Lecture 19: Stability

ECE 481: Power Electronics

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Announcements

- HW #8 due Tuesday, Nov 12
- Midterm Exam #2 posted on Tuesday
- Final Exam Schedule

Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

which is of the form

$$\boxed{\hat{v} = \hat{v}_{ref} \frac{1}{H} \cancel{\frac{T}{1+T}} \to 1 + \hat{v}_g \cancel{\frac{G_{vg}}{1+T}} \to 0 - \hat{i}_{load} \cancel{\frac{Z_{out}}{1+T}} \to 0}$$

@ f such that T is "very large"

$$with \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M \quad = \text{"loop gain"}$$

$\to \hat{v} \approx \hat{v}_{ref} \frac{1}{H}$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

@ f such that T is small

$$\hat{v} = \underbrace{\cancel{\hat{v}_{ref} \frac{T}{H}}}_{T \text{ is small}} \to 0 + \hat{v}_g G_{vg} - \hat{i}_{load} Z_{out}$$

No attenuation of disturbances

Poor tracking of reference

Approximating $1/(1+T)$ and $T/(1+T)$

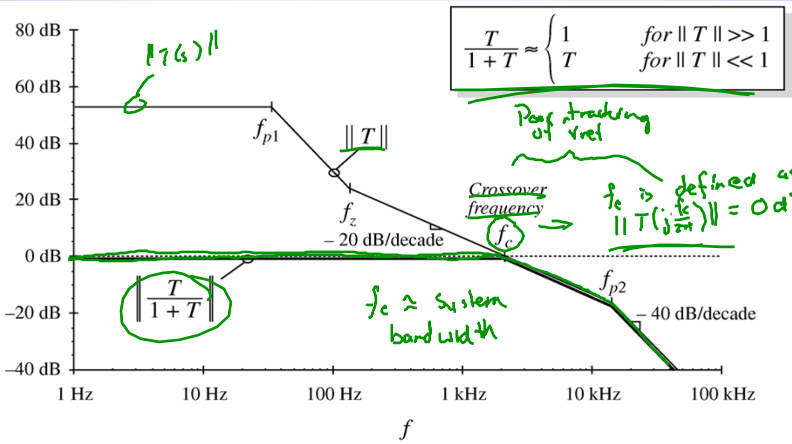
$$\frac{T}{1+T} \approx \begin{cases} 1 & for \; \| T \| >> 1 \\ T & for \; \| T \| << 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & for \; \| T \| >> 1 \\ 1 & for \; \| T \| << 1 \end{cases}$$

$\frac{\hat{v}}{\hat{v}_{ref}} = \underbrace{\frac{1}{H}}_{\text{Ideal Gain}} \frac{T}{1+T}$

Example: construction of $T/(1+T)$

H = 1



Example: analytical expressions for approximate reference to output transfer function

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \approx \frac{1}{H(s)}$$

This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

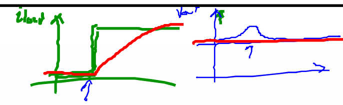
At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T/(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1+T(s)} \approx \frac{T(s)}{H(s)} = \frac{G_c(s) G_{vd}(s)}{V_M}$$

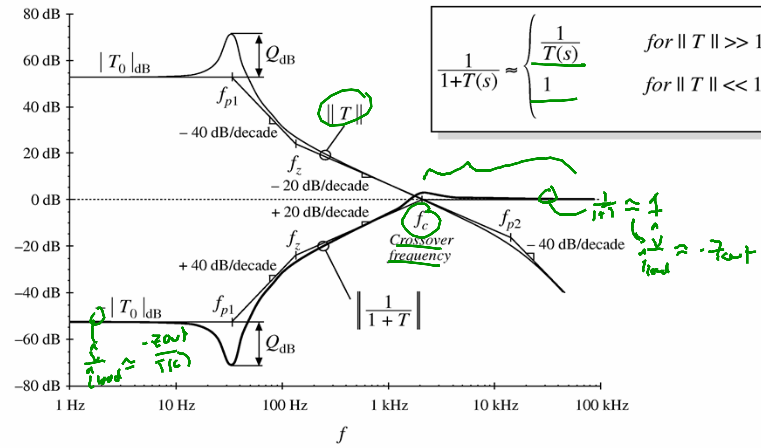
This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

$$\frac{\hat{v}}{\hat{i}_{load}} = -Z_{out}\frac{1}{1+T}$$

if $Z_{out} = 1$



Same example: construction of 1/(1+T)



$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Interpretation: how the loop rejects disturbances

Below the crossover frequency: $f < f_c$ and $\| T \| > 1$

- Then $1/(1+T) \approx 1/T$, and disturbances are reduced in magnitude by $1/\| T \|$

Above the crossover frequency: $f > f_c$ and $\| T \| < 1$

- Then $1/(1+T) \approx 1$, and the feedback loop has essentially no effect on disturbances

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & \text{for } \| T \| >> 1 \\ 1 & \text{for } \| T \| << 1 \end{cases}$$

Terminology: open-loop vs. closed-loop

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$\rightarrow$ $G_{vd}(s)$ $G_{vg}(s)$ $Z_{out}(s)$ $\rightarrow$ open loop

Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \quad \frac{G_{vg}(s)}{1+T(s)} \quad \frac{Z_{out}(s)}{1+T(s)} \rightarrow \text{closed loop}$$

The loop gain:

$T(s)$ $\rightarrow$ Designing with $G_c(s)$

f_c, φ_M defined for $T(s)$

Determination of stability directly from $T(s)$

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

  Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

  A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

  (1) f_c $\left(\|T(j\omega_c)\| = 0\,\text{dB} \right)$ is uniquely defined

  (2) $T(s)$ has no RHP poles

Defined for $T(s)$

9.4.1. The phase margin test

A test on $\boxed{T(s)}$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

The phase margin $\boxed{\varphi_m}$ is determined from the phase of $T(s)$ at f_c, as follows:

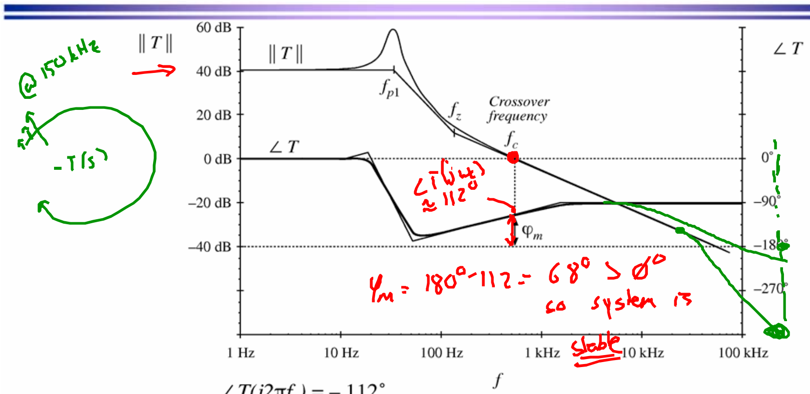
$$\boxed{\varphi_m = 180^\circ + \angle T(j2\pi f_c)}$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

> the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

System is stable if $\varphi_M > 0^\circ$

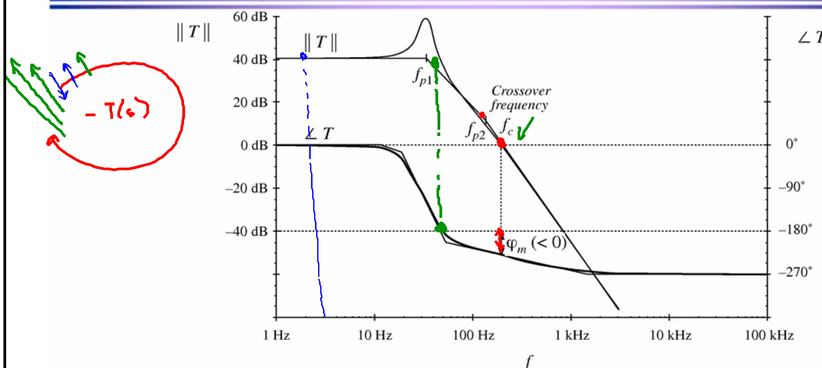
Example: a loop gain leading to a stable closed-loop system



$\angle T(j2\pi f_c) = -112^\circ$

$\varphi_m = 180^\circ - 112^\circ = +68^\circ$

Example: a loop gain leading to an unstable closed-loop system



$\angle T(j2\pi f_c) = -230°$

$\varphi_m = 180° - 230° = -50°$ < 0° → system is unstable

9.4.2. The relation between phase margin and closed-loop damping factor

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop Q is quantified in this section.

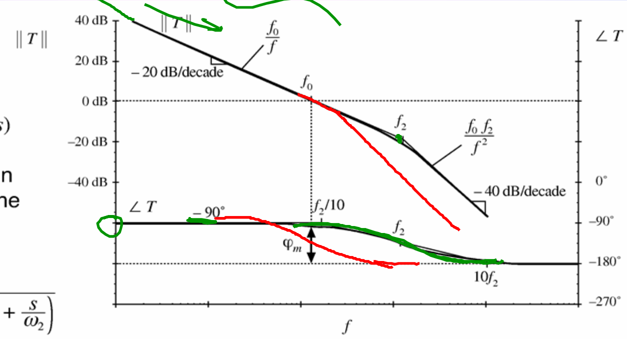
A simple second-order system

$\frac{1}{\frac{s}{\omega_0}}$

$\to T_0$

$\frac{T}{\left(\frac{s}{\omega_x}+1\right)}$ ω_x

$\frac{T}{\frac{s}{\omega_x}} \approx \frac{1}{\frac{s}{\omega_0}}$



Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Closed-loop response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1+T(s)} = \frac{1}{1+\frac{1}{T(s)}} = \frac{1}{1+\frac{s}{\omega_0}+\frac{s^2}{\omega_0\omega_2}}$$

or,

$$\frac{T(s)}{1+T(s)} = \frac{1}{1+\frac{s}{Q\omega_c}+\left(\frac{s}{\omega_c}\right)^2}$$

ω_c not necessarily equal to crossover frequency

where

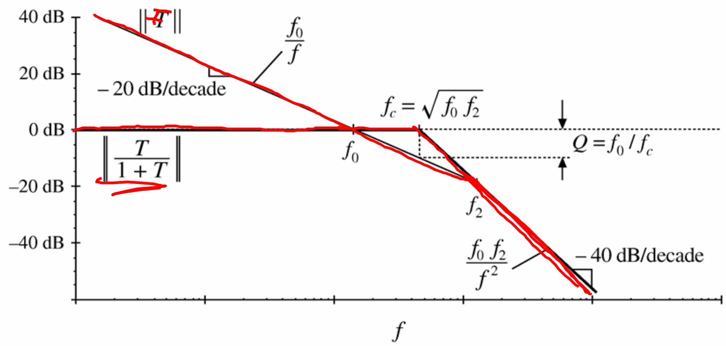
$$\omega_c = \sqrt{\omega_0\omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

if $\omega_0 >> \omega_2$ Q is large

if $\omega_0 << \omega_2$ Q is small

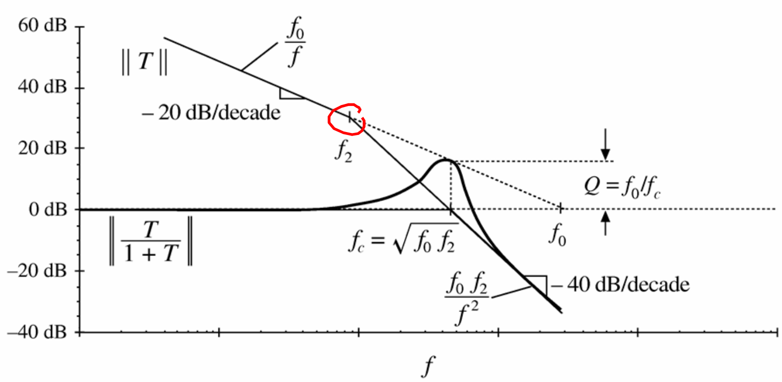
Low-Q case

$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$ low-Q approximation: $Q\,\omega_c = \omega_0$ $\frac{\omega_c}{Q} = \omega_2$



High-Q case

$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c$ $Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$



Q vs. φ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

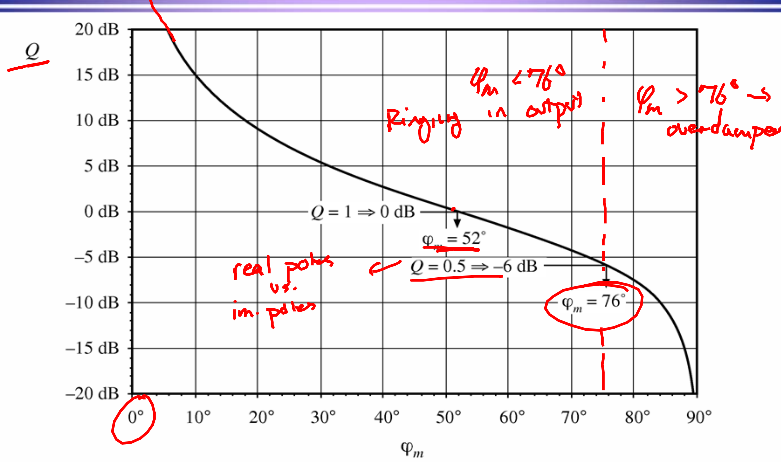
of closed loop system

$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

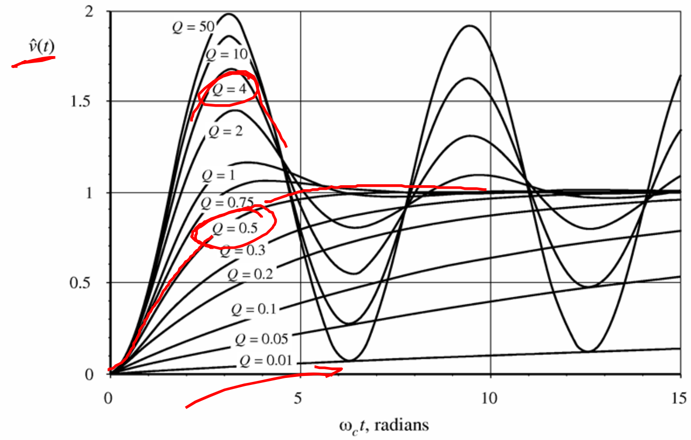
φ_m defined for $T(s)$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1+\sqrt{1+4Q^4}}{2Q^4}}$$

Q vs. φ_m



Transient response vs. damping factor



Shape $T(s)$ on a bode plot using $G_c(s)$

(1) Large $T(s)$ at low frequencies
(2) High $f_c \leftrightarrow (BW)_{CL}$
(3) Adequate φ_m

9.5. Regulator design

Typical specifications:

Reject Disturbance via (1) & (2)

- Minimize Effect of load current variations on output voltage regulation
  - This is a limit on the maximum allowable output impedance $- Z_{out} \frac{1}{1+T}$
- Minimize Effect of input voltage variations on the output voltage regulation
  - This limits the maximum allowable line-to-output transfer function $G_{vg} \frac{1}{1+T}$
- fast Transient response time $\rightarrow \uparrow f_c$
  - This requires a sufficiently high crossover frequency
- Low Overshoot and ringing $\rightarrow \uparrow \varphi_m$
  - An adequate phase margin must be obtained

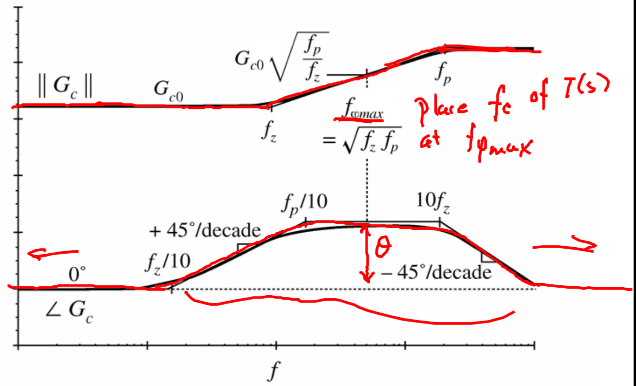
The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

9.5.1. Lead (PD) compensator

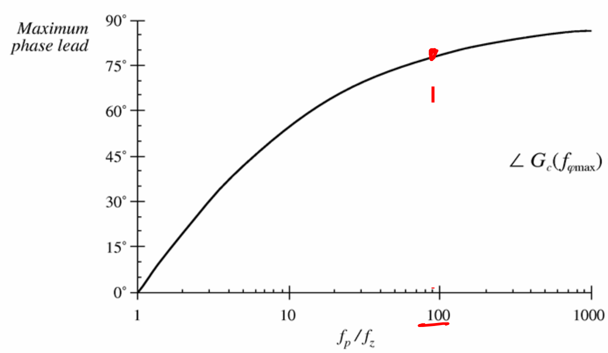
$$G_c(s) = G_{c0}\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$

$\omega_z < \omega_p$

Improves phase margin



Lead compensator: maximum phase lead



$$f_{\varphi\max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi\max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2}\right)$$

$$\frac{f_p}{f_z} = \frac{1+\sin(\theta)}{1-\sin(\theta)}$$

Lead compensator design

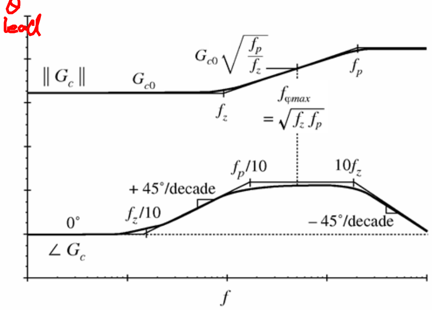
↳ provides phase lead

To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

want θ phase lead

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}}$$



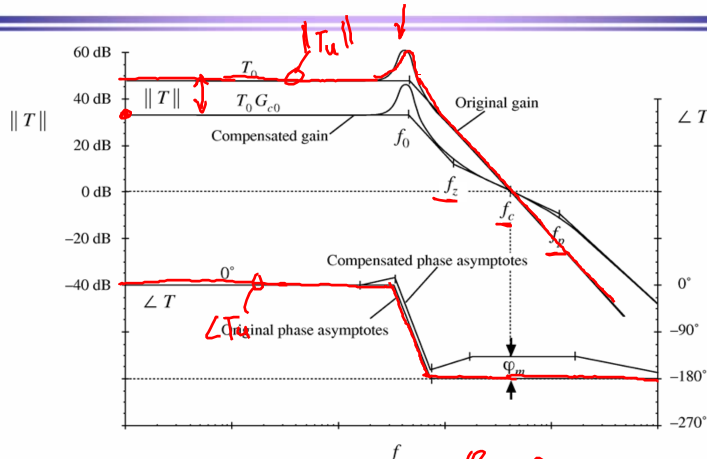
If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

$\tilde{v} = V_{ref} \frac{1}{1+T} \frac{T}{1+T}$

First, plot $T_u(s)$ → uncompensated loop gain (Loop gain with $G_c(s) = 1$)
Next, design $G_c(s)$ to shape $T(s)$

Example: lead compensation

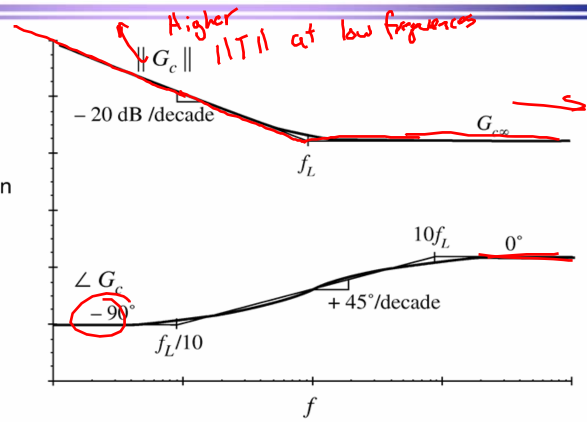


$\varphi_M = 0$

9.5.2. Lag (PI) compensation

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



Example: lag compensation

original (uncompensated) loop gain is

$$T_u(s) = \frac{T_{u0}}{\left(1 + \frac{s}{\omega_0}\right)}$$

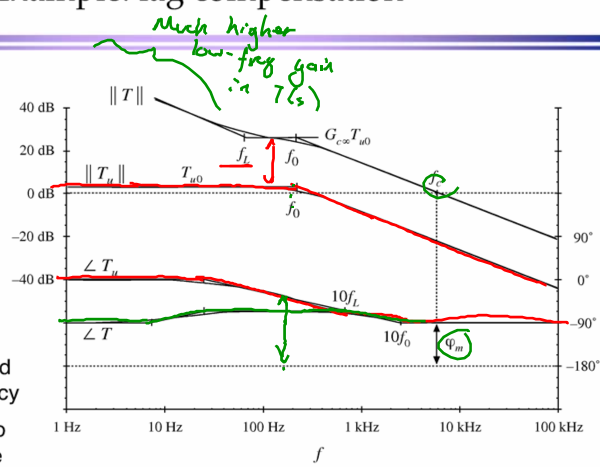
compensator:

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Design strategy: choose

$G_{c\infty}$ to obtain desired crossover frequency

ω_L sufficiently low to maintain adequate phase margin



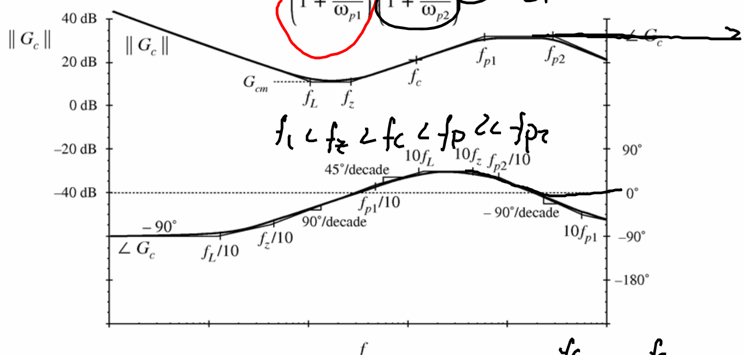
9.5.3. Combined (PID) compensator

inverted zero (PI)

(PD)

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

realistic representation of the circuitry



How high can f_c be? $f_c < \frac{f_s}{5}$ or $\frac{f_s}{10}$