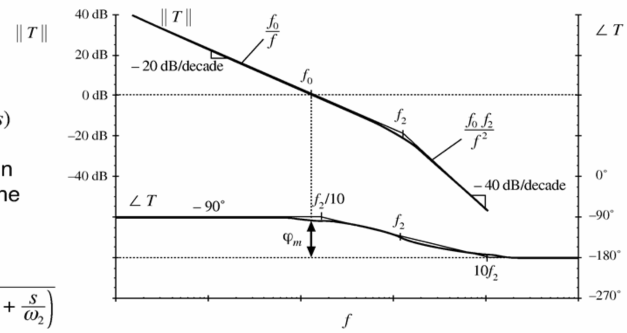
Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop Q is quantified in this section.

A simple second-order system

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$



Closed-loop response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0 \omega_2}}$$

or,

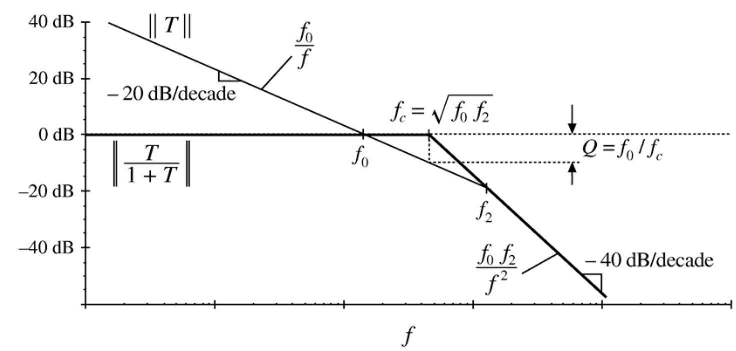
$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

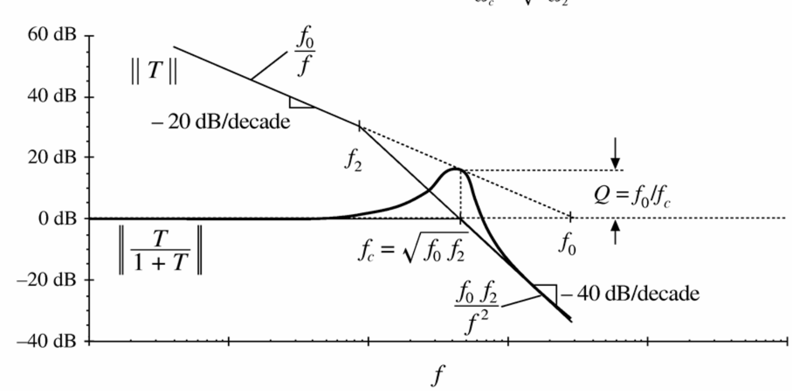
Low-Q case

$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$ low-Q approximation: $Q\,\omega_c = \omega_0$ $\frac{\omega_c}{Q} = \omega_2$



High-Q case

$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c$ $Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$



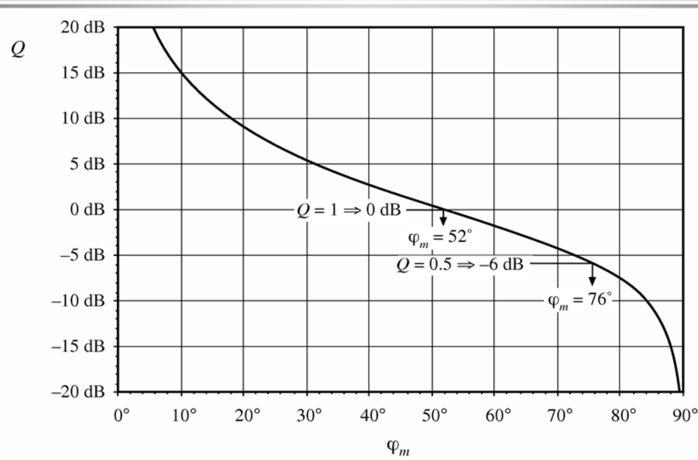
Q vs. φ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}$$

Q vs. φ_m



9.4.3. Transient response vs. damping factor

Unit-step response of second-order system $T(s)/(1+T(s))$

$$\hat{v}(t) = 1 + \frac{2Q\, e^{-\omega_c t/2Q}}{\sqrt{4Q^2-1}} \sin\left[\frac{\sqrt{4Q^2-1}}{2Q}\,\omega_c\, t + \tan^{-1}\left(\sqrt{4Q^2-1}\right)\right] \qquad Q > 0.5$$

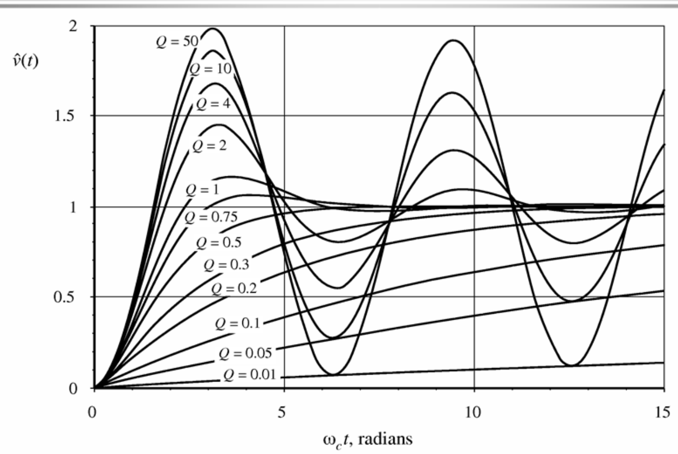
$$\hat{v}(t) = 1 - \frac{\omega_2}{\omega_2-\omega_1} e^{-\omega_1 t} - \frac{\omega_1}{\omega_1-\omega_2} e^{-\omega_2 t} \qquad Q < 0.5$$

$$\omega_1,\, \omega_2 = \frac{\omega_c}{2Q}\left(1 \pm \sqrt{1-4Q^2}\right)$$

For $Q > 0.5$, the peak value is

$$peak\ \hat{v}(t) = 1 + e^{-\pi/\sqrt{4Q^2-1}}$$

Transient response vs. damping factor



9.5. Regulator design

Typical specifications:

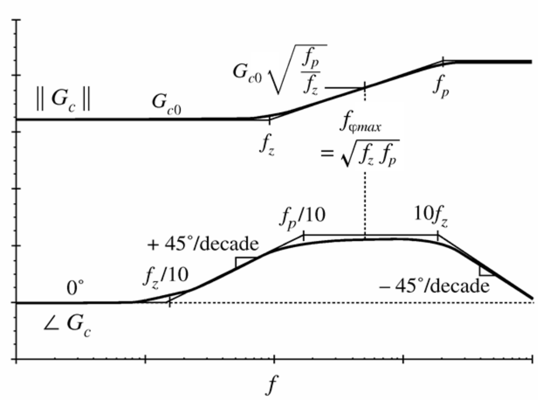
- Effect of load current variations on output voltage regulation
  - This is a limit on the maximum allowable output impedance
- Effect of input voltage variations on the output voltage regulation
  - This limits the maximum allowable line-to-output transfer function
- Transient response time
  - This requires a sufficiently high crossover frequency
- Overshoot and ringing
  - An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

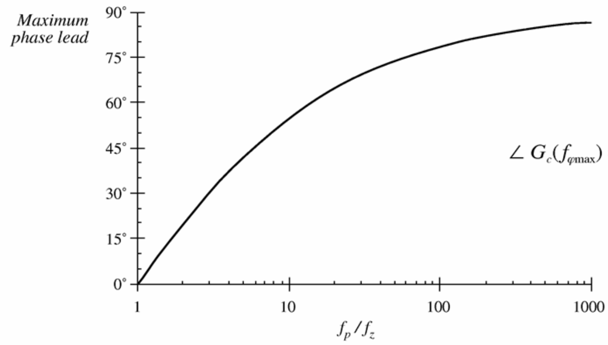
9.5.1. Lead (PD) compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Lead compensator: maximum phase lead



$$f_{\varphi\max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi\max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin\left(\theta\right)}{1 - \sin\left(\theta\right)}$$

Lead compensator design

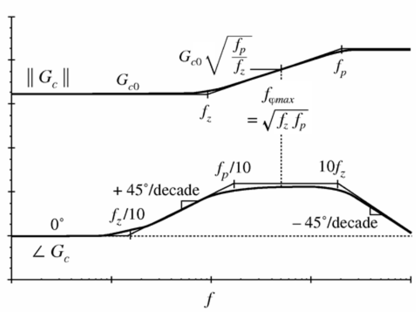
To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin\left(\theta\right)}{1 + \sin\left(\theta\right)}}$$

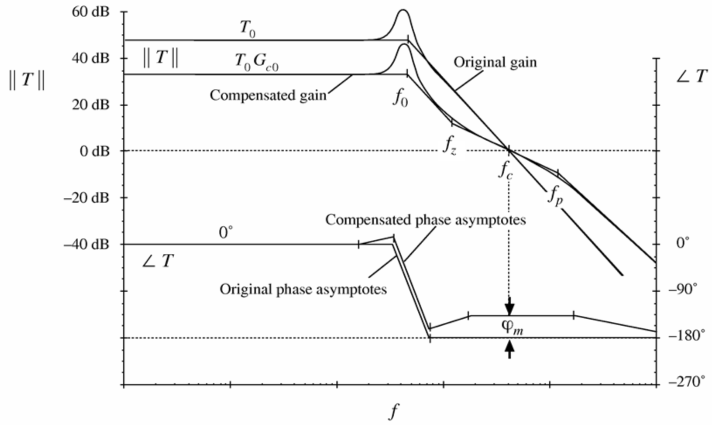
$$f_p = f_c \sqrt{\frac{1 + \sin\left(\theta\right)}{1 - \sin\left(\theta\right)}}$$

If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



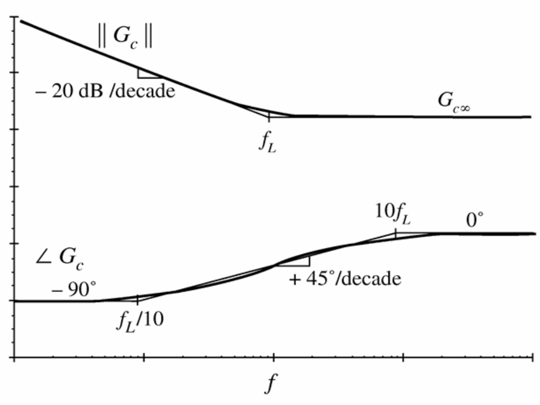
Example: lead compensation



9.5.2. Lag (PI) compensation

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



Example: lag compensation

original (uncompensated) loop gain is

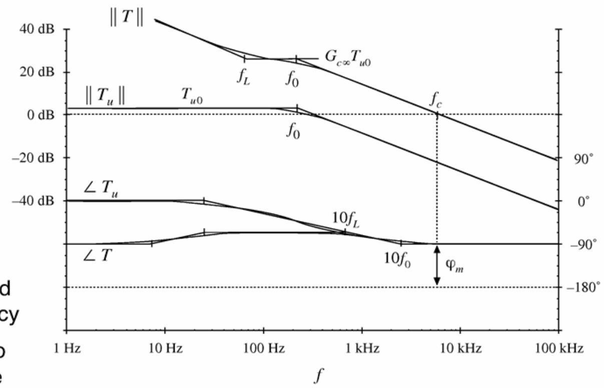
$$T_u(s) = \frac{T_{u0}}{\left(1 + \frac{s}{\omega_0}\right)}$$

compensator:

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Design strategy: choose

- $G_{c\infty}$ to obtain desired crossover frequency
- ω_L sufficiently low to maintain adequate phase margin



9.5.3. Combined (PID) compensator

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$