Lecture 21: Feedback Loop Compensation

ECE 481: Power Electronics

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Announcements

- Midterm due Tuesday
- ECE 482: Power Electronic Circuits
- http://oira.tennessee.edu/sais/

Midterm:
Impedance paper is A-ok!

Shape $T(s)$ on a bode plot using $G_c(s)$

{ (1) Large $T(s)$ at low frequencies
(2) High $f_c \leftrightarrow (BW)_{CL}$
(3) Adequate ϕ_m

9.5. Regulator design

Typical specifications:

Reject Disturbance via (1) & (2)

- Minimize Effect of load current variations on output voltage regulation
  - This is a limit on the maximum allowable output impedance — $Z_{out}\ \frac{1}{1+T}$
- Minimize Effect of input voltage variations on the output voltage regulation
  - This limits the maximum allowable line-to-output transfer function $G_{vg}\ \frac{1}{1+T}$
- fast Transient response time → $\uparrow f_c \approx (BW)_{CL}$
  - This requires a sufficiently high crossover frequency
- Low Overshoot and ringing → $\uparrow \varphi_m$
  - An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

Lead compensator design

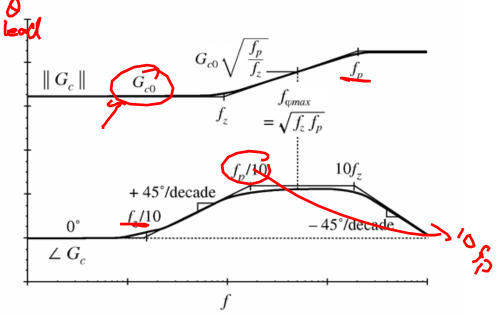
↳ provides phase lead

To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

want θ phase lead

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$



If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$

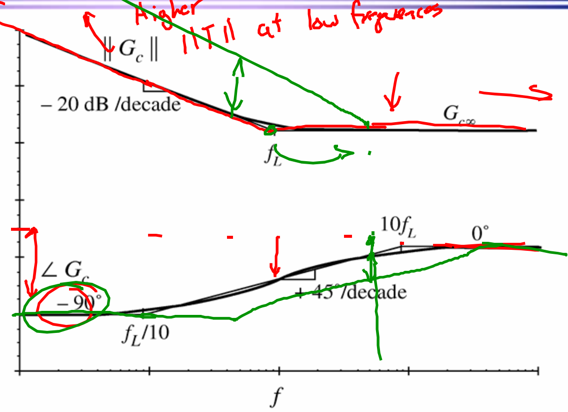
$\|T_0\| \to \infty$

9.5.2. Lag (PI) compensation

Higher $\|T\|$ at low frequencies

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



$V = V_{ref}\frac{T_0}{1+T_0}$

9.5.3. Combined (PID) compensator

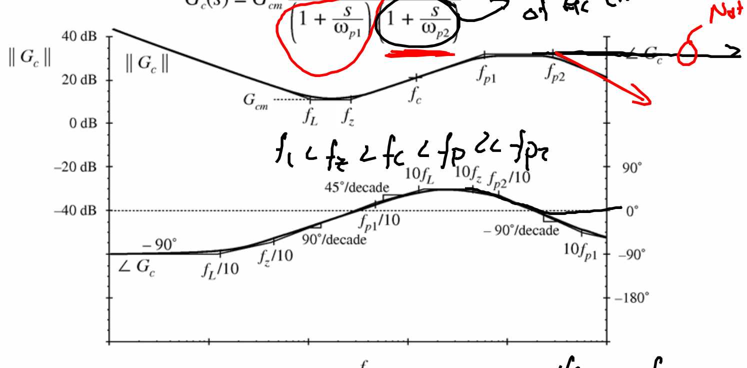
inverted zero (PI)

(PD)

$$G_c(s) = G_{cm}\frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

realistic representation of the circuitry

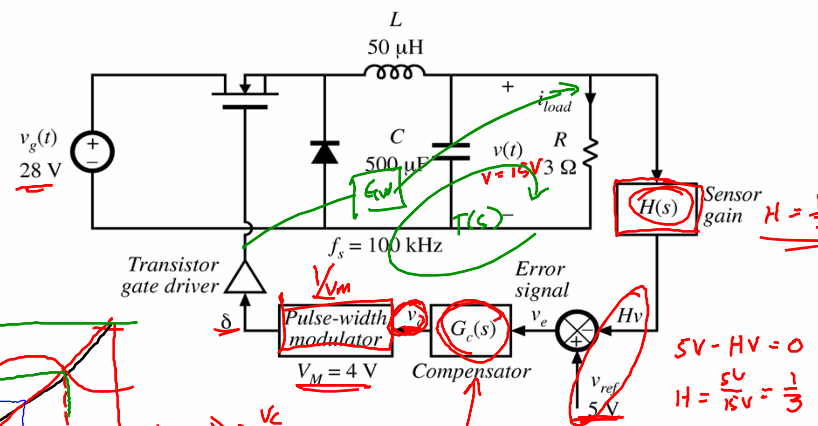
Not possible

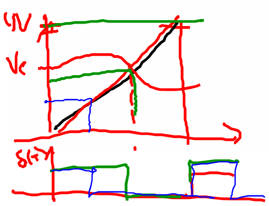


How high can f_c be? $f_c < \frac{f_s}{5}$ or $\frac{f_s}{10}$

Goal: Design $G_c(s)$ to shape $T(s)$

9.5.4. Design example





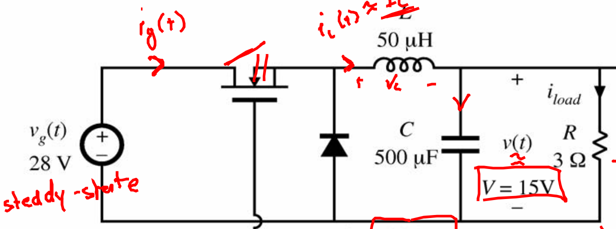
$H = \frac{1}{3}$

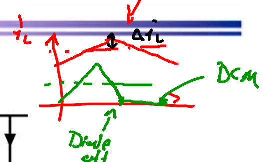
$D = \frac{V_c}{V_M}$

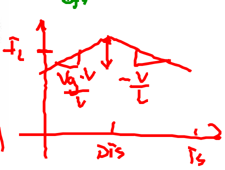
$5V - HV = 0$

$H = \frac{5V}{15V} = \frac{1}{3}$

Quiescent operating point @ DC







In steady-state

$\langle v_L \rangle = 0 = D(V_g - V) + D'(-V)$

$0 = DV_g - V$

$D = \frac{V}{V_g} = \frac{15}{28} \approx 0.5$

$\langle i_C \rangle = D(I_L - \frac{V}{R}) + D'(I_L - \frac{V}{R})$

$\langle i_C \rangle = 0 = I_L - \frac{V}{R}$

$I_L = \frac{V}{R} = 3A$

$\langle i_g \rangle = D I_L + D'(0)$

$I_g = D I_L \approx 2.5A$

Check if converter is in CCM
- Assume CCM & check that $I_L > \Delta i_L$

$2\Delta i_L = \frac{V_g - V}{L} DT_s$

$\Delta i_L = \frac{V_g - V}{2L} DT_s = \frac{13}{2(50\mu)} \frac{1}{2} 10\mu s \approx 0.7A$

$\Delta i_L < I_L$ ✓

Need $G_{vd} = \frac{\hat{v}}{\hat{d}}$

$\langle v_L \rangle = L\frac{d\langle i_L\rangle}{dt} = d(t)\langle v_g\rangle - \langle v\rangle$

$\langle i_C \rangle = C\frac{d\langle v\rangle}{dt} = \langle i_L\rangle - \frac{\langle v\rangle}{R}$

$\langle i_g \rangle = d(t)\langle i_L\rangle$

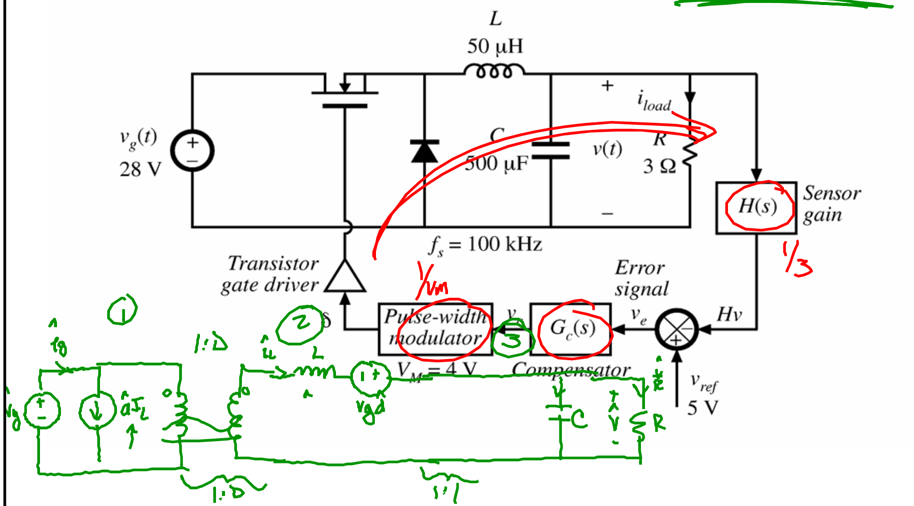
Linearize

$L\frac{d\hat{i}_L}{dt} = D\hat{v}_g + V_g\hat{d} - \hat{v}$ (2)

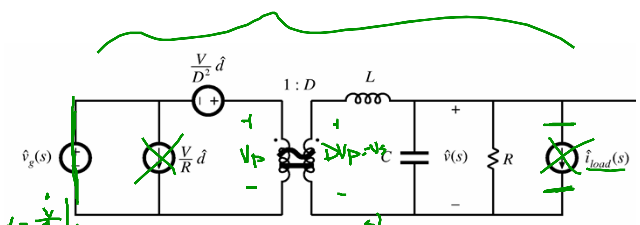
$C\frac{d\hat{v}}{dt} = \hat{i}_L - \frac{\hat{v}}{R}$ (3)

$\hat{i}_g = D\hat{i}_L + \hat{d}I_L$ (1)

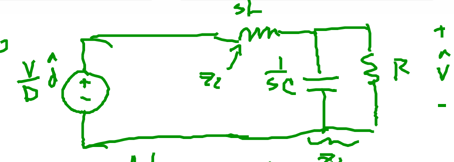
Small-signal model



Open-Loop Control-to-Output Transfer Function



Find $G_{vd} = \frac{\hat{v}}{\hat{d}}\Big|_{\hat{v}_g = 0,\ \hat{i}_{load} = 0}$



$\hat{v} = \frac{V}{D}\hat{d}\,\frac{Z_1}{Z_1 + Z_2}$, $Z_1 = R \,\|\, \frac{1}{sC}$ $\quad Z_2 = sL$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \frac{R}{\sqrt{L/C}}$

$G_{vd} = \frac{\hat{v}}{\hat{d}} = \frac{V}{D}\frac{R\,\|\,\frac{1}{sC}}{sL + R\,\|\,\frac{1}{sC}} = \frac{V}{D}\frac{1}{s^2LC + s\frac{L}{R} + 1} = G_{d0}\frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$ $\quad G_{d0} = \frac{V}{D}$

Open-loop control-to-output transfer function $G_{vd}(s)$

$$G_{vd}(s) = \frac{V}{D}\,\frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

standard form:

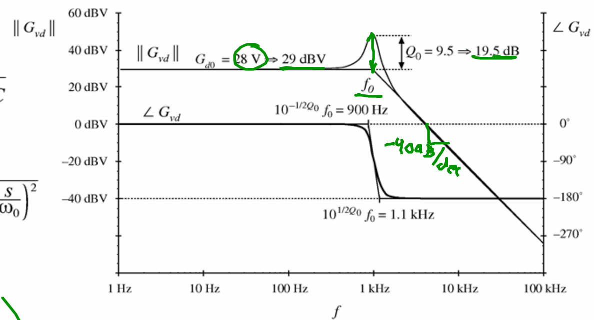
$$G_{vd}(s) = G_{d0}\,\frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$



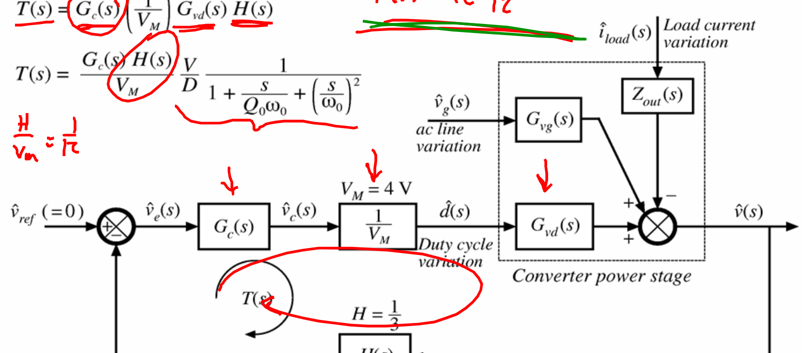
System block diagram

$$T(s) = G_c(s)\left(\frac{1}{V_M}\right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\,H(s)}{V_M}\,\frac{V}{D}\,\frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\frac{H}{V_m} = \frac{1}{12}$$

$$T(s) = G_c \frac{1}{12} G_{vd}$$



Design G_c to get: $\left\{ \begin{array}{l} \text{High } f_c \\ \text{Large } \varphi_m \\ \text{Large } \|T_0\| \end{array} \right.$

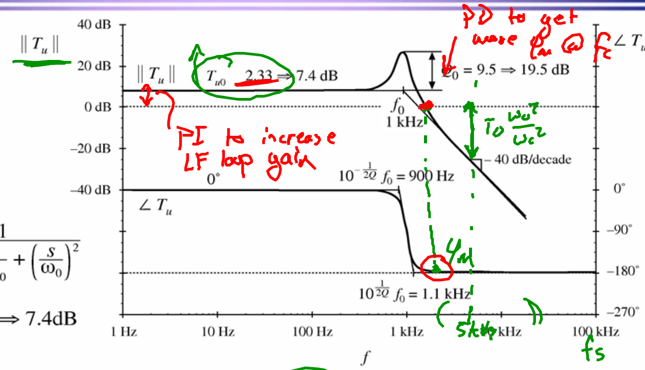
First, look at $G_c = 1$

consider $T_u(s) = \frac{1}{12} G_{vd}$

$V = \frac{1}{H}\frac{T}{1+T} V_{ref}$

$= 3 \frac{2.3}{1+2.3} 5 \approx 10V$ wanted $V = 15V$

Uncompensated loop gain (with $G_c = 1$)



With $G_c = 1$ the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H\,V}{D\,V_M} = 2.33 \Rightarrow 7.4\text{dB}$$

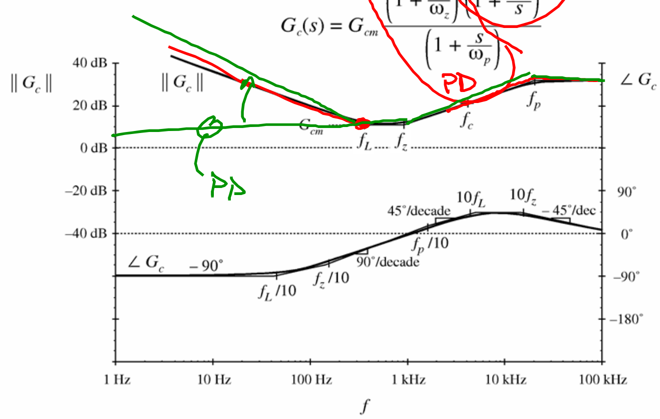
f_c = 1.8 kHz, $\varphi_m = 5°$

$\varphi_M \approx \emptyset$

Improved compensator (PID)

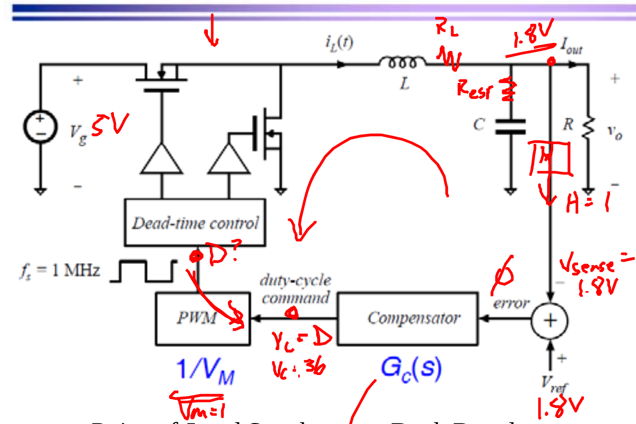
$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

PD



- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

Another Compensator Design Example



Point-of-Load Synchronous Buck Regulator

For Buck:

$\frac{V}{V_g} = M(D) = D$

$D = \frac{1.8}{5} \approx .36$

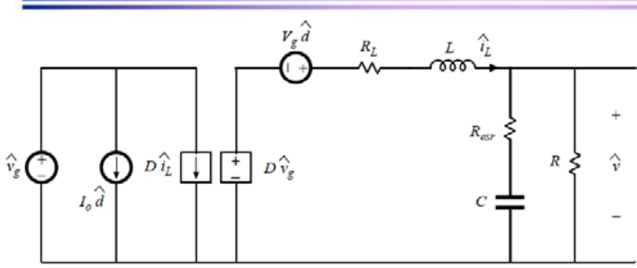
Pact D ≈ .36

Assume G_c such that $T(s)$ has
- Large ϕ_m
- Large $|T_0|$
- Large f_c

Power stage parameters

- Switching frequency: f_s = 1MHz
- V_{ref} = 1.8 V
- I_{out} = 0 to 5 A
- V_g = 5 V
- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ
- V_M = 1 V
- H = 1

Buck Averaged Small-Signal Model



$$G_{vd}(s) = \frac{\hat{v}_o}{\hat{d}}$$

@ $s = j\omega$

$\omega_0 \ll \omega \ll \omega_{esr}$

$$G_{vd}(s) = V_g \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{1}{Q}\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

$j\omega$, $j\omega$

Pair of poles:

$$f_o = \frac{1}{2\pi\sqrt{CL}} = 11\,\text{kHz}$$

$$Q_{loss} = \frac{\sqrt{L/C}}{R_{esr} + R_L} = 2.3 \rightarrow 7.2\,\text{dB} \qquad Q_{load} = \frac{R}{\sqrt{L/C}} > 5$$

$$Q = Q_{loss} \| Q_{load} = \frac{Q_{loss} Q_{load}}{Q_{loss} + Q_{load}} < 2.3 \rightarrow 7.2\,\text{dB}$$

Low-frequency gain (including PWM gain):

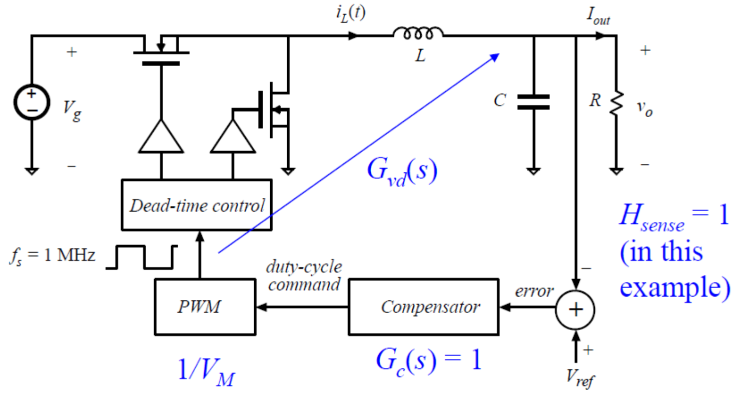
$$G_{vdo} \frac{1}{V_M} = 5 \rightarrow 14\text{dB}$$

$G_{vd} \approx V_g \frac{1}{\frac{\omega^2}{\omega_0^2}} = V_g \frac{\omega_0^2}{\omega^2}$

ESR zero:

$$f_{esr} = \frac{1}{2\pi C R_{esr}} = 1\,\text{MHz}$$

Uncompensated loop gain T_u



$T_u(s) = H_{sense}(1/V_M)G_{vd}(s)$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

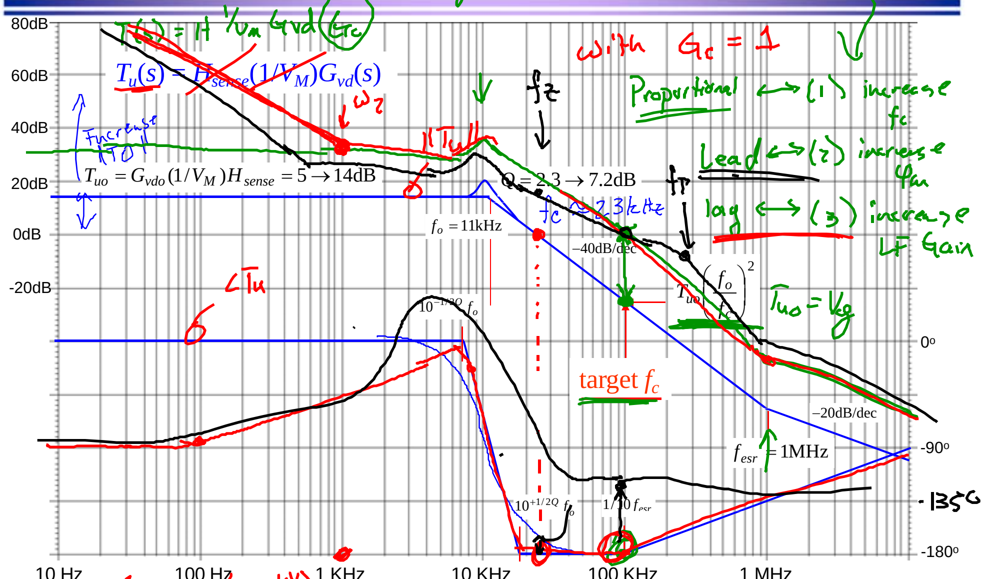
$T_u =$

→ If $G_c = k$, $k \in \mathbb{R}$

$\|T\| = \|T_u G_c\| = \|T_u k\| = \|T_u\| + \|k\|$ $\quad k = T_{uo}\left(\frac{f_o}{f_c}\right)^2$

Magnitude and phase Bode plots of T_u

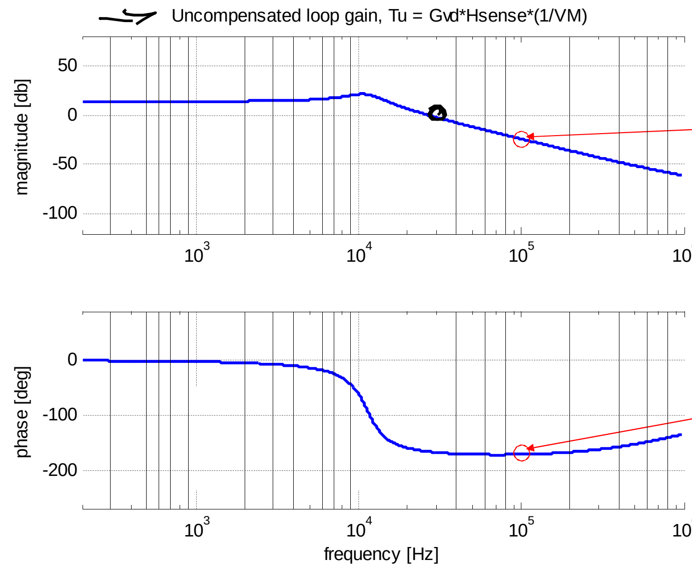
→ Design to meet (1–3)



$G_c = k\left(1 + \frac{\omega_z}{s}\right)$ $\quad \varphi_m \approx 0^\circ$ $\quad \varphi_m = 0^\circ$ $\quad$ want $\varphi_m \gtrsim 52^\circ$

→ $\boxed{\varphi_m = 180^\circ + \angle T(j\omega_c)}$ $\quad$ Need $\theta = 52^\circ$

Magnitude and phase Bode plots of T_u



Exact magnitude and phase responses (MATLAB)

Target cross-over frequency $f_c = f_s/10 = 100$ kHz

No phase margin: a lead (PD) compensator is required

Lead (PD) compensator design

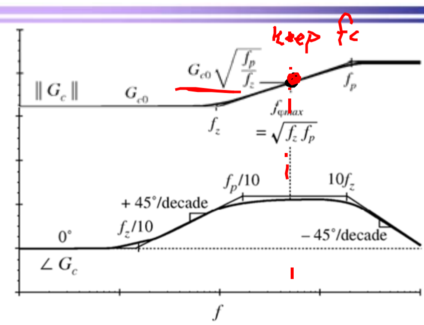
1. Choose: $f_c = 100\text{ kHz}$

   $\theta = \varphi_m = 53^o$

2. Compute:

$$f_z = f_c\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 33\text{ kHz}$$

$$f_p = f_c\sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 300\text{ kHz}$$



3. Find G_{co} to position the crossover frequency:

$$\underbrace{T_{uo}\left(\frac{f_o}{f_c}\right)^2}_{\text{Magnitude of } T_u \text{ at } f_c} \underbrace{G_{co}\sqrt{\frac{f_p}{f_z}}}_{\text{Magnitude of } G_c \text{ at } f_c} = 1$$

$$G_{co} = \frac{1}{T_{uo}}\left(\frac{f_c}{f_o}\right)^2\sqrt{\frac{f_z}{f_p}} = 5.45 \to 15\text{ dB}$$

→ K PD to keep fc

Lead (PD) compensator summary

$$G_c(s) = G_{co} \underbrace{\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)}}_{\text{Lead compensator}} \underbrace{\frac{1}{\left(1+\frac{s}{\omega_{p2}}\right)}}_{\text{HF pole}}$$

Practical limitation

$G_{co} = 5.45 \rightarrow 15\,\text{dB}$

$f_z = 33\,\text{kHz}$

$f_{p1} = 300\,\text{kHz}$

$f_c = 100\,\text{kHz}$ (=1/10 of f_s)

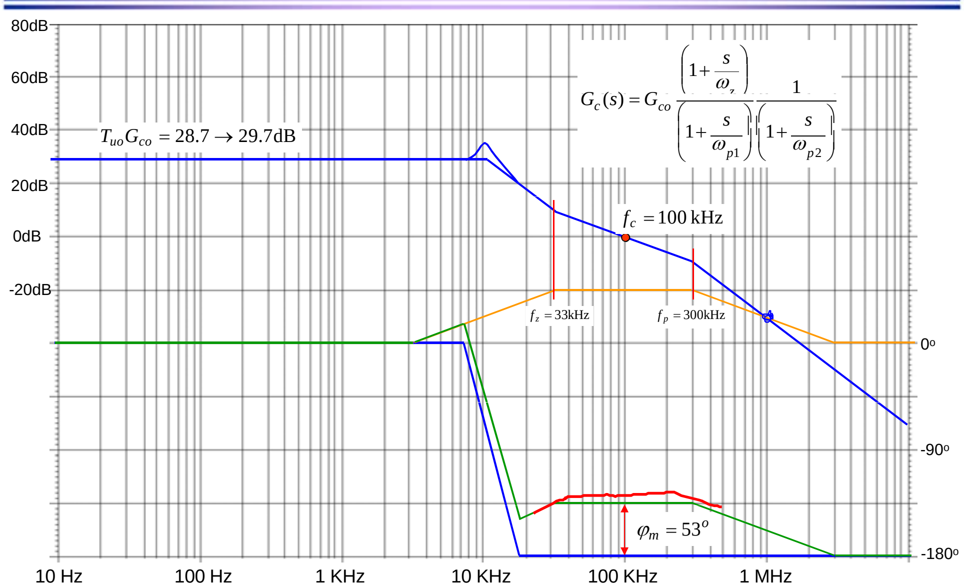
High-frequency gain of the lead compensator: $G_{co} f_{p1}/f_z$ = 49 (34 dB)

Added high-frequency pole: $f_{p2} = 1\,\text{MHz}$ (= f_{esr} = f_s in this example)

Practical implementation would require an op-amp with a gain bandwidth product (GBW) of at least 49*f_{p2} = 49 MHz

K & lead

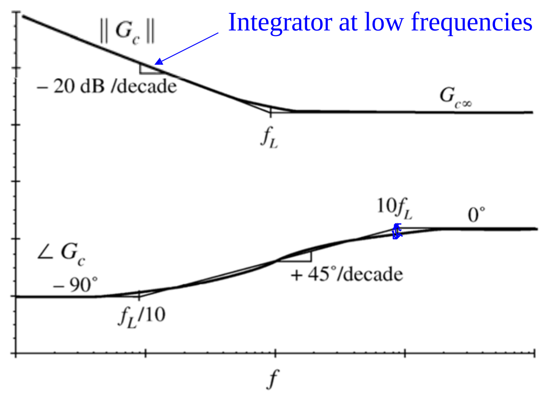
Loop gain with lead (PD) compensator



Add lag (PI) compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

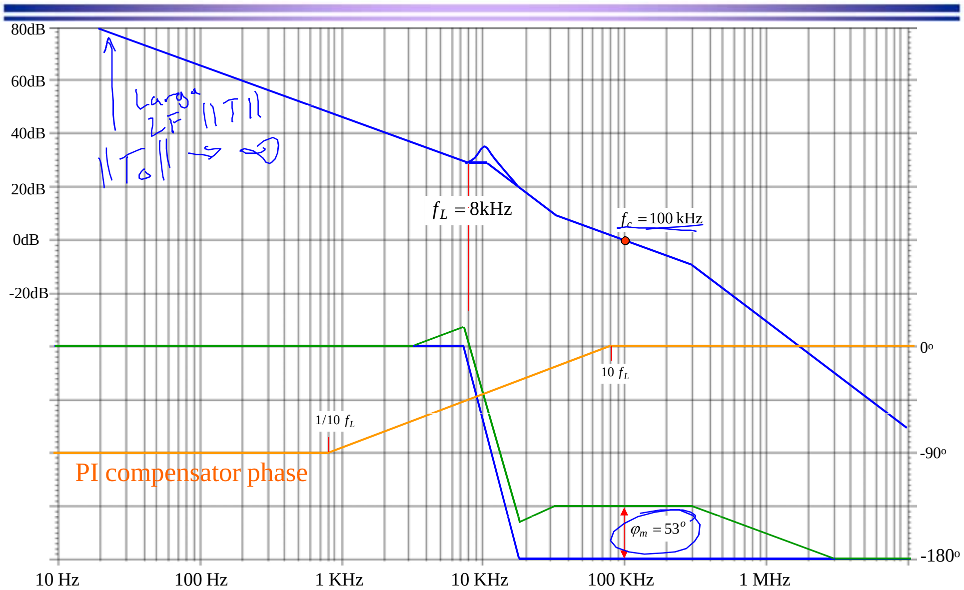
Improves low-frequency loop gain and regulation



Choose $10f_L < f_c$ so that phase margin stays approximately the same: f_L = 8 kHz

Keep the same cross-over frequency: $G_{c\infty} = G_{co} = G_{cm} = 5.45 \rightarrow 15\,\mathrm{dB}$

Adding PI Compensator

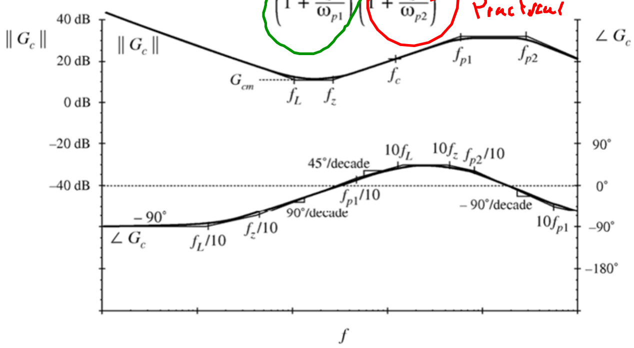


Complete analog PID compensator: summary

PI PD

$$G_c(s) = G_{cm}\frac{\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

Practical



$G_{cm} = 5.45 \rightarrow 15\,\text{dB}$

$f_L = 8\,\text{kHz}$

$f_z = 33\,\text{kHz}$

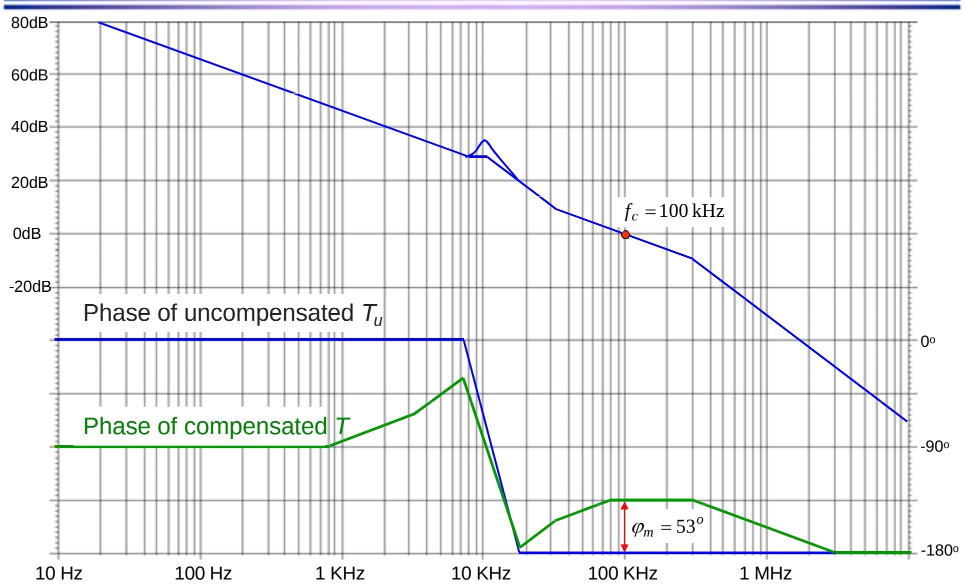
$f_{p1} = 300\,\text{kHz}$

$f_{p2} = 1\,\text{MHz}$

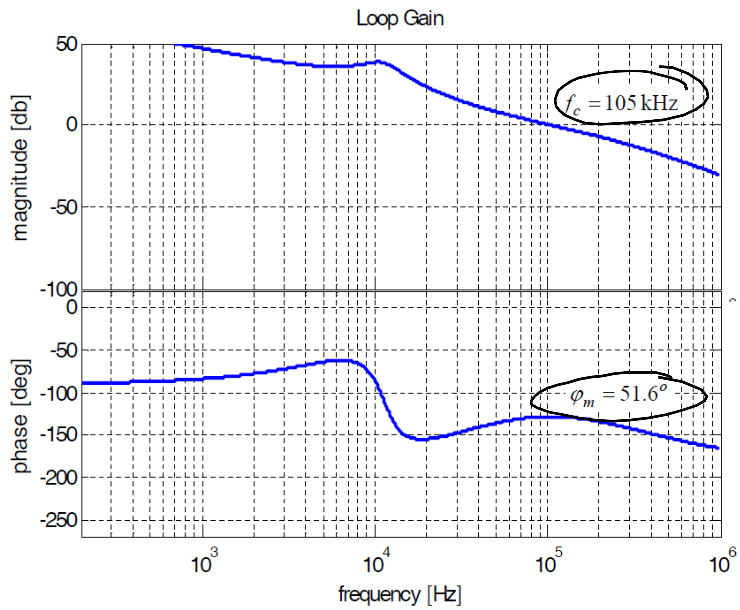
Crossover frequency: $f_c = 100\,\text{kHz}$ (=1/10 of f_s)

Phase margin: $\varphi_m = 53^o$

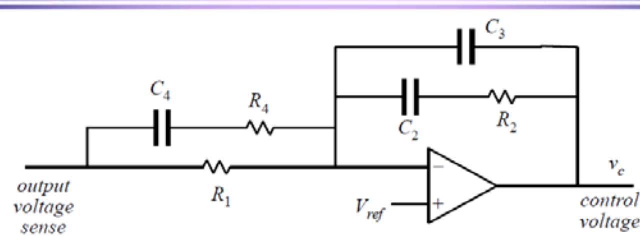
Magnitude and phase Bode plots of T



Verification: exact loop gain magnitude and phase responses (MATLAB)



Analog PID compensator implementation



Design equations (approximate)

$$G_{cm} = \frac{R_2}{R_1} \qquad f_L = \frac{1}{2\pi R_2 C_2}$$

$$f_z = \frac{1}{2\pi (R_1 + R_4) C_4} \qquad f_{p1} = \frac{1}{2\pi R_4 C_4}$$

$$f_{p2} = \frac{1}{2\pi R_2 C_3}$$

$$G_c(s) = G_{cm} \frac{\left(1+\frac{\omega_L}{s}\right)\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$\omega_L < \omega_z < \omega_{p1} < \omega_{p2}$

Verification of closed-loop responses

Closed-loop reference-to-output response

Closed-loop transfer function from $\hat{v}_{ref}$ to $\hat{v}(s)$ is:

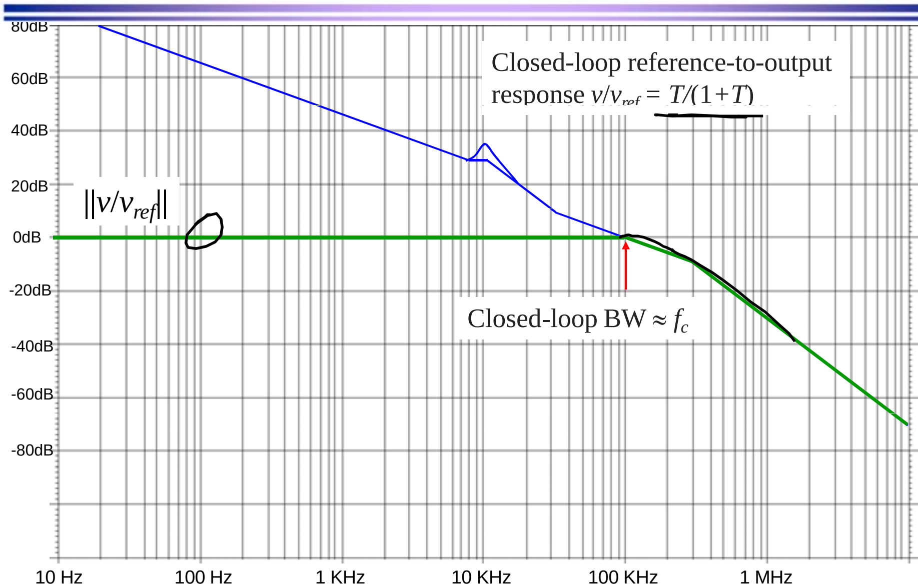
$$\left.\frac{\hat{v}(s)}{\hat{v}_{ref}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}$$

Closed-loop output impedance

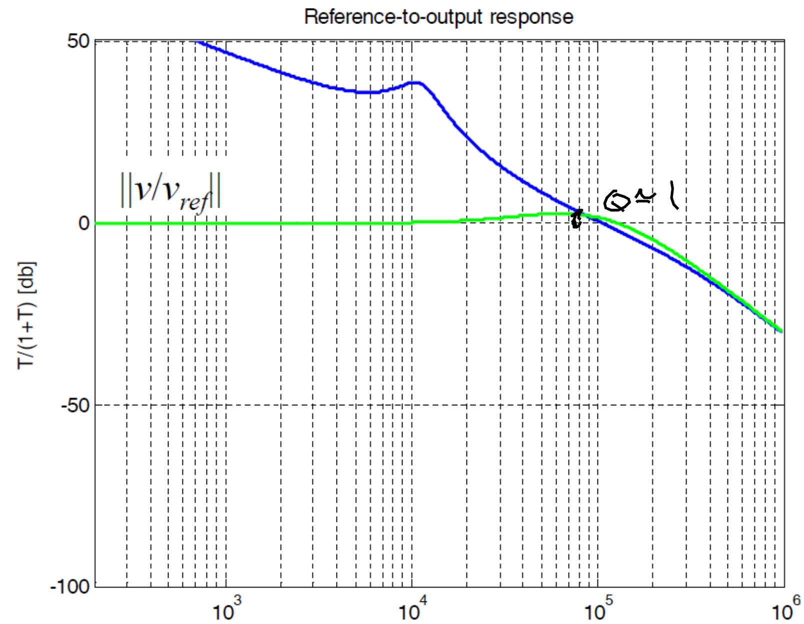
$$\left.\frac{\hat{v}(s)}{-\hat{i}_{load}(s)}\right|_{\substack{\hat{v}_{ref} = 0 \\ \hat{v}_g = 0}} = \frac{Z_{out}(s)}{1 + T(s)}$$

and step-load transient response

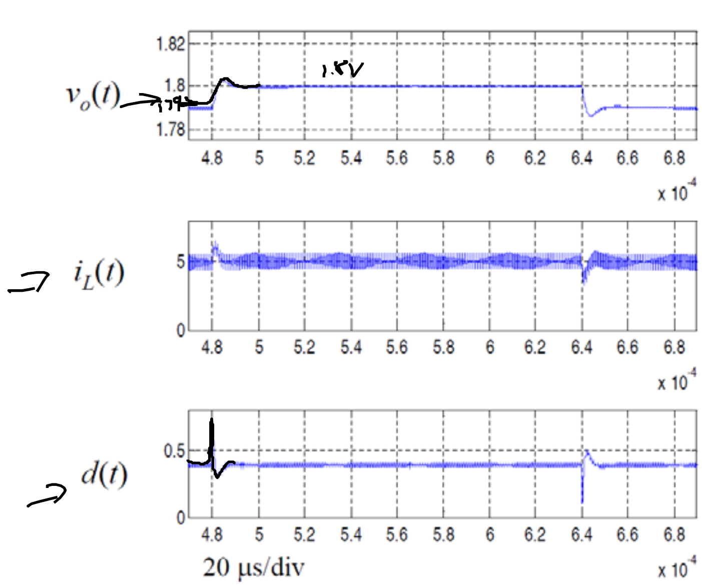
Construction of closed-loop $T/(1+T)$ response



Closed-loop reference-to-output response



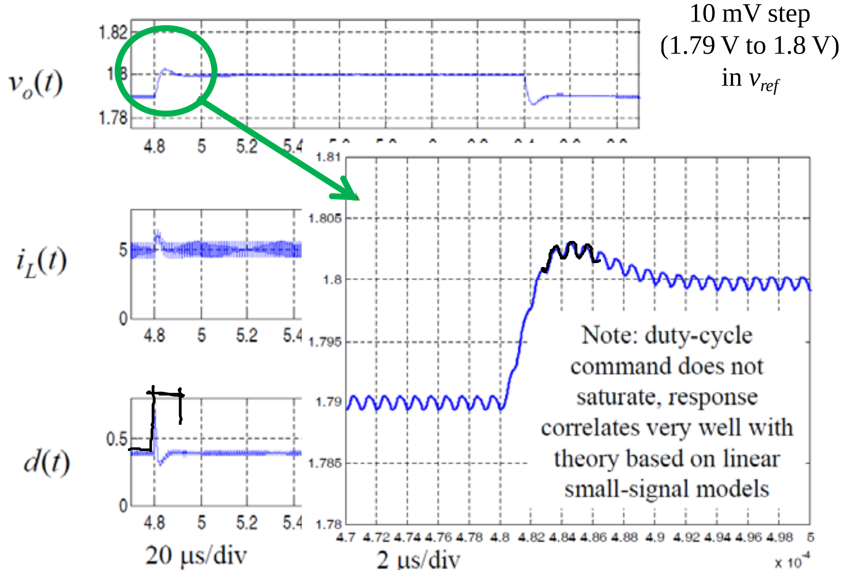
Small-signal step-reference response



10 mV step
(1.79 V to 1.8 V)
in v_{ref}

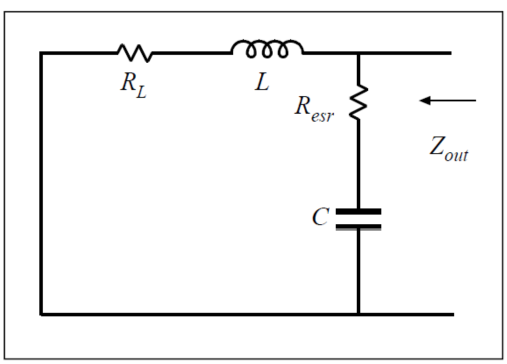
Small-signal step-reference response

10 mV step
(1.79 V to 1.8 V)
in v_{ref}



Output impedance

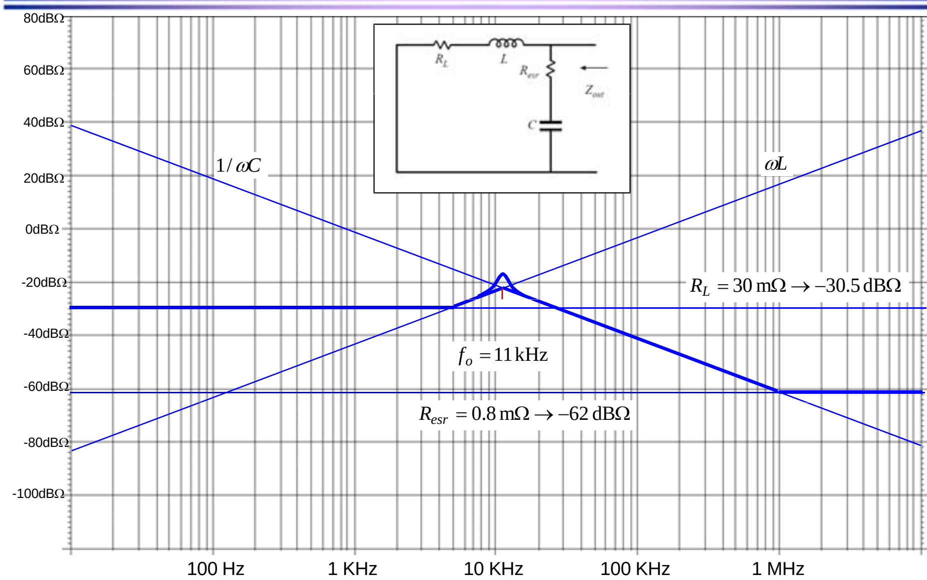
Synchronous buck open-loop output impedance



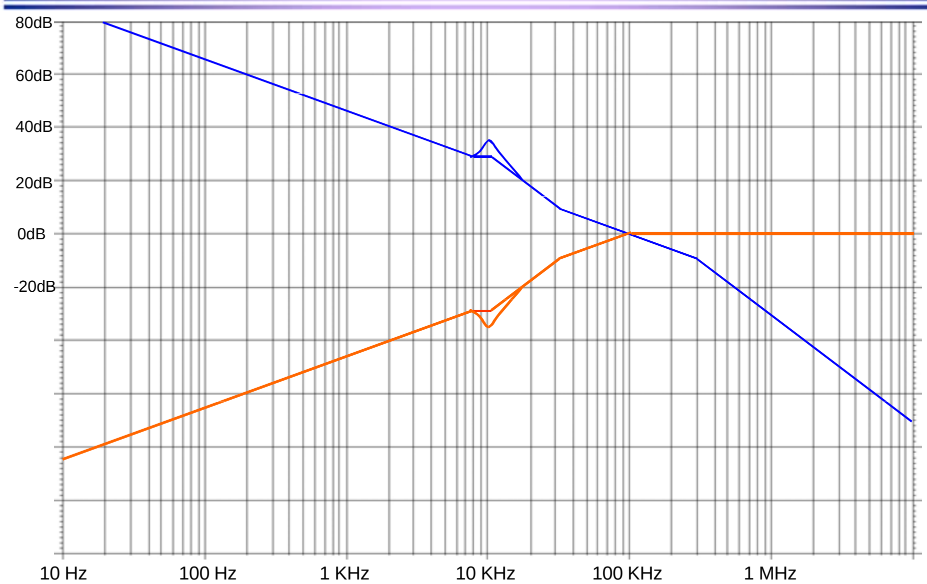
$$Z_{out}(s) = \left(R_{esr} + \frac{1}{sC}\right) \| \left(R_L + sL\right)$$

- L = 1 μH
- R_L = 30 mΩ
- C = 200 μF
- R_{esr} = 0.8 mΩ

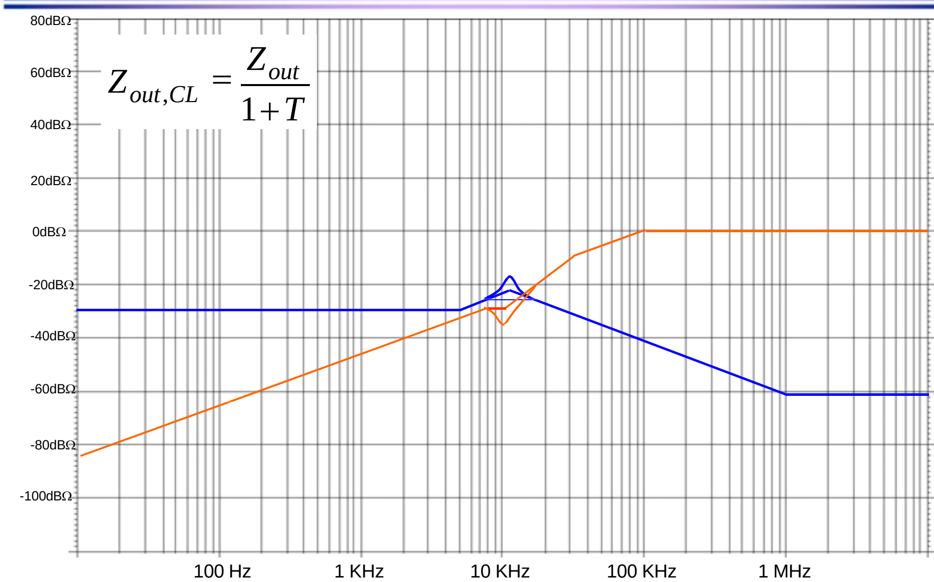
Open-loop output impedance: algebra on the graph



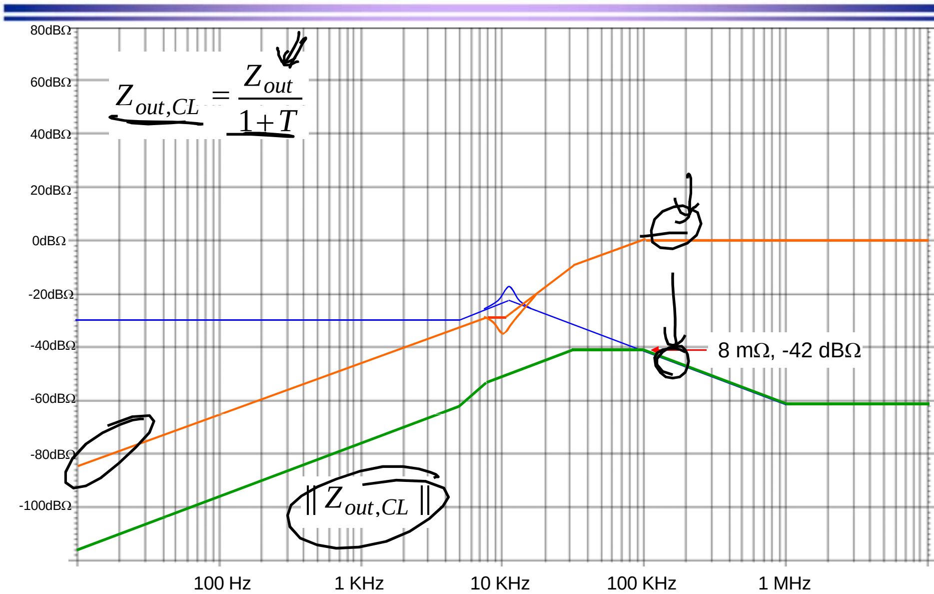
Construction of $1/(1+T)$



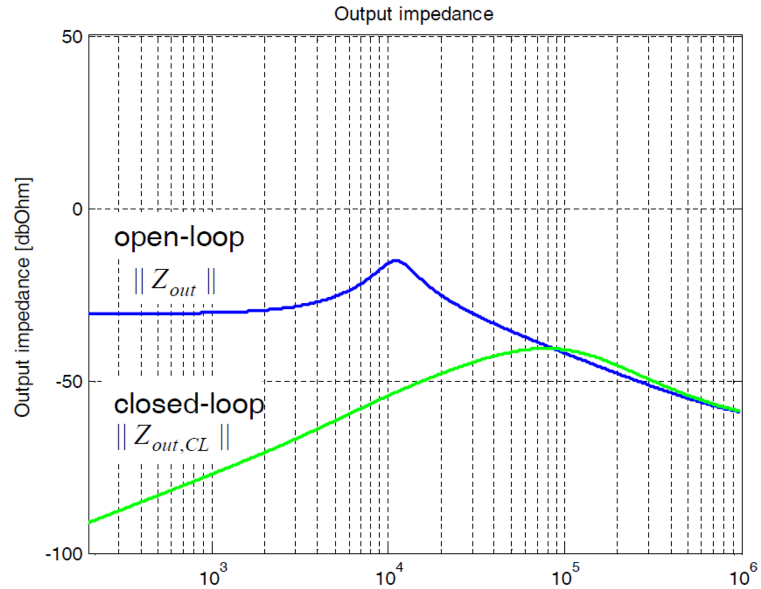
Construction of closed-loop output impedance



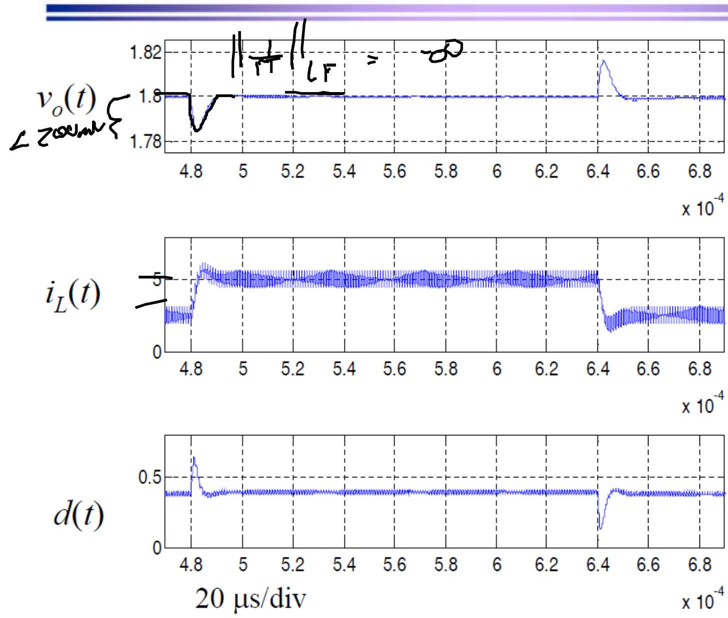
Closed-loop output impedance $Z_{out,CL}$



Verification: closed-loop output impedance



Step-load transient responses



2.5-5 A step-load transient

Step-load transient responses