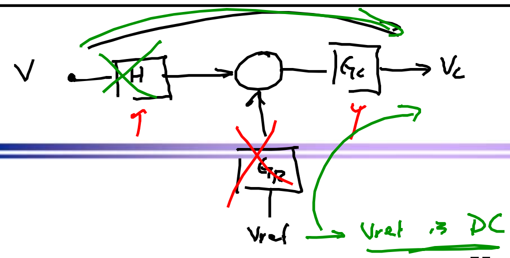

Lecture 20: Feedback Loop Compensation

ECE 481: Power Electronics

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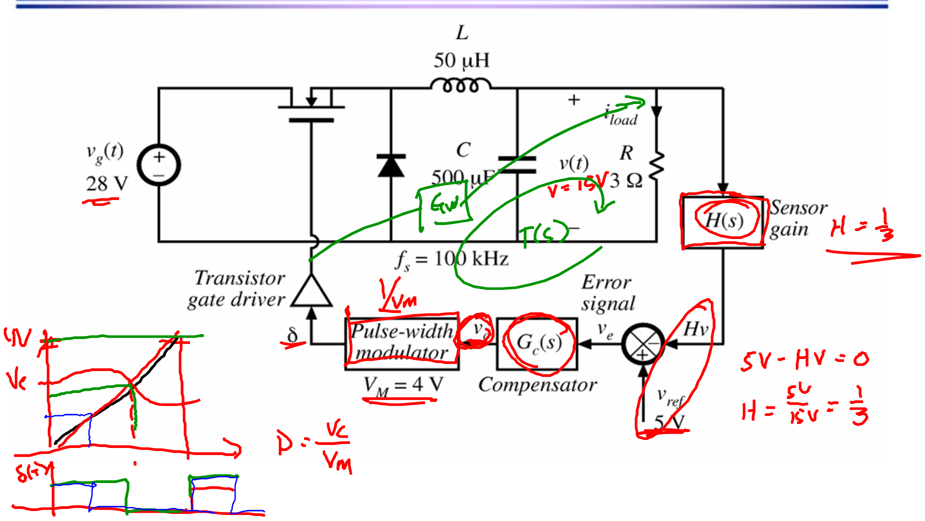
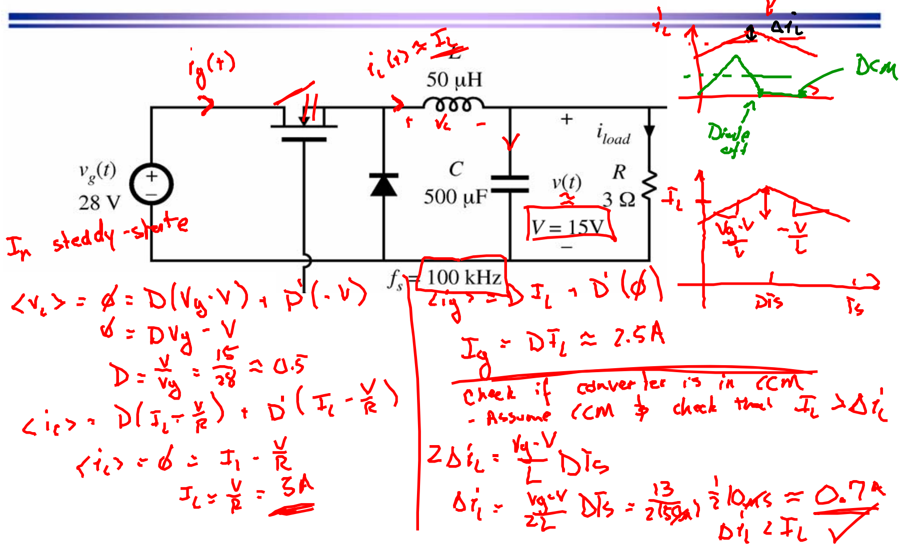
Fall 2013

9.5.4. Design example



Quiescent operating point



Quiescent operating point

Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

Need $G_{vd} = \frac{\hat{v}}{\hat{d}}$

$\langle v_L \rangle = L \frac{d\langle i_L \rangle}{dt} = d(t)\langle v_g \rangle - \langle v \rangle$

$\langle i_c \rangle = C \frac{d\langle v \rangle}{dt} = \langle i_L \rangle - \frac{\langle v \rangle}{R}$

$\langle i_g \rangle = d(t)\langle i_L \rangle$

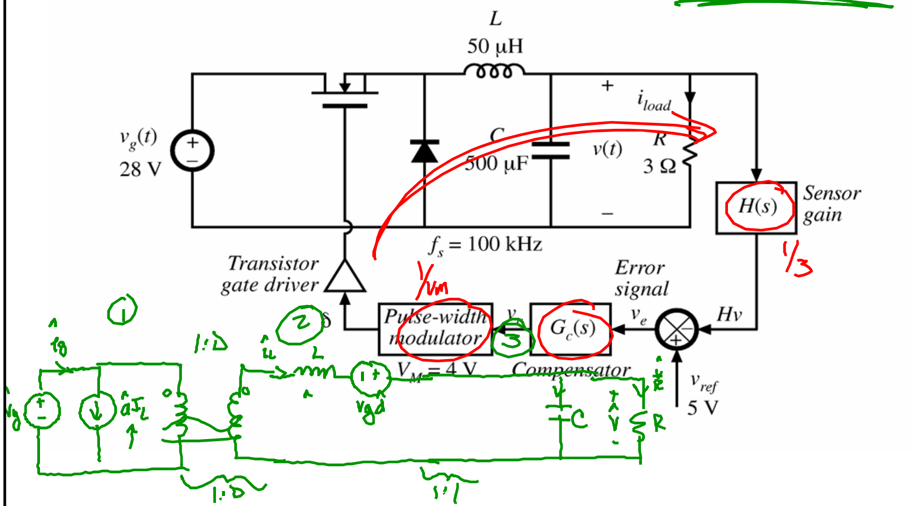
Linearize

$L \frac{d\hat{i}_L}{dt} = D\hat{v}_g + V_g \hat{d} - \hat{v}$ (2)

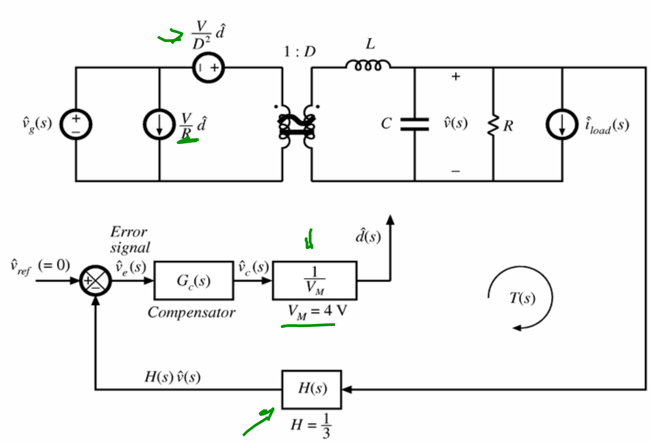
$C \frac{d\hat{v}}{dt} = \hat{i}_L - \frac{\hat{v}}{R}$ (3)

$\hat{i}_g = D\hat{i}_L + \hat{d} I_L$ (1)

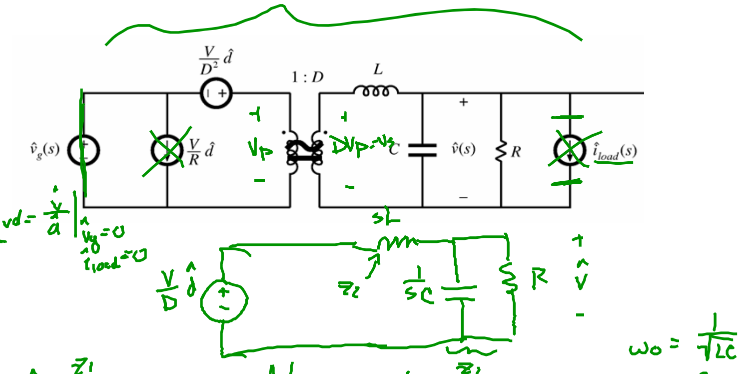
Small-signal model



Small-signal model



Open-Loop Control-to-Output Transfer Function



Find $G_{vd} = \frac{\hat v}{\hat d}\Big|_{\hat v_g = 0,\ \hat i_{load} = 0}$

$\hat v = \frac{V}{D}\hat d \frac{Z_1}{Z_1 + Z_2}$, $Z_1 = R \| \frac{1}{sC}$ $\quad Z_2 = sL$

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad Q = \frac{R}{\sqrt{L/C}}$

$$G_{vd} = \frac{\hat v}{\hat d} = \frac{V}{D}\frac{R\|\frac{1}{sC}}{sL + R\|\frac{1}{sC}} = \frac{V}{D}\frac{1}{s^2LC + s\frac{L}{R} + 1} = G_{d0}\frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$$

$G_{d0} = \frac{V}{D}$

Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D}\frac{1}{1 + s\frac{L}{R} + s^2LC}$$

standard form:

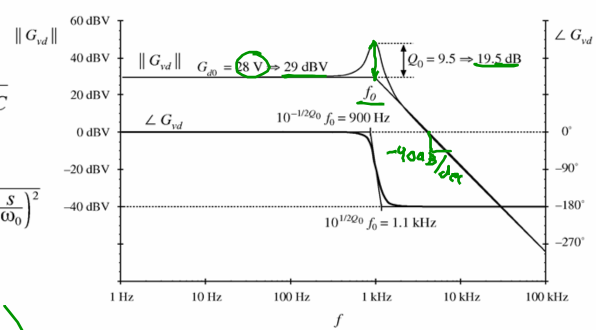
$$G_{vd}(s) = G_{d0}\frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

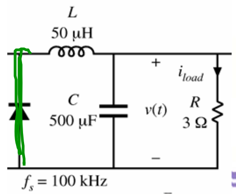
salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$



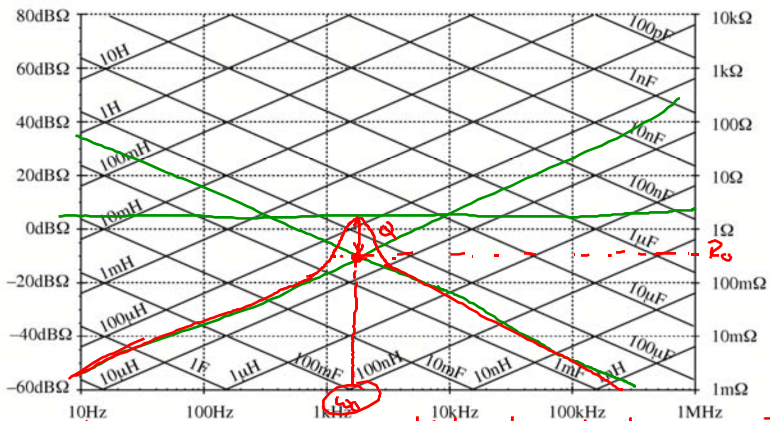


$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$

$D = 15/28 = 0.536$

$Z_{out} = R \parallel \frac{1}{sC} \parallel sL$

Open-Loop Output Impedance



$\| \frac{1}{j\omega_0 C} \| = \| j\omega_0 L \|$

$\frac{1}{\omega_0 C} = \omega_0 L \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \approx 1\text{kHz}$

$\|Q\|_{dB} = \|R\|_{dB} - \|R_0\|$ $\quad R_0 = \sqrt{\frac{L}{C}}$

$\|Q\|_{dB} = \|R\|_{dB} - \|\sqrt{\tfrac{L}{C}}\|_{dB}$

$Q = R/\sqrt{\tfrac{L}{C}}$

Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

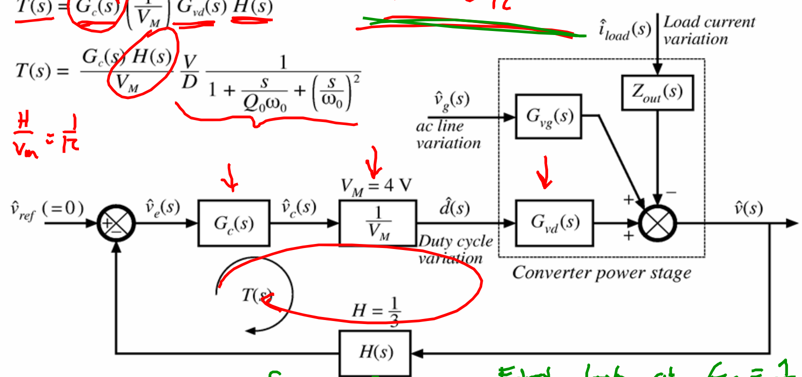
System block diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$T(s) = G_c \frac{1}{12} G_{vd}$

$\frac{H}{V_m} = \frac{1}{12}$



Design G_c to get: { High f_c, Large φ_m, Large $\|T_0\|$

First, look at $G_c = 1$ consider $T_u(s) = \frac{1}{12} G_{vd}$

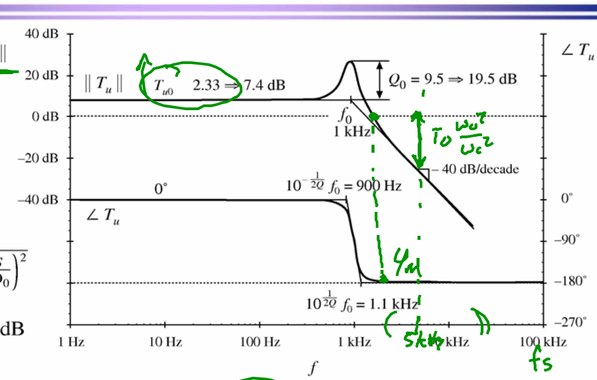
$V = \frac{1}{H} \frac{T}{1+T} V_{ref}$
$= 3 \frac{2.3}{1+2.3} 5 \approx 10V$ wanted $V = 15V$

Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$ the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

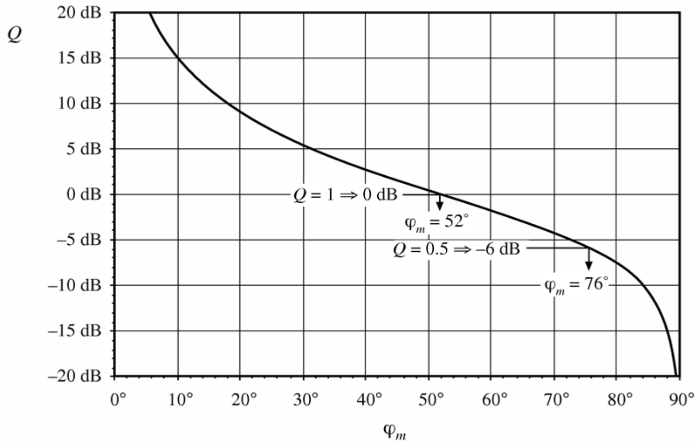
$$T_{u0} = \frac{H\, V}{D\, V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



$f_c = 1.8$ kHz, $\varphi_m = 5°$

$\varphi_M \approx 0$

Q vs. φ_m



Lead compensator design

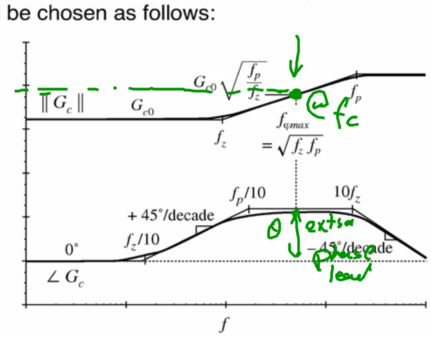
To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



Lead compensator design

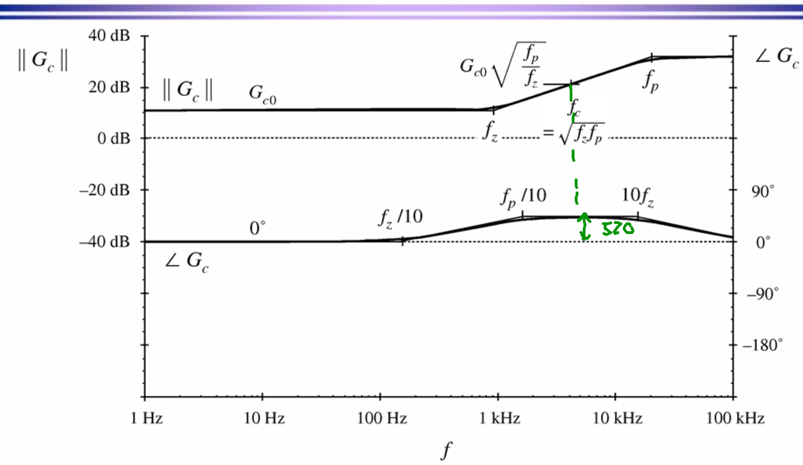
- Obtain a crossover frequency of 5 kHz, with phase margin of 52°
- T_u has phase of approximately – 180° at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52° at 5 kHz
- T_u has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of + 20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz})\sqrt{\frac{1-\sin(52°)}{1+\sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz})\sqrt{\frac{1+\sin(52°)}{1-\sin(52°)}} = 14.5\text{kHz}$$

- Compensator dc gain should be $G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

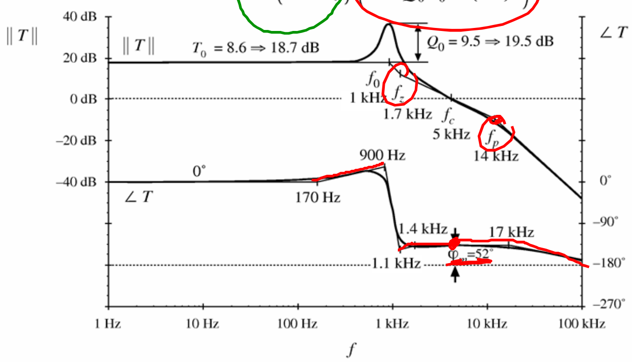
Lead compensator Bode plot



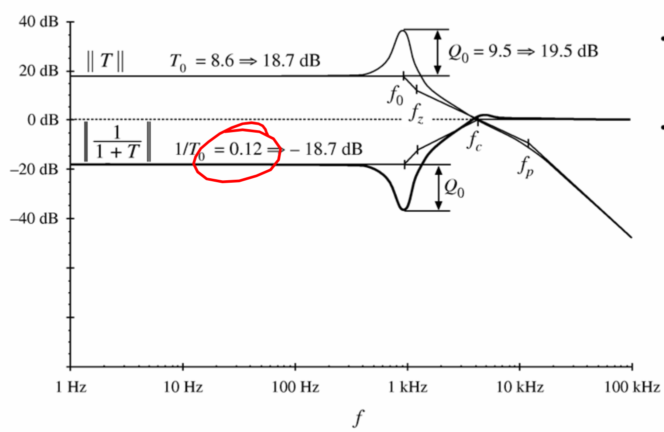
Loop gain, with lead compensator

$$T(s) = T_{u0} G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

PD compensator



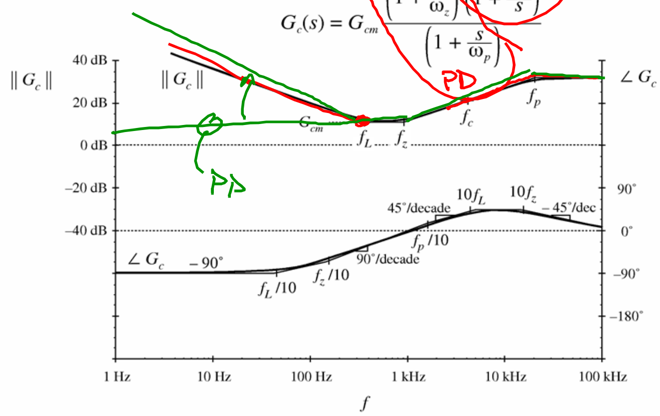
1/(1+T), with lead compensator



- need more low-frequency loop gain
- hence, add inverted zero (PID controller)

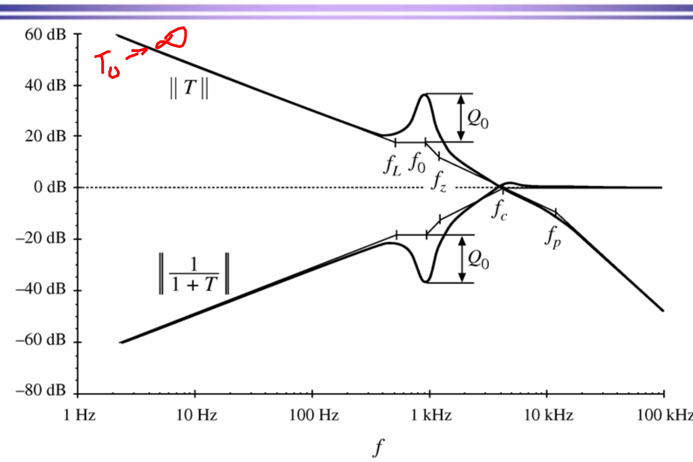
Improved compensator (PID)

$$G_c(s) = G_{cm}\frac{\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{\omega_L}{s}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$



- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

$T(s)$ and $1/(1+T(s))$, with PID compensator



Line-to-output transfer function