Lecture 22: Design of Power Magnetics

ECE 481: Power Electronics

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9.7. Summary of key points

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $1/H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.
2. The loop gain $T(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency f_c is the frequency at which the loop gain T has unity magnitude, and is a measure of the bandwidth of the control system.

Summary of key points

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where T is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where T is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.
4. Stability can be assessed using the phase margin test. The phase of T is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

Summary of key points

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or *PD* controllers, are added to improve the phase margin and extend the control system bandwidth. *PI* controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.

& PID

Experimentally measuring T(s)

6. Loop gains can be experimentally measured by use of voltage or current injection. This approach avoids the problem of establishing the correct quiescent operating conditions in the system, a common difficulty in systems having a large dc loop gain. An injection point must be found where interstage loading is not significant. Unstable loop gains can also be measured.

Part III. Magnetics

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Chapter 13 Basic Magnetics Theory

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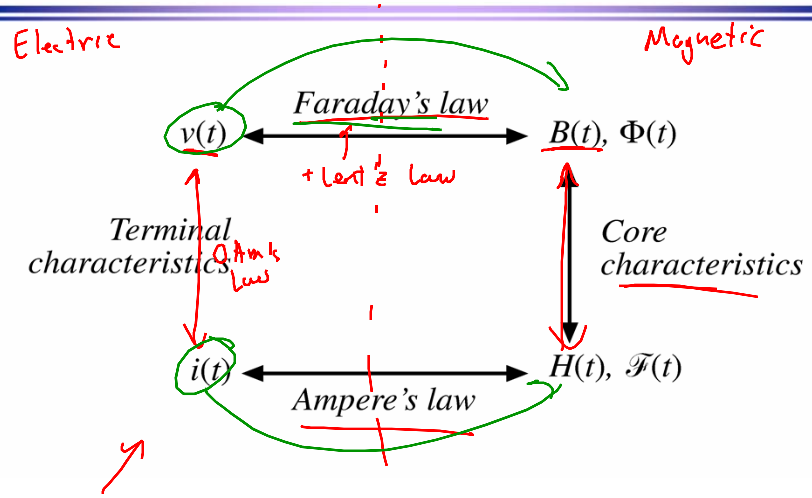
Chapter 13 Basic Magnetics Theory

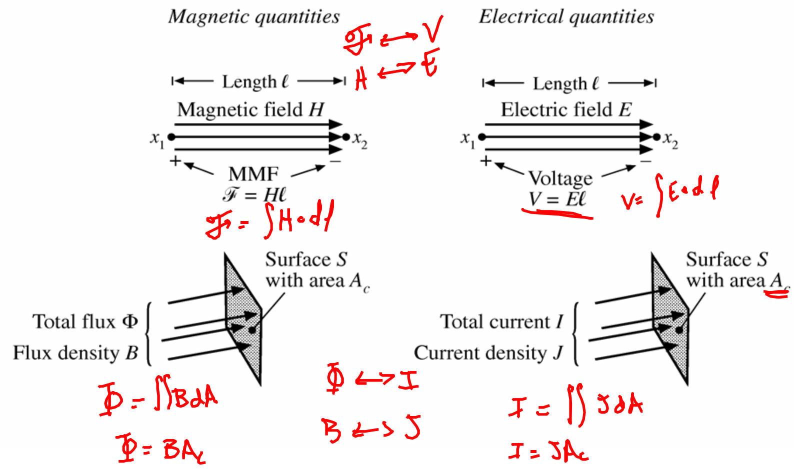
Lm was relatively small

13.1 Review of Basic Magnetics

13.1.1 Basic relationships



Basic quantities

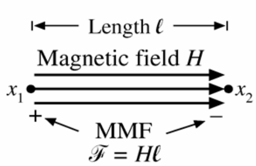


Magnetic field $\boldsymbol{H}$ and magnetomotive force $\mathcal{F}$

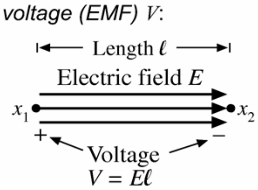
Magnetomotive force (MMF) $\mathcal{F}$ between points x_1 and x_2 is related to the magnetic field $\boldsymbol{H}$ according to

$$\mathcal{F} = \int_{x_1}^{x_2} \boldsymbol{H} \cdot d\boldsymbol{\ell}$$

Example: uniform magnetic field of magnitude H



Analogous to electric field of strength E, which induces voltage (EMF) V:

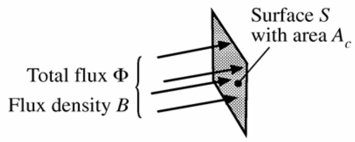


Flux density B and total flux Φ

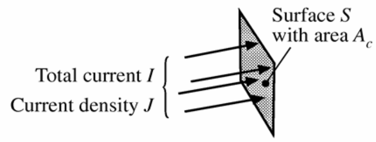
The total magnetic flux Φ passing through a surface of area A_c is related to the flux density $\boldsymbol{B}$ according to

$$\Phi = \int_{surface\ S} \boldsymbol{B} \cdot d\boldsymbol{A}$$

Example: uniform flux density of magnitude B



Analogous to electrical conductor current density of magnitude J, which leads to total conductor current I:



Faraday's law

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

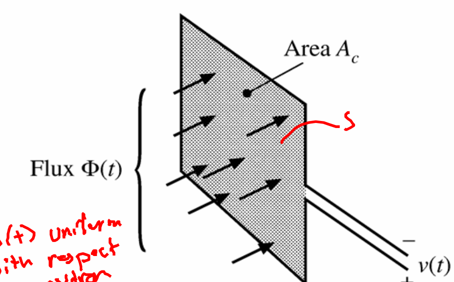
$$\oint E \cdot dl = v(t) = \frac{d\Phi(t)}{dt}$$

Assumption/Simplification

For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$\boxed{v(t) = A_c \frac{dB(t)}{dt}}$$

B(t) uniform with respect to position

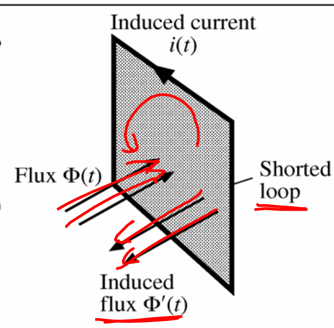


Lenz's law

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

Example: a shorted loop of wire

- Changing flux $\Phi(t)$ induces a voltage $v(t)$ around the loop
- This voltage, divided by the impedance of the loop conductor, leads to current $i(t)$
- This current induces a flux $\Phi'(t)$, which tends to oppose changes in $\Phi(t)$



Ampere's law

The net MMF around a closed path is equal to the total current passing through the interior of the path:

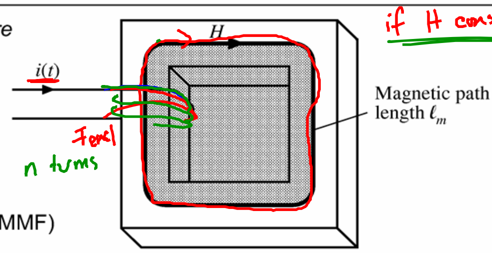
$$\mathcal{F} = \oint_{\text{closed path}} \boldsymbol{H} \cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

Example: magnetic core. Wire carrying current $i(t)$ passes through core window.

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength $H(t)$, the integral (MMF) is $H(t)\ell_m$. So

$$\mathcal{F}(t) = H(t)\ell_m = i(t)$$

if H constant



$H(t)\ell_m = n\,i(t)$

Ampere's law: discussion

- Relates magnetic field strength $H(t)$ to winding current $i(t)$
- We can view winding currents as sources of MMF
- Previous example: total MMF around core, $\mathscr{F}(t) = H(t)\ell_m$, is equal to the winding current MMF $i(t)$
- The total MMF around a closed loop, accounting for winding current MMF's, is zero (like voltage)

$\Phi(t)$ only flows in a closed path (like current)

Faraday: $v(t) = A_c \frac{dB(t)}{dt}$

in saturation ↓ $v(t) = A_c \frac{dB_{sat}}{dt}$

$v(t) = 0$

Ampere: $H = \frac{n\,i(t)}{l_m}$

$H_{sat} = \frac{n}{l_m} I_{sat}$

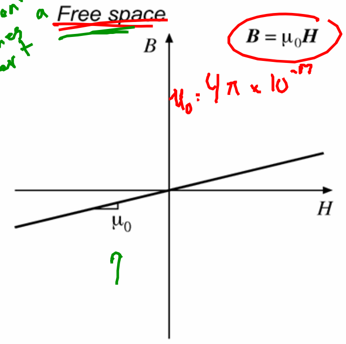
Core material characteristics: the relation between *B* and *H*

Magnetic component becomes a short

Free space

$B = \mu_0 H$

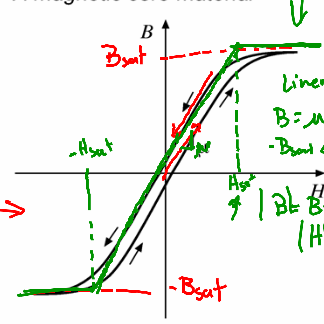
$\mu_0 = 4\pi \times 10^{-7}$



μ_0 = permeability of free space

= $4\pi \cdot 10^{-7}$ Henries per meter

A magnetic core material



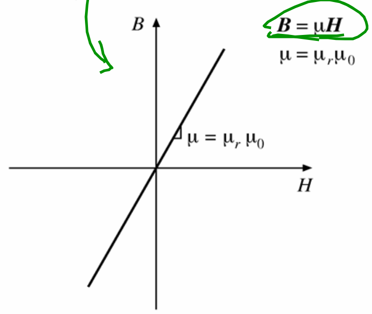
Linear model $B = \mu H$ if $-B_{sat} < B < B_{sat}$

$|B| \leq B_{sat}$ if $|H| \geq H_{sat}$

Highly nonlinear, with ~~hysteresis~~ and saturation

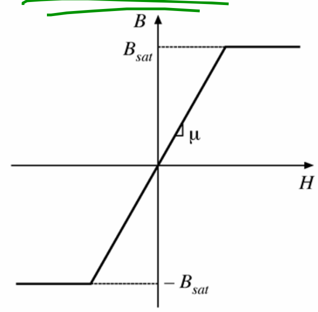
Piecewise-linear modeling of core material characteristics

No hysteresis or saturation



Typical $\mu_r = 10^3$ to 10^5

Saturation, no hysteresis



Typical B_{sat} = 0.3 to 0.5T, ferrite
0.5 to 1T, powdered iron
1 to 2T, iron laminations

Units

Table 12.1. Units for magnetic quantities

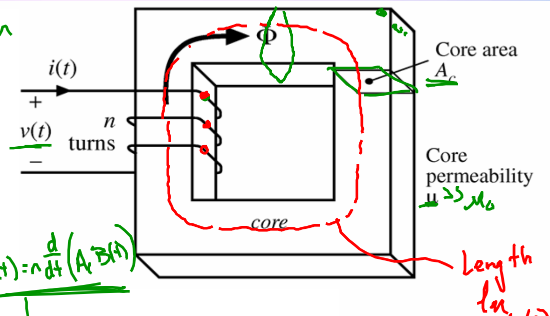
quantity	*MKS*	*unrationalized cgs*	*conversions*
core material equation	$B = \mu_0 \mu_r H$	$B = \mu_r H$	
B	Tesla	Gauss	$1\text{T} = 10^4\text{G}$
H	Ampere / meter	Oersted	$1\text{A/m} = 4\pi \cdot 10^{-3}$ Oe
Φ	Weber	Maxwell	$1\text{Wb} = 10^8$ Mx $1\text{T} = 1\text{Wb} / \text{m}^2$

Example: a simple inductor

Because $\mu \gg \mu_0$

→ Assume Φ flows through the core

→ Assume B & H are uniform throughout the core material



Faraday's: $v_{turn}(t) = \frac{d\Phi(t)}{dt}$

$$\left.\begin{aligned} v(t) &= n v_{turn}(t) = n\frac{d\Phi(t)}{dt} \\ \Phi(t) &= A_c B(t) \end{aligned}\right\} v(t) = n\frac{d}{dt}\left(A_c B(t)\right)$$

Core Material

(1) $B = \mu H$ if not saturated → $v(t) = n\frac{d}{dt}\left(A_c \mu H(t)\right)$

Ampere's: $H(t) l_m = n i(t)$

$v_L(t) = L \frac{d i_L(t)}{dt}$

(2) $B = B_{sat}$ if saturated

$v(t) = \phi \quad (L = \phi)$

$$v(t) = n\frac{d}{dt}\left(A_c \mu \frac{n}{l_m} i(t)\right) = \frac{n^2 \mu A_c}{l_m}\frac{di(t)}{dt}$$

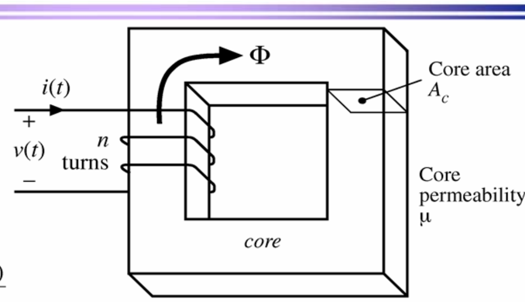
$$\boxed{L = \frac{n^2 \mu A_c}{l_m}}$$

Example: a simple inductor

Faraday's law:

For each turn of wire, we can write

$$v_{turn}(t) = \frac{d\Phi(t)}{dt}$$



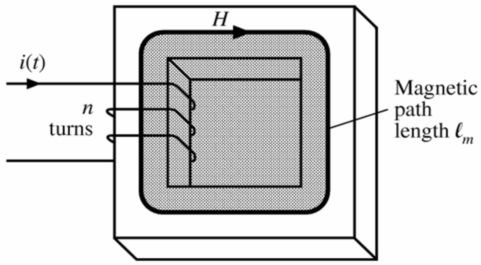
Total winding voltage is

$$v(t) = n v_{turn}(t) = n\frac{d\Phi(t)}{dt}$$

Express in terms of the average flux density $B(t) = \mathscr{F}(t)/A_c$

$$v(t) = nA_c \frac{dB(t)}{dt}$$

Inductor example: Ampere's law



Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the *mean magnetic path length* ℓ_m.

For uniform field strength $H(t)$, the core MMF around the path is $H\,\ell_m$.

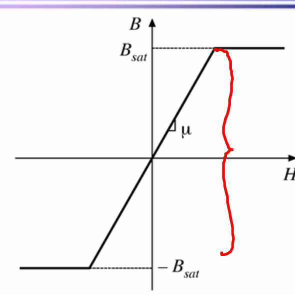
Winding contains n turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $ni(t)$.

From Ampere's law, we have

$$\underline{H(t)\,\ell_m = n\,i(t)}$$

Inductor example: core material model

$$B = \begin{cases} B_{sat} & \text{for } H \ge B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \le -B_{sat}/\mu \end{cases}$$



Find winding current at onset of saturation: substitute $i = I_{sat}$ and $H = B_{sat}/\mu$ into equation previously derived via Ampere's law. Result is

$$\boxed{I_{sat} = \frac{B_{sat}\ell_m}{\mu n}}$$

$$L = \frac{\mu A_c n^2}{l_m}$$

Electrical terminal characteristics

We have:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\,\ell_m = n\,i(t) \qquad B = \begin{cases} B_{sat} & \text{for } H \ge B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \le -B_{sat}/\mu \end{cases}$$

Eliminate B and H, and solve for relation between v and i. For $|i| < I_{sat}$,

$$v(t) = \mu n A_c \frac{dH(t)}{dt} \quad \longrightarrow \quad v(t) = \frac{\mu n^2 A_c}{\ell_m}\frac{di(t)}{dt}$$

which is of the form

$$v(t) = L\frac{di(t)}{dt} \qquad \text{with} \qquad \boxed{L = \frac{\mu n^2 A_c}{\ell_m}}$$

—an inductor

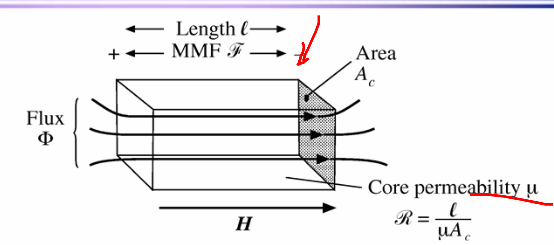
For $|i| > I_{sat}$ the flux density is constant and equal to B_{sat}. Faraday's law then predicts

$$v(t) = nA_c\frac{dB_{sat}}{dt} = 0$$

—saturation leads to short circuit

13.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:



MMF between ends of element is

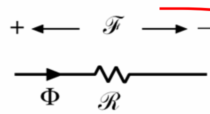
$$\mathscr{F} = H\ell$$

Since $H = B/\mu$ and $B = \Phi / A_c$, we can express $\mathscr{F}$ as

Ohm's Law analogue

$$\boxed{\mathscr{F} = \Phi\mathscr{R}} \qquad \text{with} \qquad \boxed{\mathscr{R} = \frac{\ell}{\mu A_c}}$$

A corresponding model:



$\mathscr{R}$ = reluctance of element

Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

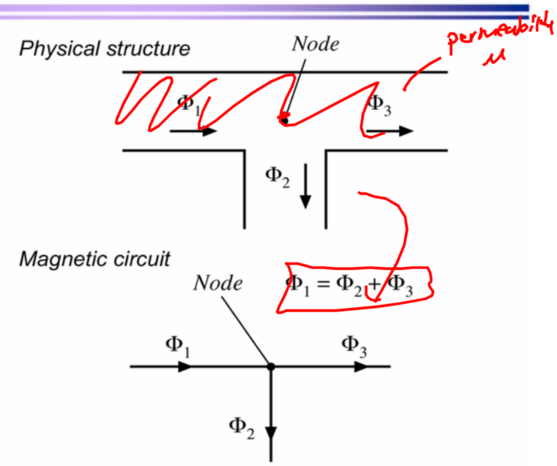
- Represent each element with reluctance
- Windings are sources of MMF
- MMF → voltage, flux → current
- Solve magnetic circuit using Kirchoff's laws, etc.

Magnetic analog of Kirchoff's current law

Divergence of $\boldsymbol{B} = 0$

Flux lines are continuous and cannot end

Total flux entering a node must be zero



Magnetic analog of Kirchoff's voltage law

Follows from Ampere's law:

$$\mathcal{F}_1 = \oint_{closed\ path} \boldsymbol{H} \cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

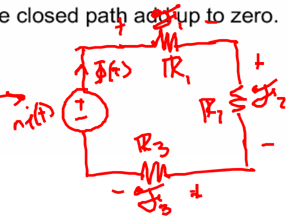
Left-hand side: sum of MMF's across the reluctances around the closed path

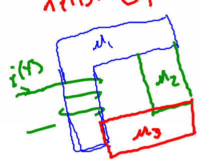
Right-hand side: currents in windings are sources of MMF's. An n-turn winding carrying current $i(t)$ is modeled as an MMF (voltage) source, of value $ni(t)$.

Total MMF's around the closed path add up to zero.

$n i(t) = \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3$

$\mathcal{F}_{winding} = n i(t)$



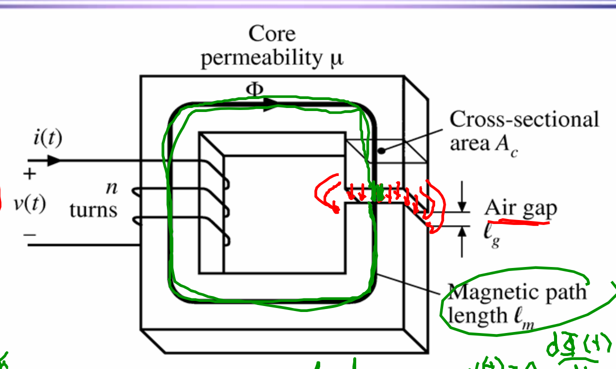


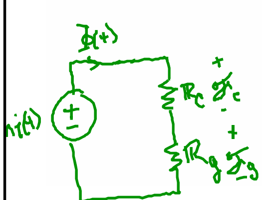
Example: inductor with air gap

$L = \frac{n^2}{\frac{l_m}{\mu A_c} + \frac{l_g}{\mu_0 A_c}}$

if $\mu >> \mu_0$

$\boxed{L \approx \frac{\mu_0 A_c n^2}{l_g}}$





$\Phi(t) = \frac{n i(t)}{\mathcal{R}_c + \mathcal{R}_g}$

$\mathcal{R}_c = \frac{l_m - l_g}{\mu A_c}$

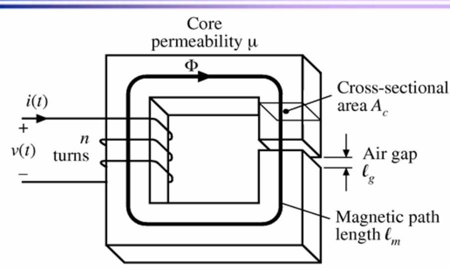
$\mathcal{R}_g = \frac{l_g}{\mu_0 A_c}$

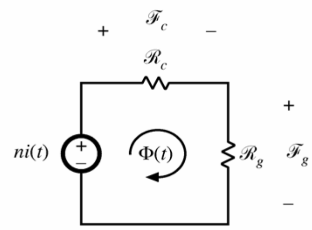
$v(t) = n \frac{d\Phi(t)}{dt}$

$v(t) = \boxed{\frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}} \frac{di(t)}{dt}$

$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$

Magnetic circuit model

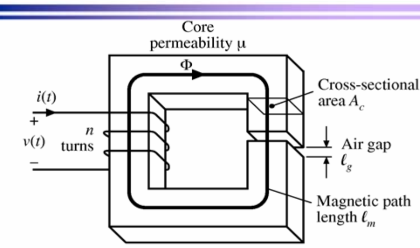


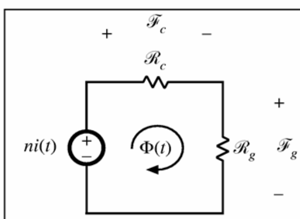


$$\mathscr{F}_c + \mathscr{F}_g = ni \qquad \mathscr{R}_c = \frac{\ell_c}{\mu A_c}$$

$$ni = \Phi\left(\mathscr{R}_c + \mathscr{R}_g\right) \qquad \mathscr{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Solution of model





$$ni = \Phi\left(\mathscr{R}_c + \mathscr{R}_g\right)$$

$$\mathscr{R}_c = \frac{\ell_c}{\mu A_c}$$

$$\mathscr{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Faraday's law: $v(t) = n\dfrac{d\Phi(t)}{dt}$

Substitute for Φ: $v(t) = \dfrac{n^2}{\mathscr{R}_c + \mathscr{R}_g}\dfrac{di(t)}{dt}$

Hence inductance is

$$\boxed{L = \frac{n^2}{\mathscr{R}_c + \mathscr{R}_g}}$$

Effect of air gap

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\Phi_{sat} = B_{sat}A_c$$

$$I_{sat} = \frac{B_{sat}A_c}{n}\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

Effect of air gap:

- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability