Lecture 20: Feedback Loop Compensation

ECE 481: Power Electronics

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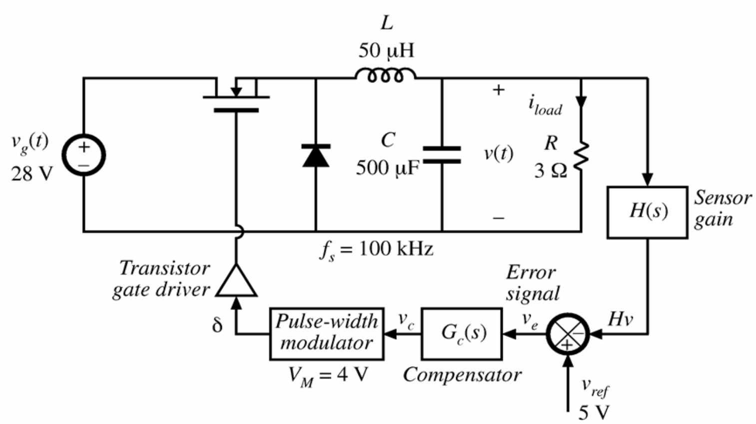
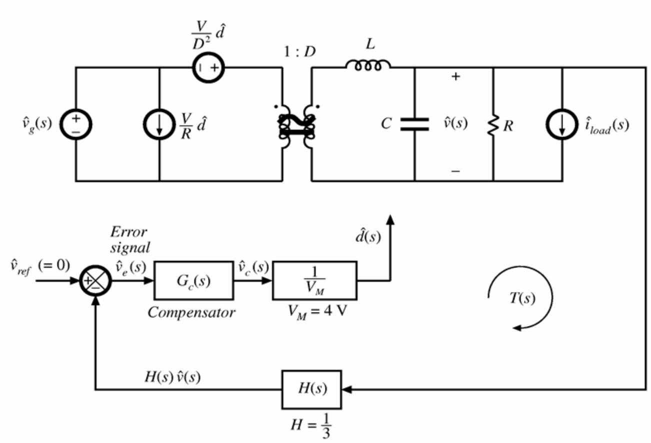
9.5.4. Design example

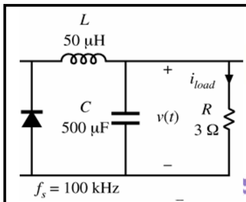


Quiescent operating point

Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain $H(s)$	$H = V_{ref}/V = 5/15 = 1/3$

Small-signal model

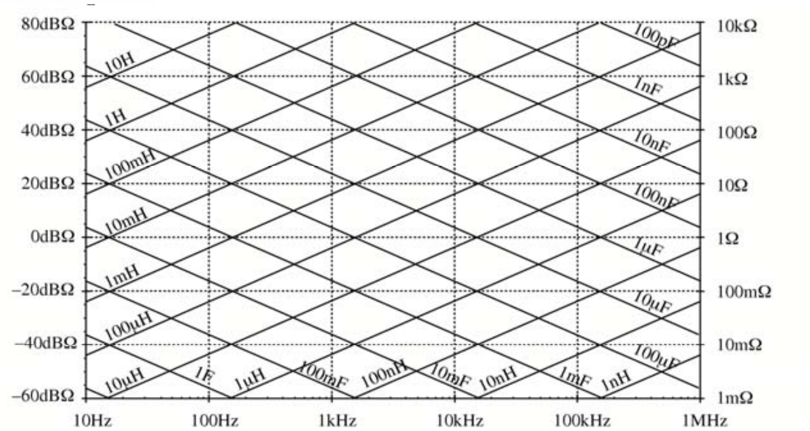




$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$

$D = 15/28 = 0.536$

Open-Loop Control-to-Output Transfer Function



Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

standard form:

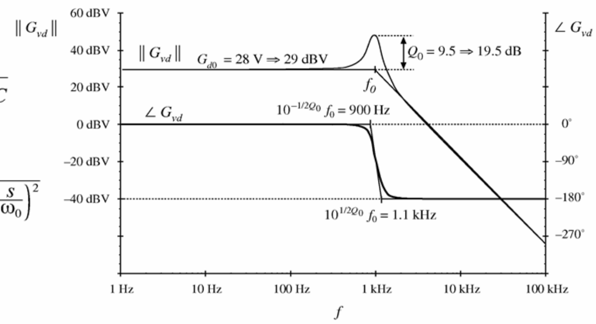
$$G_{vd}(s) = G_{d0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$



Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function
standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

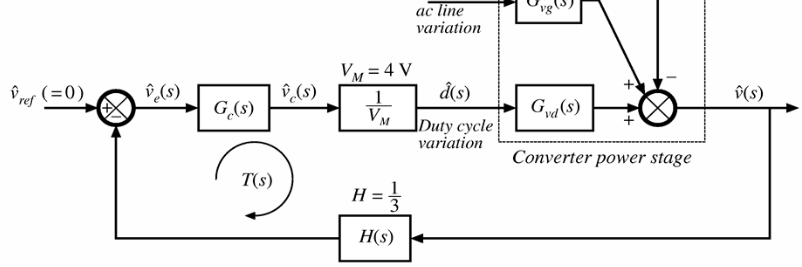
Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

System block diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M}\, \frac{V}{D}\, \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

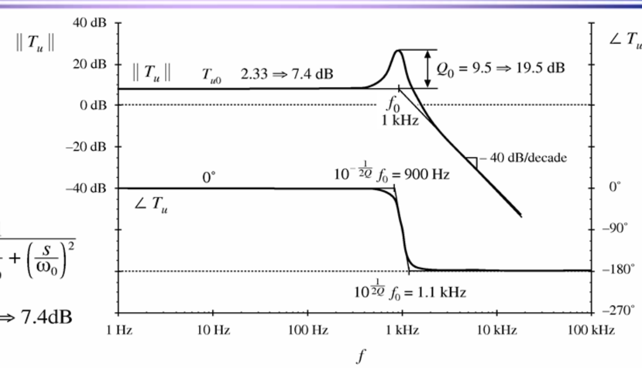


Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0}\, \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\,V}{D\,V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



$f_c = 1.8$ kHz, $\varphi_m = 5^\circ$

Lead compensator design

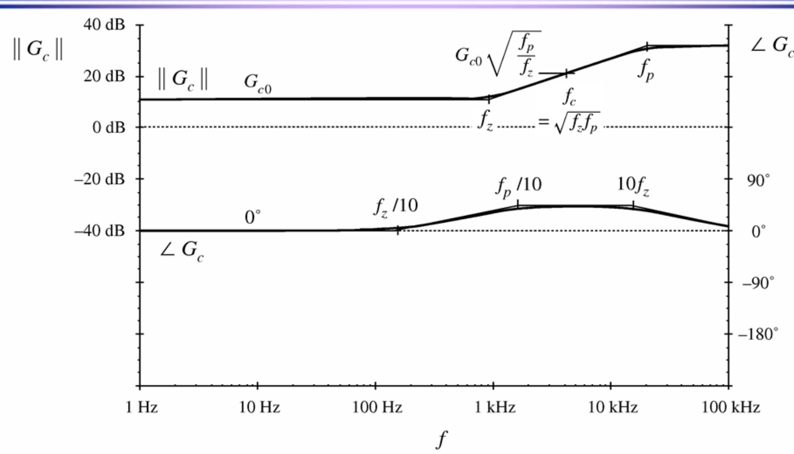
- Obtain a crossover frequency of 5 kHz, with phase margin of 52˚
- T_u has phase of approximately – 180˚ at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52˚ at 5 kHz
- T_u has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of + 20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz})\sqrt{\frac{1 - \sin(52°)}{1 + \sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz})\sqrt{\frac{1 + \sin(52°)}{1 - \sin(52°)}} = 14.5\text{kHz}$$

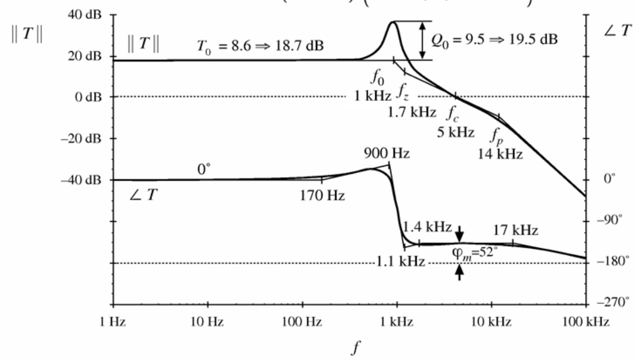
- Compensator dc gain should be $G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

Lead compensator Bode plot

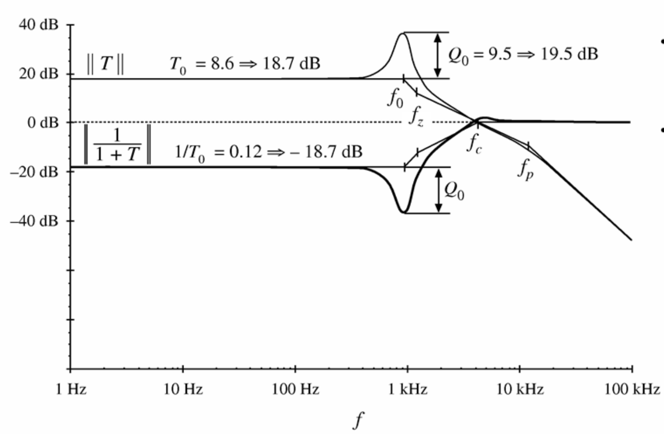


Loop gain, with lead compensator

$$T(s) = T_{u0}\, G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$



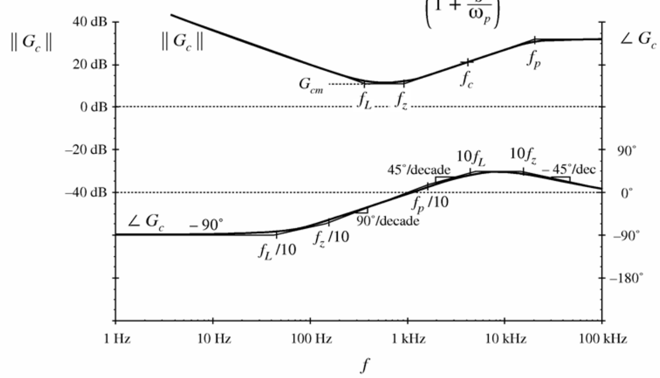
1/(1+T), with lead compensator



- need more low-frequency loop gain
- hence, add inverted zero (PID controller)

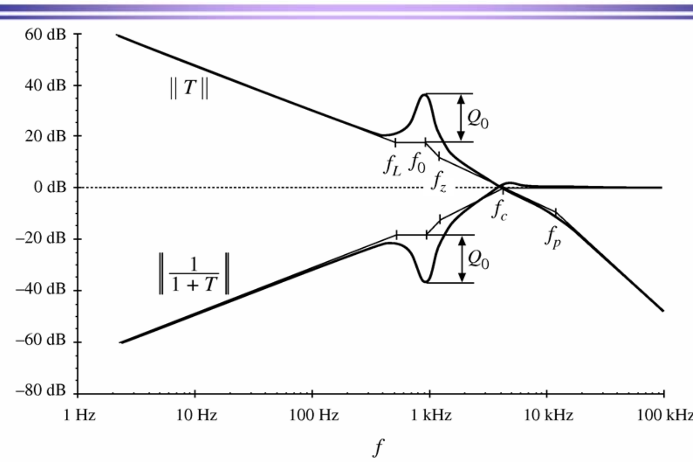
Improved compensator (PID)

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

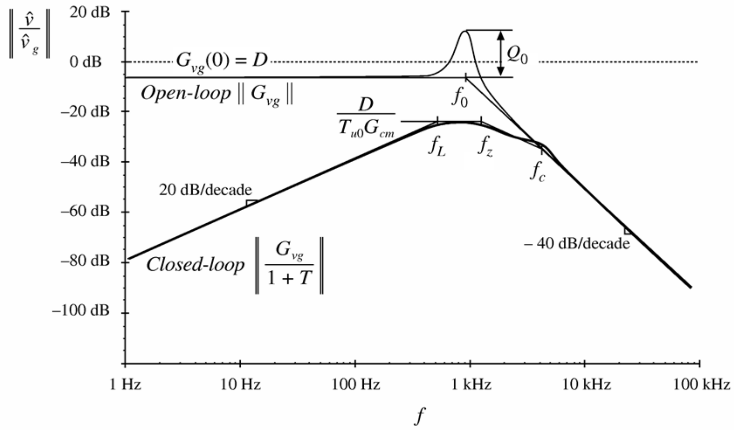


- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

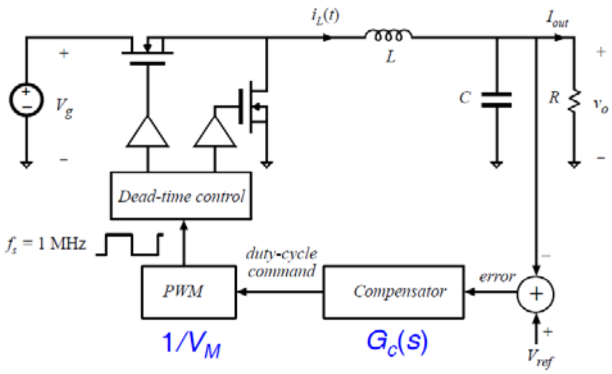
$T(s)$ and $1/(1+T(s))$, with PID compensator



Line-to-output transfer function



Another Compensator Design Example

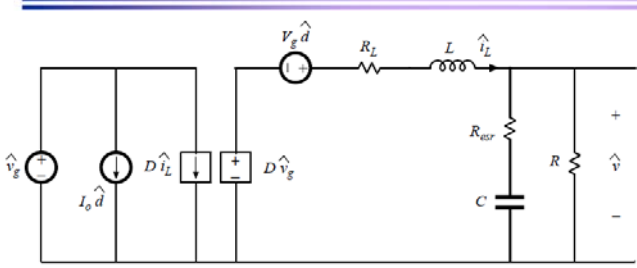


Point-of-Load Synchronous Buck Regulator

Power stage parameters

- Switching frequency: f_s = 1MHz
- V_{ref} = 1.8 V
- I_{out} = 0 to 5 A
- V_g = 5 V
- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ
- V_M = 1 V
- H = 1

Buck Averaged Small-Signal Model



$$G_{vd}(s) = \frac{\hat{v}_o}{\hat{d}}$$

$$G_{vd}(s) = V_g \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{1}{Q}\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

Pair of poles:

$$f_o = \frac{1}{2\pi\sqrt{CL}} = 11\,\text{kHz}$$

$$Q_{loss} = \frac{\sqrt{L/C}}{R_{esr} + R_L} = 2.3 \rightarrow 7.2\,\text{dB} \qquad Q_{load} = \frac{R}{\sqrt{L/C}} > 5$$

$$Q = Q_{loss} \,||\, Q_{load} = \frac{Q_{loss} Q_{load}}{Q_{loss} + Q_{load}} < 2.3 \rightarrow 7.2\,\text{dB}$$

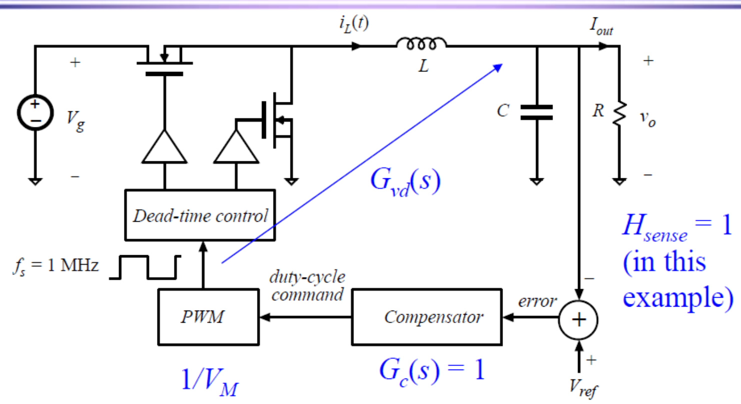
Low-frequency gain (including PWM gain):

$$G_{vdo} \frac{1}{V_M} = 5 \rightarrow 14\text{dB}$$

ESR zero:

$$f_{esr} = \frac{1}{2\pi C R_{esr}} = 1\,\text{MHz}$$

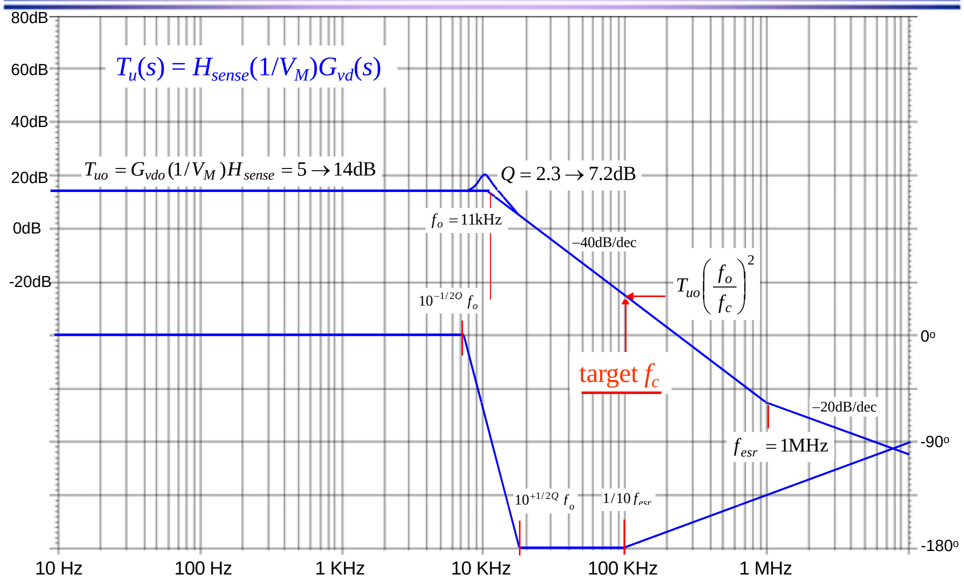
Uncompensated loop gain T_u



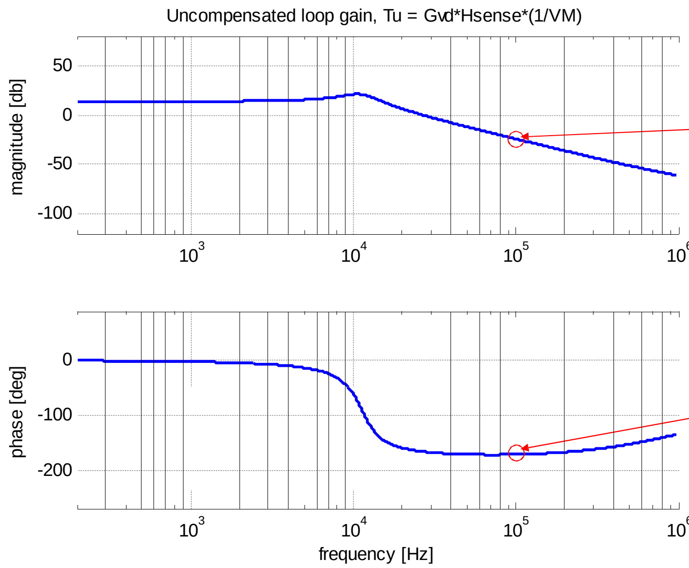
$T_u(s) = H_{sense}(1/V_M)G_{vd}(s)$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

Magnitude and phase Bode plots of T_u



Magnitude and phase Bode plots of T_u



Exact magnitude and phase responses (MATLAB)

Target cross-over frequency $f_c = f_s/10 = 100$ kHz

No phase margin: a lead (PD) compensator is required

Lead (PD) compensator design

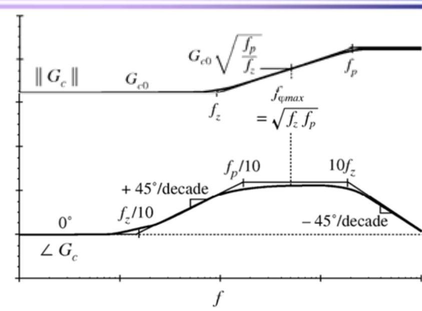
1. Choose: $f_c = 100\text{ kHz}$

$\theta = \varphi_m = 53^o$

2. Compute:

$$f_z = f_c\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 33\text{ kHz}$$

$$f_p = f_c\sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 300\text{ kHz}$$



3. Find G_{co} to position the crossover frequency:

$$\underbrace{T_{uo}\left(\frac{f_o}{f_c}\right)^2}_{\text{Magnitude of } T_u \text{ at } f_c}\underbrace{G_{co}\sqrt{\frac{f_p}{f_z}}}_{\text{Magnitude of } G_c \text{ at } f_c}=1 \qquad G_{co}=\frac{1}{T_{uo}}\left(\frac{f_c}{f_o}\right)^2\sqrt{\frac{f_z}{f_p}}=5.45\rightarrow 15\text{ dB}$$

Lead (PD) compensator summary

$$G_c(s) = G_{co} \frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)} \frac{1}{\left(1+\frac{s}{\omega_{p2}}\right)}$$

Lead compensator — HF pole

$G_{co} = 5.45 \rightarrow 15\,\text{dB}$

$f_z = 33\,\text{kHz}$

$f_{p1} = 300\,\text{kHz}$

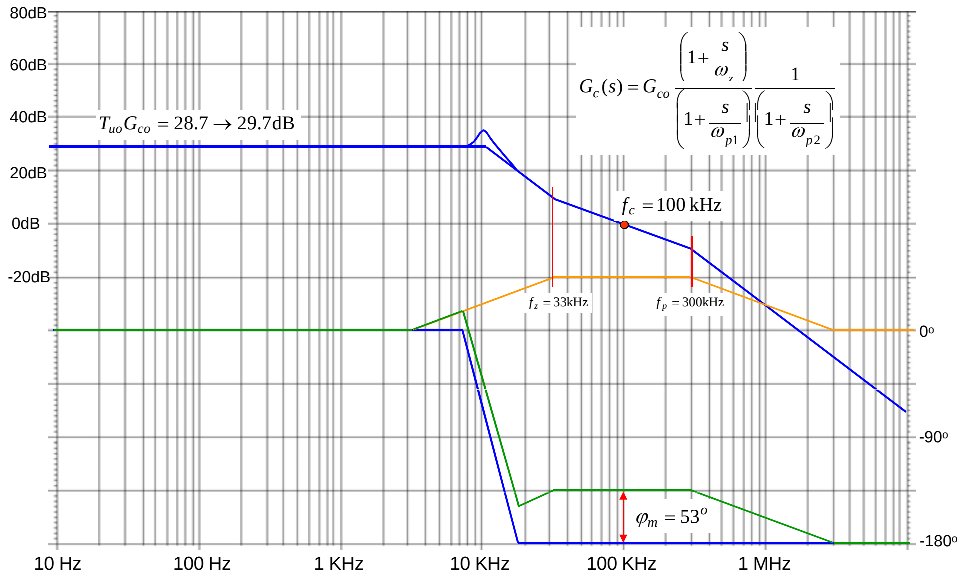
$f_c = 100\,\text{kHz}$ (=1/10 of f_s)

High-frequency gain of the lead compensator: $G_{co} f_{p1}/f_z$ = 49 (34 dB)

Added high-frequency pole: $f_{p2} = 1\,\text{MHz}$ (= f_{esr} = f_s in this example)

Practical implementation would require an op-amp with a gain bandwidth product (GBW) of at least 49*f_{p2} = 49 MHz

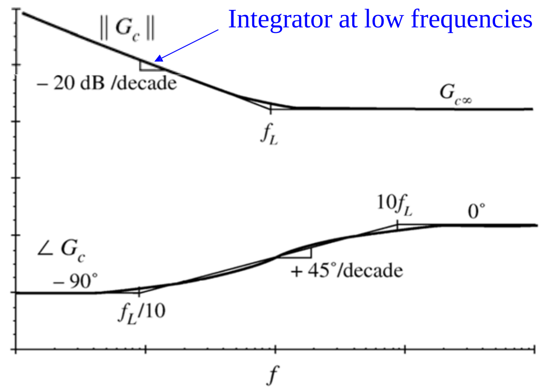
Loop gain with lead (PD) compensator



Add lag (PI) compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

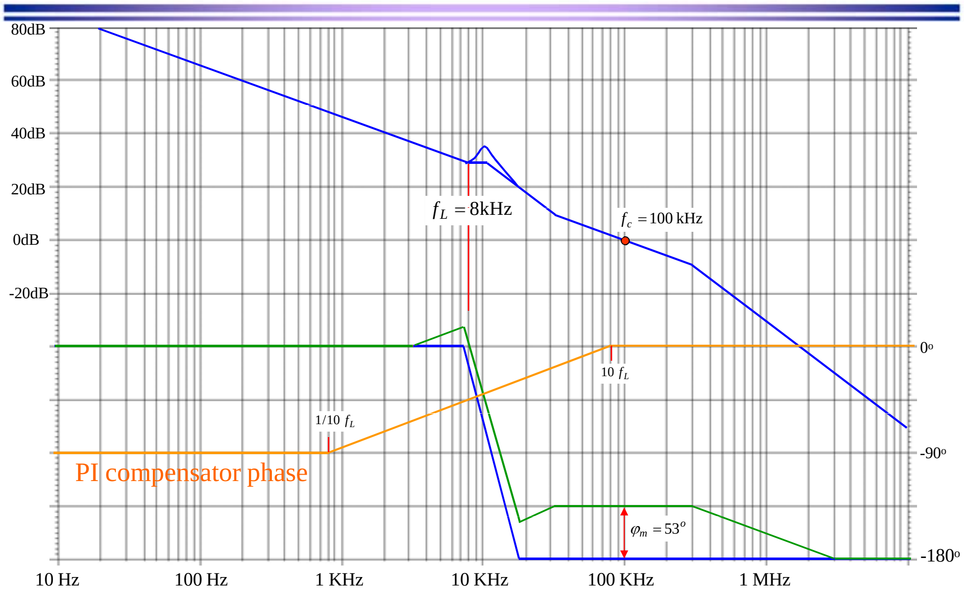
Improves low-frequency loop gain and regulation



Choose $10f_L < f_c$ so that phase margin stays approximately the same: f_L = 8 kHz

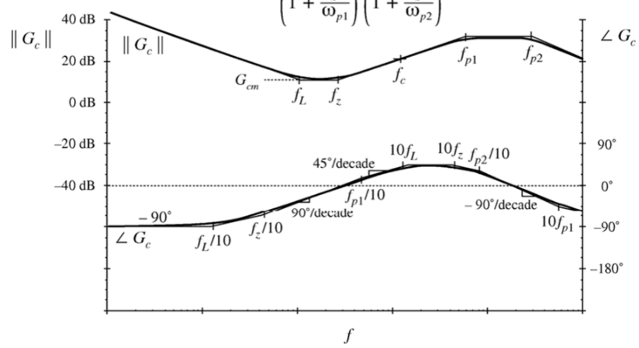
Keep the same cross-over frequency: $G_{c\infty} = G_{co} = G_{cm} = 5.45 \rightarrow 15\,\mathrm{dB}$

Adding PI Compensator



Complete analog PID compensator: summary

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



$G_{cm} = 5.45 \rightarrow 15$ dB

$f_L = 8$ kHz

$f_z = 33$ kHz

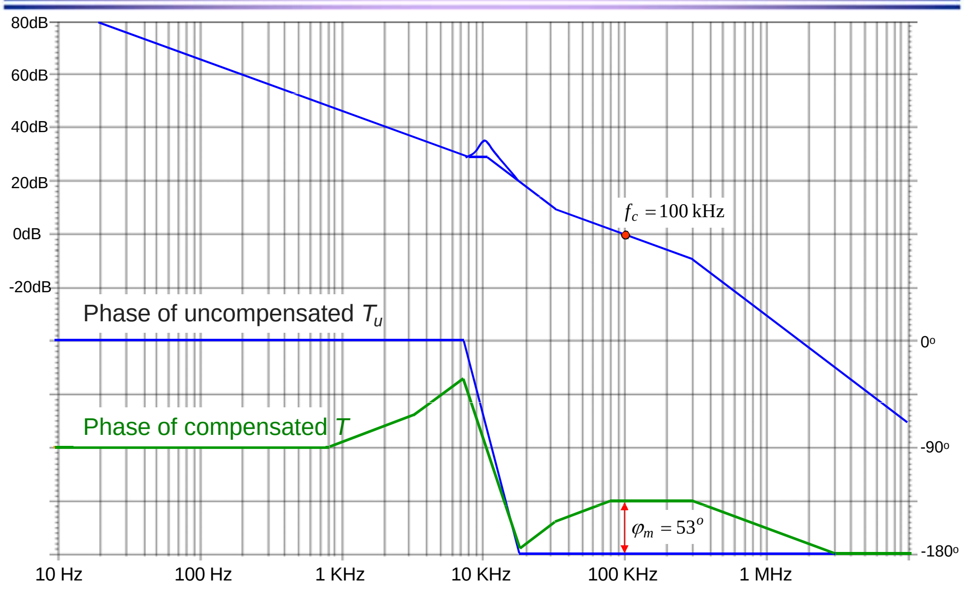
$f_{p1} = 300$ kHz

$f_{p2} = 1$ MHz

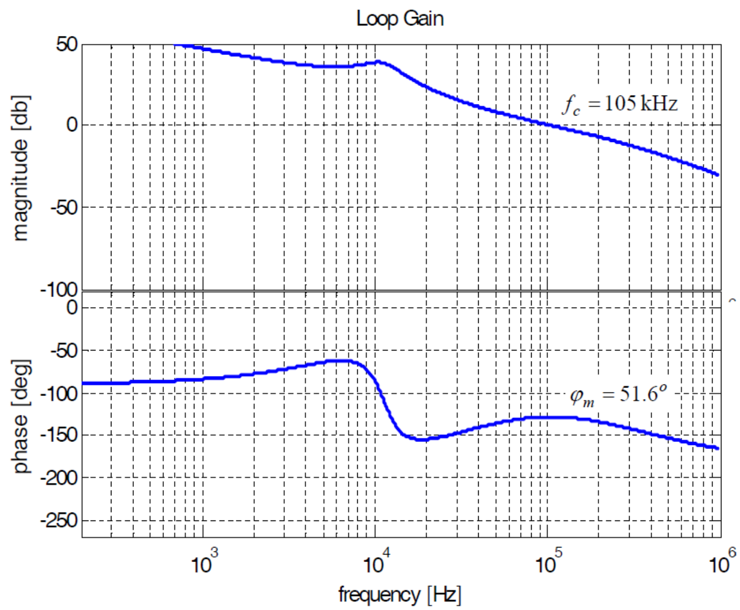
Crossover frequency: $f_c = 100$ kHz (=1/10 of f_s)

Phase margin: $\varphi_m = 53^o$

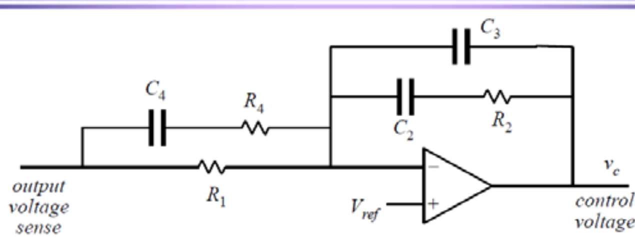
Magnitude and phase Bode plots of T



Verification: exact loop gain magnitude and phase responses (MATLAB)



Analog PID compensator implementation



$$G_c(s) = G_{cm} \frac{\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

Design equations (approximate)

$$G_{cm} = \frac{R_2}{R_1} \qquad f_L = \frac{1}{2\pi R_2 C_2}$$

$$f_z = \frac{1}{2\pi (R_1 + R_4) C_4} \qquad f_{p1} = \frac{1}{2\pi R_4 C_4}$$

$$f_{p2} = \frac{1}{2\pi R_2 C_3}$$

Verification of closed-loop responses

Closed-loop reference-to-output response

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

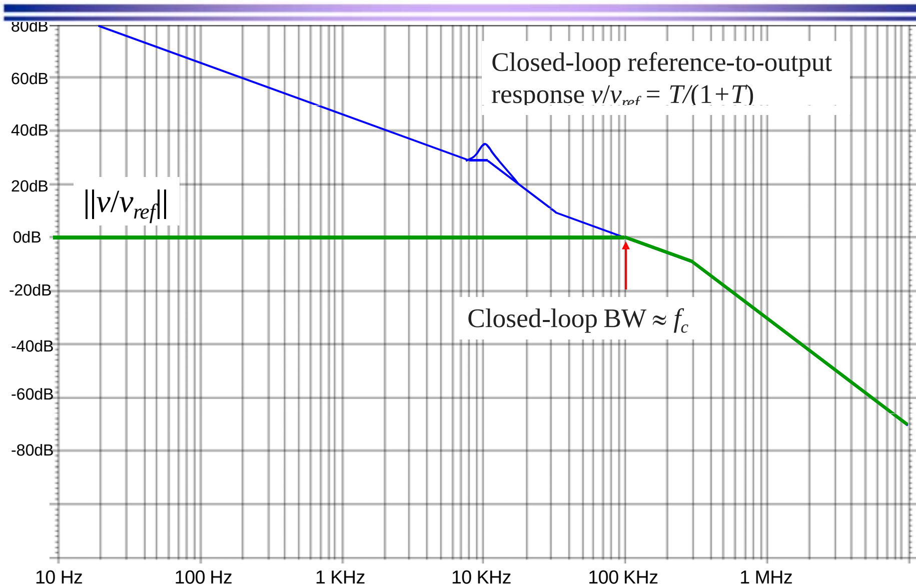
$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

Closed-loop output impedance

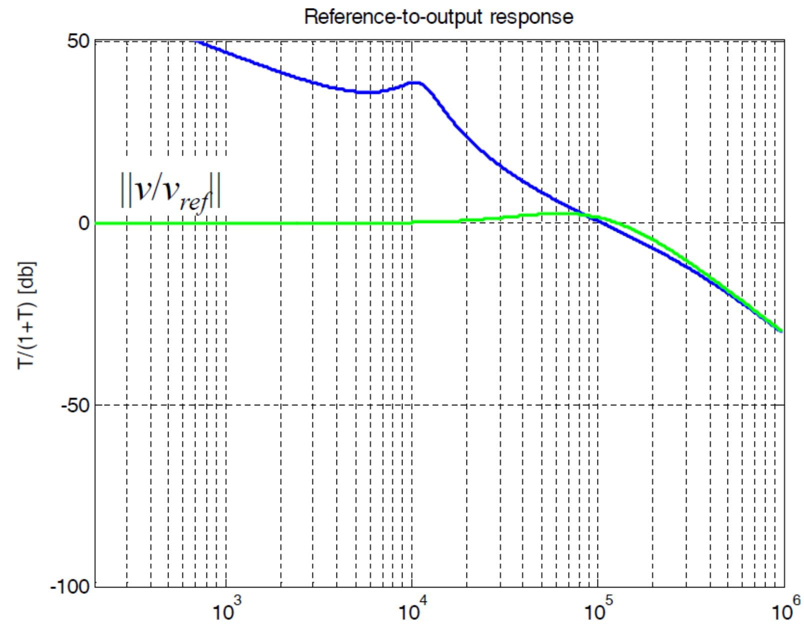
$$\left.\frac{\hat{v}(s)}{-\hat{i}_{load}(s)}\right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{v}_g = 0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

and step-load transient response

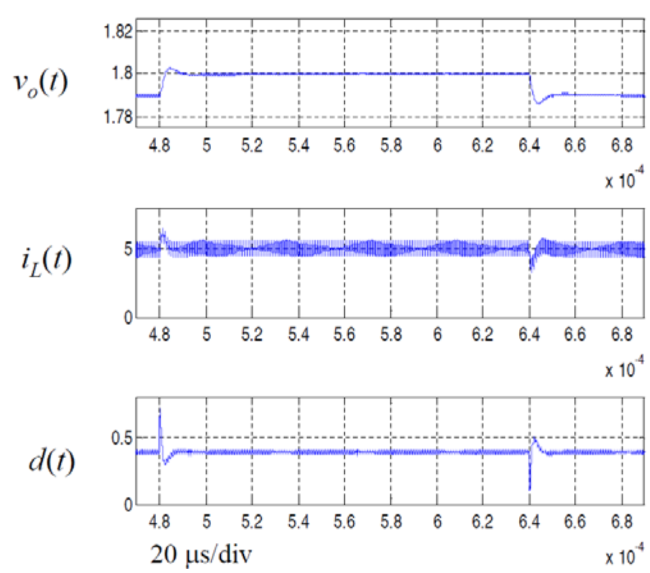
Construction of closed-loop $T/(1+T)$ response



Closed-loop reference-to-output response

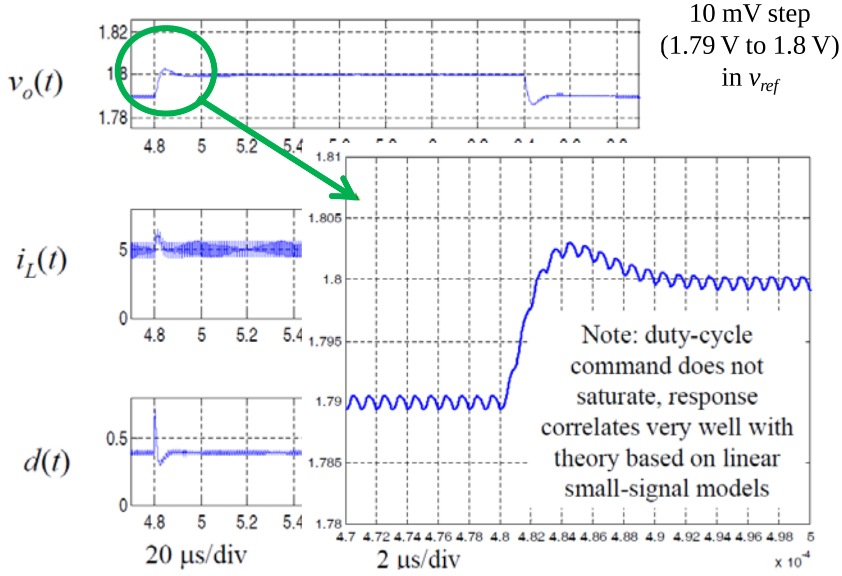


Small-signal step-reference response



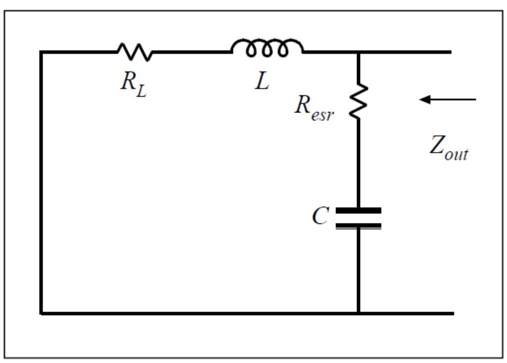
10 mV step
(1.79 V to 1.8 V)
in v_{ref}

Small-signal step-reference response



Output impedance

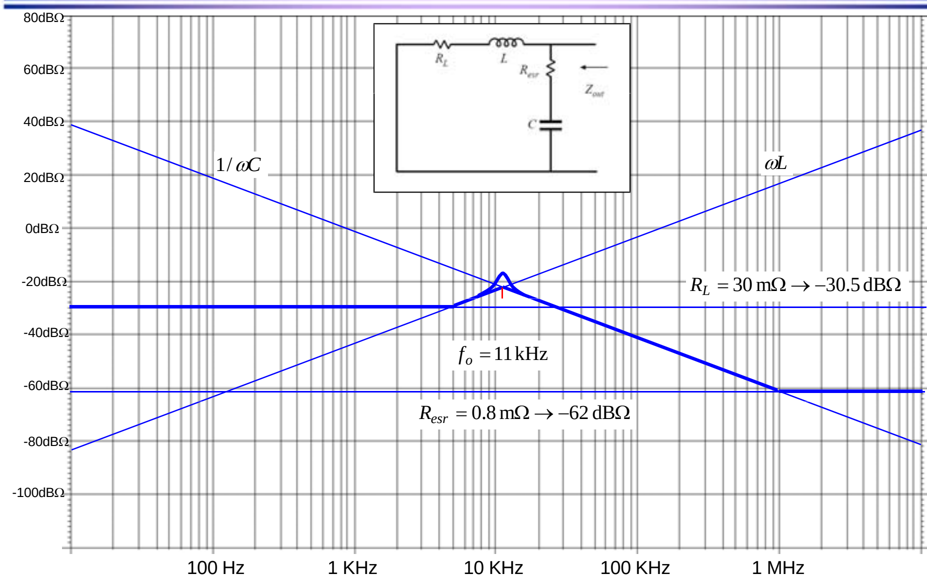
Synchronous buck open-loop output impedance



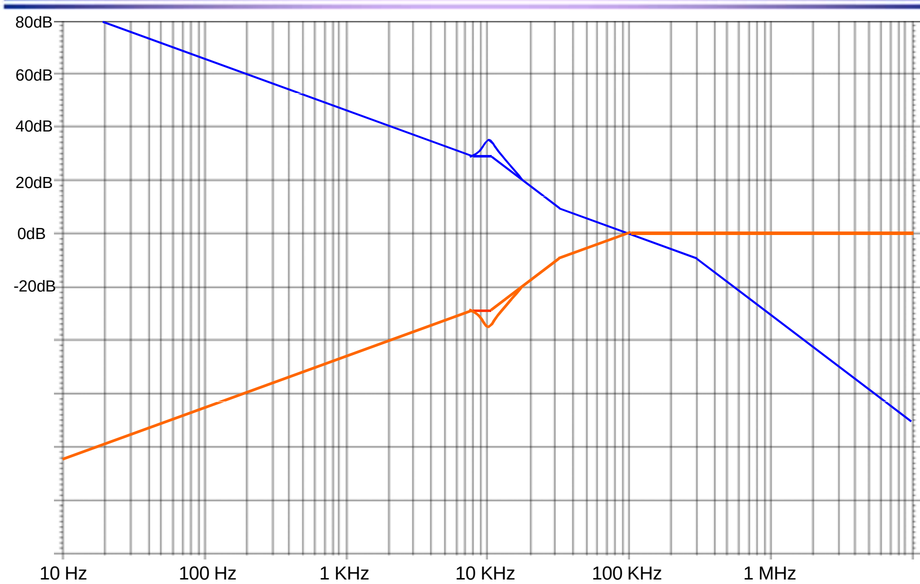
$$Z_{out}(s) = \left(R_{esr} + \frac{1}{sC}\right) \| \left(R_L + sL\right)$$

- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ

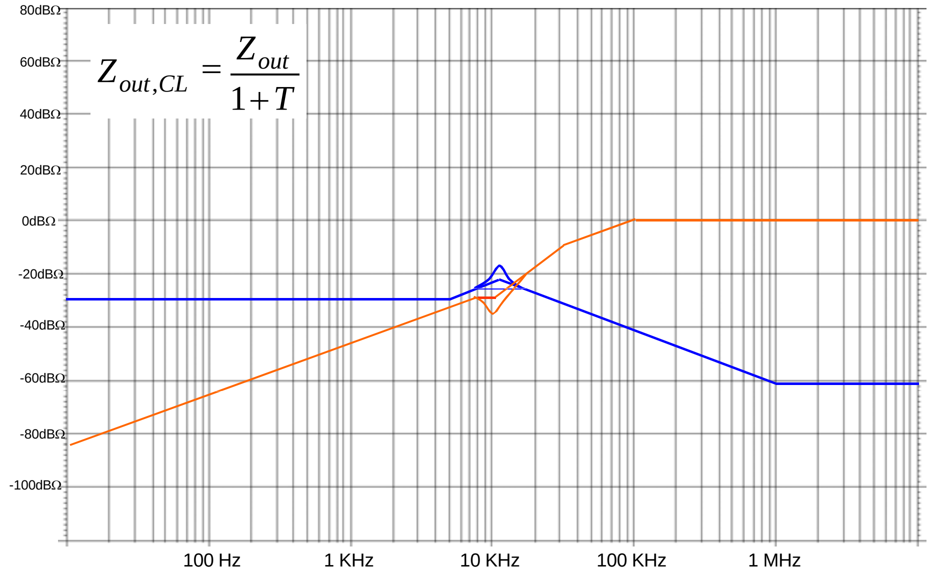
Open-loop output impedance: algebra on the graph



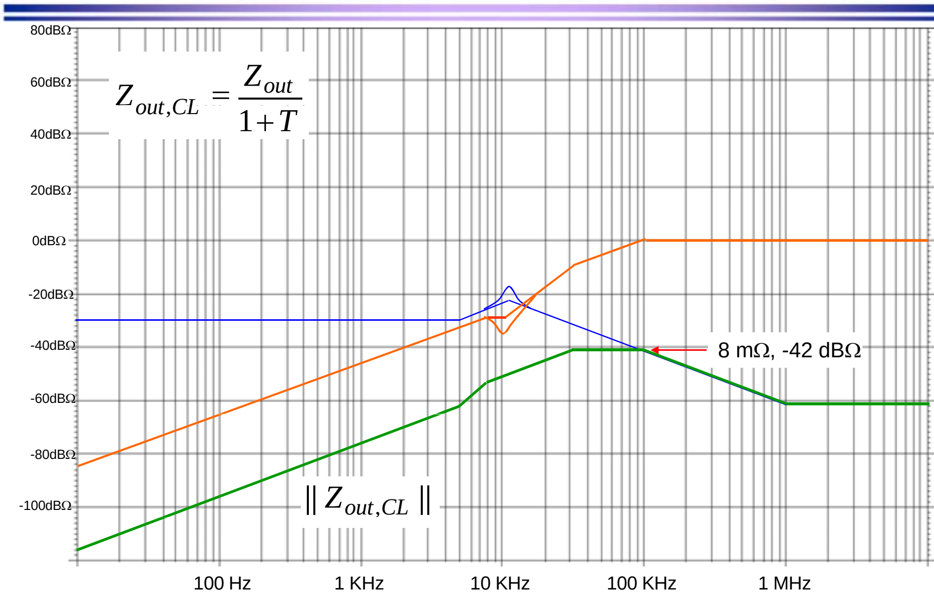
Construction of 1/(1+T)



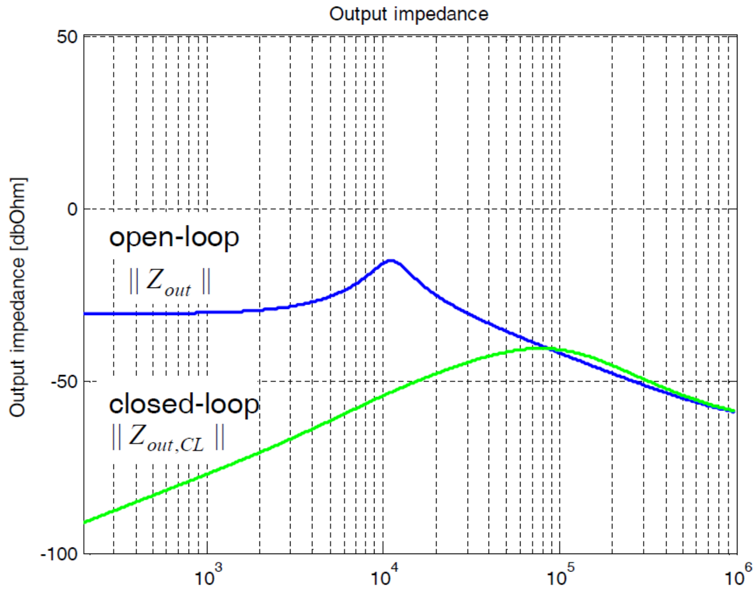
Construction of closed-loop output impedance



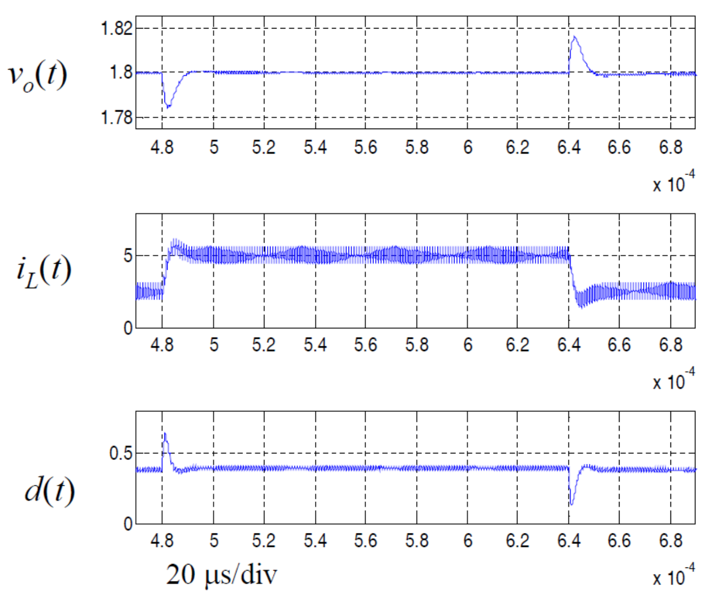
Closed-loop output impedance $Z_{out,CL}$



Verification: closed-loop output impedance



Step-load transient responses



2.5-5 A step-load transient

Step-load transient responses