Lecture 22: Design of Power Magnetics

ECE 481: Power Electronics

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9.7. Summary of key points

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $1/H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.
2. The loop gain $T(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency f_c is the frequency at which the loop gain T has unity magnitude, and is a measure of the bandwidth of the control system.

Summary of key points

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where T is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where T is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.
4. Stability can be assessed using the phase margin test. The phase of T is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

Summary of key points

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or *PD* controllers, are added to improve the phase margin and extend the control system bandwidth. *PI* controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.
6. Loop gains can be experimentally measured by use of voltage or current injection. This approach avoids the problem of establishing the correct quiescent operating conditions in the system, a common difficulty in systems having a large dc loop gain. An injection point must be found where interstage loading is not significant. Unstable loop gains can also be measured.

Part III. Magnetics

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Chapter 13 Basic Magnetics Theory

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Chapter 13 Basic Magnetics Theory

13.5 Several Types of Magnetic Devices, Their *B–H* Loops, and Core vs. Copper Loss

- 13.5.1 Filter inductor
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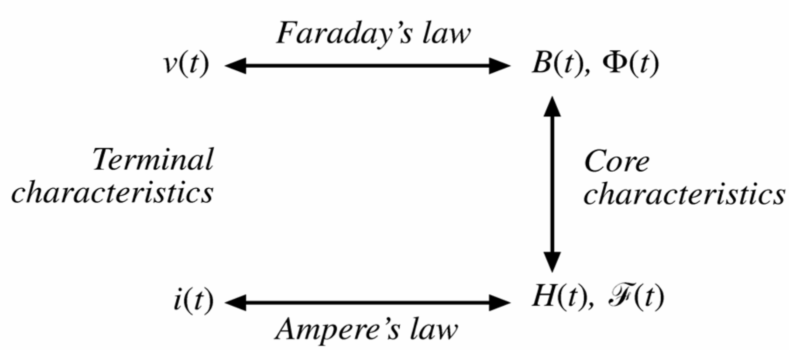
- 13.5.3 Transformer
- 13.5.4 Coupled inductor
- 13.5.5 Flyback transformer

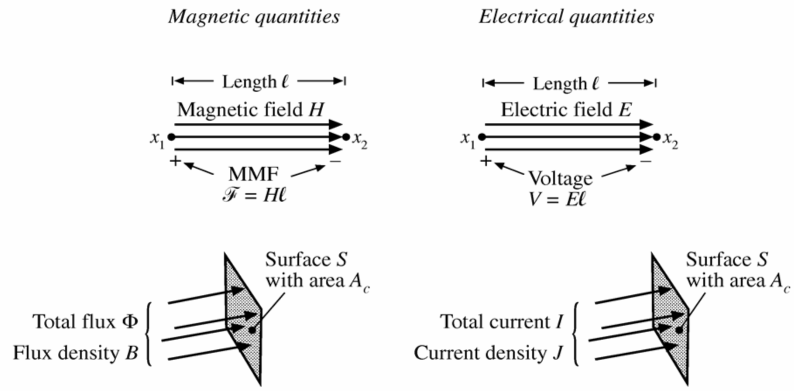
13.6 Summary of Key Points

13.1 Review of Basic Magnetics

13.1.1 Basic relationships



Basic quantities

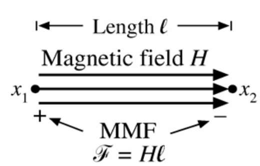


Magnetic field $\boldsymbol{H}$ and magnetomotive force $\mathscr{F}$

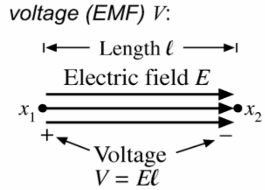
Magnetomotive force (MMF) $\mathscr{F}$ between points x_1 and x_2 is related to the magnetic field $\boldsymbol{H}$ according to

$$\mathscr{F} = \int_{x_1}^{x_2} \boldsymbol{H}\cdot d\boldsymbol{\ell}$$

Example: uniform magnetic field of magnitude H



Analogous to electric field of strength E, which induces voltage (EMF) V:

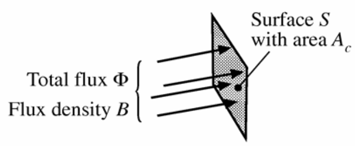


Flux density $\boldsymbol{B}$ and total flux Φ

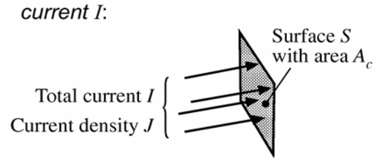
The total magnetic flux Φ passing through a surface of area A_c is related to the flux density $\boldsymbol{B}$ according to

$$\Phi = \int_{surface\ S} \boldsymbol{B} \cdot \boldsymbol{dA}$$

Example: uniform flux density of magnitude B



Analogous to electrical conductor current density of magnitude J, which leads to total conductor current I:



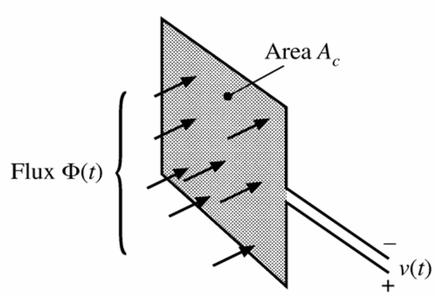
Faraday's law

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

$$v(t) = \frac{d\Phi(t)}{dt}$$

For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$v(t) = A_c \frac{dB(t)}{dt}$$

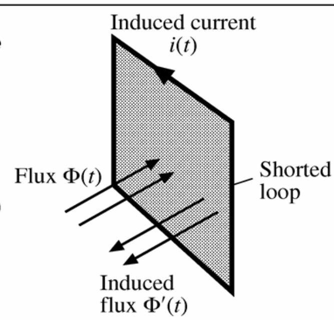


Lenz's law

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

Example: a shorted loop of wire

- Changing flux $\Phi(t)$ induces a voltage $v(t)$ around the loop
- This voltage, divided by the impedance of the loop conductor, leads to current $i(t)$
- This current induces a flux $\Phi'(t)$, which tends to oppose changes in $\Phi(t)$



Ampere's law

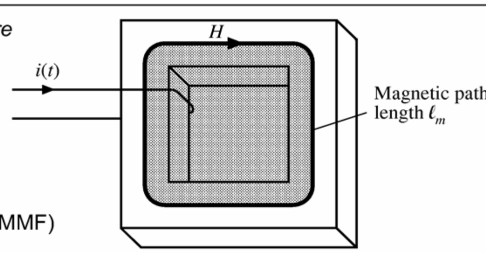
The net MMF around a closed path is equal to the total current passing through the interior of the path:

$$\oint_{closed\ path} \boldsymbol{H} \cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

Example: magnetic core. Wire carrying current $i(t)$ passes through core window.

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength $H(t)$, the integral (MMF) is $H(t)\ell_m$. So

$$\mathscr{F}(t) = H(t)\ell_m = i(t)$$

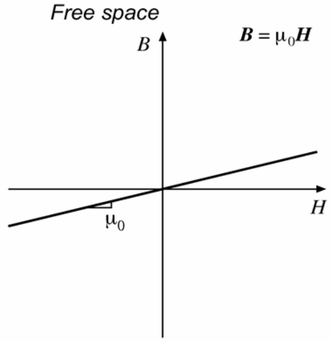


Ampere's law: discussion

- Relates magnetic field strength $H(t)$ to winding current $i(t)$
- We can view winding currents as sources of MMF
- Previous example: total MMF around core, $\mathcal{F}(t) = H(t)\ell_m$, is equal to the winding current MMF $i(t)$
- The total MMF around a closed loop, accounting for winding current MMF's, is zero

Core material characteristics: the relation between *B* and *H*

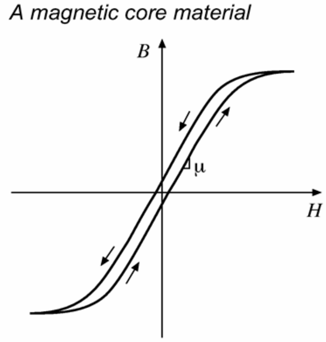
Free space



μ_0 = permeability of free space

= $4\pi \cdot 10^{-7}$ Henries per meter

A magnetic core material



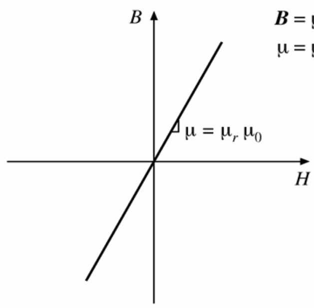
Highly nonlinear, with hysteresis and saturation

Piecewise-linear modeling of core material characteristics

No hysteresis or saturation

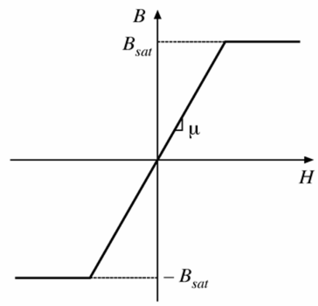
$B = \mu H$

$\mu = \mu_r \mu_0$



Typical $\mu_r = 10^3$ to 10^5

Saturation, no hysteresis



Typical B_{sat} = 0.3 to 0.5T, ferrite
0.5 to 1T, powdered iron
1 to 2T, iron laminations

Units

Table 12.1. Units for magnetic quantities

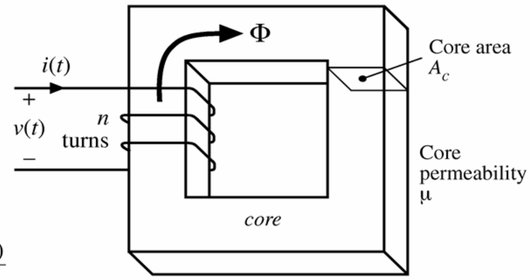
quantity	*MKS*	*unrationalized cgs*	*conversions*
core material equation	$B = \mu_0\ \mu_r\ H$	$B = \mu_r\ H$	
B	Tesla	Gauss	$1\text{T} = 10^4\text{G}$
H	Ampere / meter	Oersted	$1\text{A/m} = 4\pi \cdot 10^{-3}$ Oe
Φ	Weber	Maxwell	$1\text{Wb} = 10^8$ Mx $1\text{T} = 1\text{Wb} / \text{m}^2$

Example: a simple inductor

Faraday's law:

For each turn of wire, we can write

$$v_{turn}(t) = \frac{d\Phi(t)}{dt}$$



Total winding voltage is

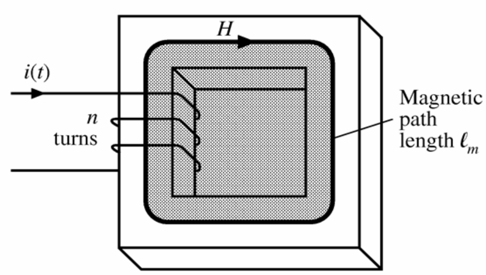
$$v(t) = nv_{turn}(t) = n\,\frac{d\Phi(t)}{dt}$$

Express in terms of the average flux density $B(t) = \mathcal{F}(t)/A_c$

$$v(t) = nA_c\,\frac{dB(t)}{dt}$$

Inductor example: Ampere's law

Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the *mean magnetic path length* ℓ_m.



For uniform field strength $H(t)$, the core MMF around the path is $H\,\ell_m$.

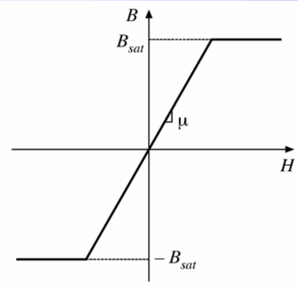
Winding contains n turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $ni(t)$.

From Ampere's law, we have

$$H(t)\,\ell_m = n\,i(t)$$

Inductor example: core material model

$$B = \begin{cases} B_{sat} & \text{for } H \ge B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \le -B_{sat}/\mu \end{cases}$$



Find winding current at onset of saturation: substitute $i = I_{sat}$ and $H = B_{sat}/\mu$ into equation previously derived via Ampere's law. Result is

$$I_{sat} = \frac{B_{sat}\ell_m}{\mu n}$$

Electrical terminal characteristics

We have:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\,\ell_m = n\,i(t) \qquad B = \begin{cases} B_{sat} & \text{for } H \ge B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \le -B_{sat}/\mu \end{cases}$$

Eliminate B and H, and solve for relation between v and i. For $|i| < I_{sat}$,

$$v(t) = \mu n A_c \frac{dH(t)}{dt} \longrightarrow v(t) = \frac{\mu n^2 A_c}{\ell_m}\frac{di(t)}{dt}$$

which is of the form

$$v(t) = L\frac{di(t)}{dt} \qquad \text{with} \qquad L = \frac{\mu n^2 A_c}{\ell_m}$$

—an inductor

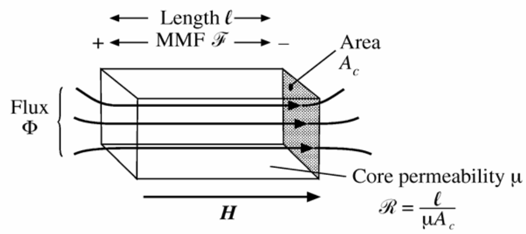
For $|i| > I_{sat}$ the flux density is constant and equal to B_{sat}. Faraday's law then predicts

$$v(t) = nA_c \frac{dB_{sat}}{dt} = 0$$

—saturation leads to short circuit

13.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:



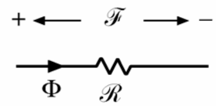
MMF between ends of element is

$$\mathscr{F} = H\ell$$

Since $H = B / \mu$ and $B = \Phi / A_c$, we can express $\mathscr{F}$ as

$$\mathscr{F} = \Phi\mathscr{R} \qquad \text{with} \qquad \mathscr{R} = \frac{\ell}{\mu A_c}$$

A corresponding model:



$\mathscr{R}$ = reluctance of element

Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

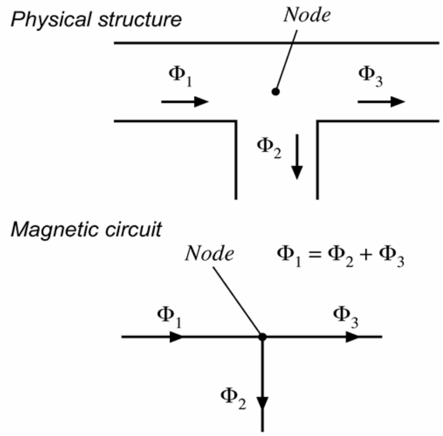
- Represent each element with reluctance
- Windings are sources of MMF
- MMF → voltage, flux → current
- Solve magnetic circuit using Kirchoff's laws, etc.

Magnetic analog of Kirchoff's current law

Divergence of $\boldsymbol{B} = 0$

Flux lines are continuous and cannot end

Total flux entering a node must be zero



$$\Phi_1 = \Phi_2 + \Phi_3$$

Magnetic analog of Kirchoff's voltage law

Follows from Ampere's law:

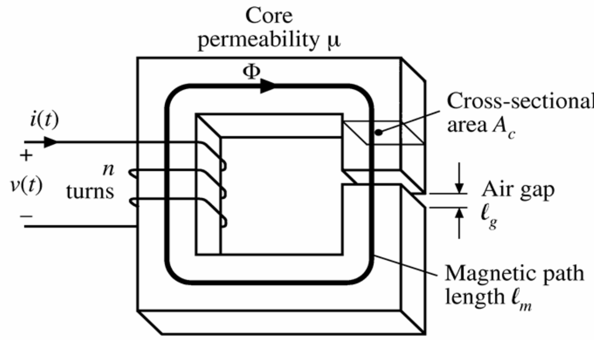
$$\oint_{closed\ path} \boldsymbol{H} \cdot d\boldsymbol{\ell} = \text{total current passing through interior of path}$$

Left-hand side: sum of MMF's across the reluctances around the closed path

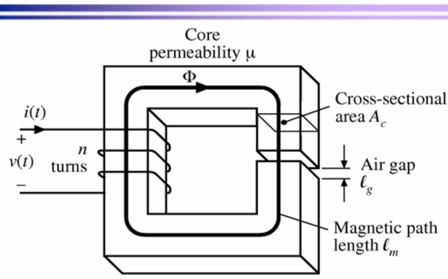
Right-hand side: currents in windings are sources of MMF's. An n-turn winding carrying current $i(t)$ is modeled as an MMF (voltage) source, of value $ni(t)$.

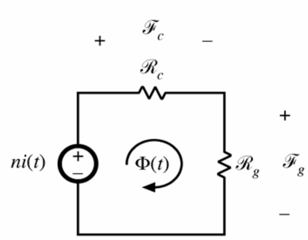
Total MMF's around the closed path add up to zero.

Example: inductor with air gap



Magnetic circuit model





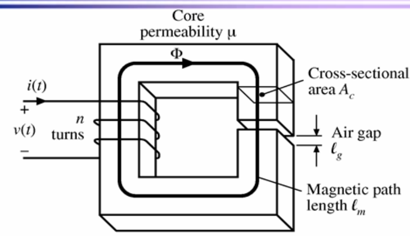
$$\mathscr{F}_c + \mathscr{F}_g = ni$$

$$\mathscr{R}_c = \frac{\ell_c}{\mu A_c}$$

$$ni = \Phi\left(\mathscr{R}_c + \mathscr{R}_g\right)$$

$$\mathscr{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Solution of model

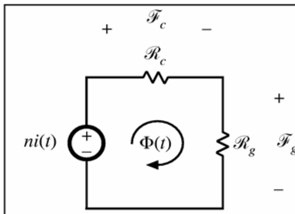


Faraday's law: $v(t) = n \frac{d\Phi(t)}{dt}$

Substitute for Φ: $v(t) = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g} \frac{di(t)}{dt}$

Hence inductance is

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$



$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$\mathcal{R}_c = \frac{\ell_c}{\mu A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Effect of air gap

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

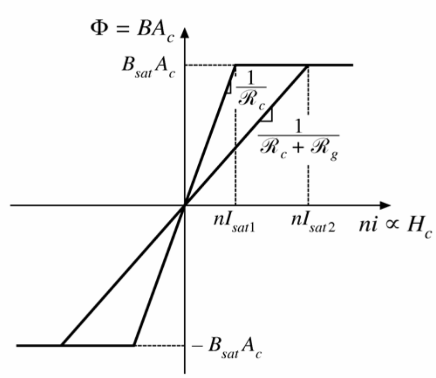
$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\Phi_{sat} = B_{sat} A_c$$

$$I_{sat} = \frac{B_{sat} A_c}{n}\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

Effect of air gap:

- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability



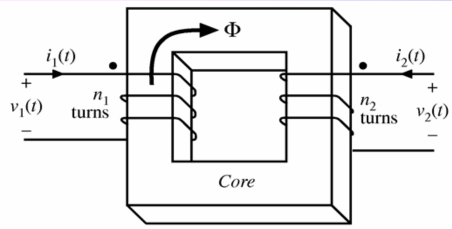
13.2 Transformer modeling

Two windings, no air gap:

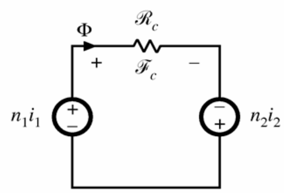
$$\mathscr{R} = \frac{\ell_m}{\mu A_c}$$

$$\mathscr{F}_c = n_1 i_1 + n_2 i_2$$

$$\Phi \mathscr{R} = n_1 i_1 + n_2 i_2$$



Magnetic circuit model:



13.2.1 The ideal transformer

In the ideal transformer, the core reluctance $\mathscr{R}$ approaches zero.

MMF $\mathscr{F}_c = \Phi \mathscr{R}$ also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday's law,

$$v_1 = n_1 \frac{d\Phi}{dt}$$

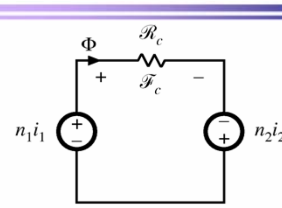
$$v_2 = n_2 \frac{d\Phi}{dt}$$

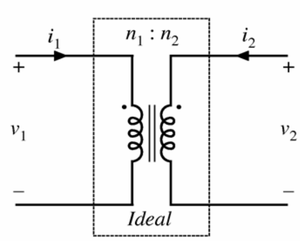
Eliminate Φ :

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2} \quad \text{and} \quad n_1 i_1 + n_2 i_2 = 0$$

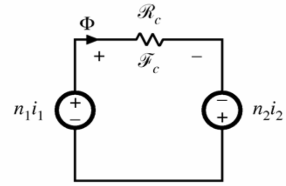




13.2.2 The magnetizing inductance

For nonzero core reluctance, we obtain

$$\Phi \mathscr{R} = n_1 i_1 + n_2 i_2 \quad \text{with} \quad v_1 = n_1 \frac{d\Phi}{dt}$$



Eliminate Φ:

$$v_1 = \frac{n_1^2}{\mathscr{R}} \frac{d}{dt}\left[i_1 + \frac{n_2}{n_1} i_2 \right]$$

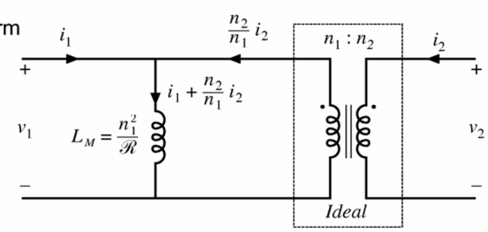
This equation is of the form

$$v_1 = L_M \frac{di_M}{dt}$$

with

$$L_M = \frac{n_1^2}{\mathscr{R}}$$

$$i_M = i_1 + \frac{n_2}{n_1} i_2$$



Magnetizing inductance: discussion

- Models magnetization of core material
- A real, physical inductor, that exhibits saturation and hysteresis
- If the secondary winding is disconnected:

  we are left with the primary winding on the core

  primary winding then behaves as an inductor

  the resulting inductor is the magnetizing inductance, referred to the primary winding
- Magnetizing current causes the ratio of winding currents to differ from the turns ratio

Transformer saturation

- Saturation occurs when core flux density $B(t)$ exceeds saturation flux density B_{sat}.
- When core saturates, the magnetizing current becomes large, the impedance of the magnetizing inductance becomes small, and the windings are effectively shorted out.
- Large winding currents $i_1(t)$ and $i_2(t)$ ***do not*** necessarily lead to saturation. If

$$0 = n_1 i_1 + n_2 i_2$$

then the magnetizing current is zero, and there is no net magnetization of the core.

- Saturation is caused by excessive applied volt-seconds

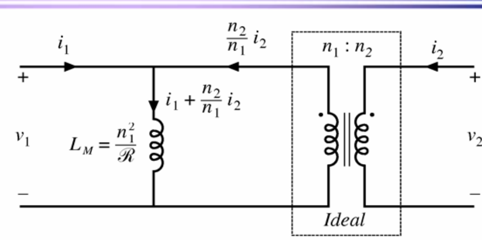
Saturation vs. applied volt-seconds

Magnetizing current depends on the integral of the applied winding voltage:

$$i_M(t) = \frac{1}{L_M} \int v_1(t)dt$$

Flux density is proportional:

$$B(t) = \frac{1}{n_1 A_c} \int v_1(t)dt$$

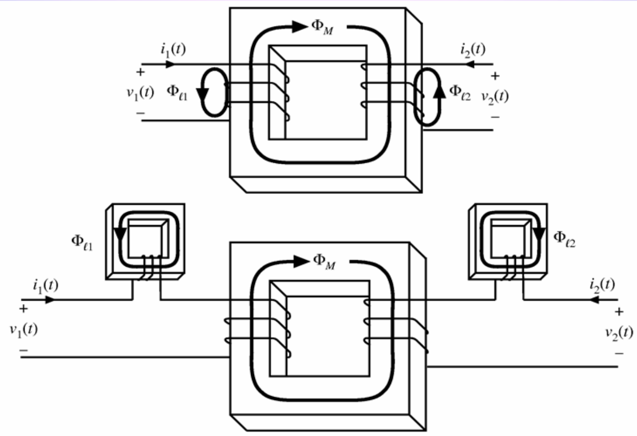


Flux density becomes large, and core saturates, when the applied volt-seconds λ_1 are too large, where

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt$$

limits of integration chosen to coincide with positive portion of applied voltage waveform

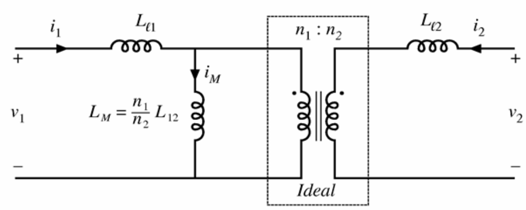
13.2.3 Leakage inductances



Transformer model, including leakage inductance

Terminal equations can be written in the form

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$



mutual inductance:

$$L_{12} = \frac{n_1 n_2}{\mathscr{R}} = \frac{n_2}{n_1} L_M$$

primary and secondary self-inductances:

$$L_{11} = L_{\ell 1} + \frac{n_1}{n_2} L_{12}$$

$$L_{22} = L_{\ell 2} + \frac{n_2}{n_1} L_{12}$$

effective turns ratio $n_e = \sqrt{\frac{L_{22}}{L_{11}}}$

coupling coefficient $k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}}$

13.3 Loss mechanisms in magnetic devices

Low-frequency losses:

- Dc copper loss
- Core loss: hysteresis loss

High-frequency losses: the skin effect

- Core loss: classical eddy current losses
- Eddy current losses in ferrite cores

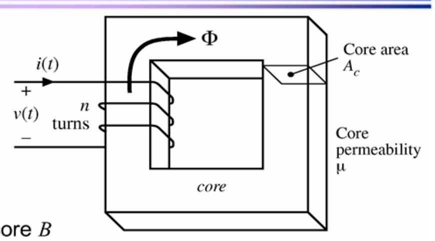
High frequency copper loss: the proximity effect

- Proximity effect: high frequency limit
- MMF diagrams, losses in a layer, and losses in basic multilayer windings
- Effect of PWM waveform harmonics

13.3.1 Core loss

Energy per cycle W flowing into n-turn winding of an inductor, excited by periodic waveforms of frequency f:

$$W = \int_{one\ cycle} v(t)i(t)\,dt$$



Relate winding voltage and current to core B and H via Faraday's law and Ampere's law:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\ell_m = ni(t)$$

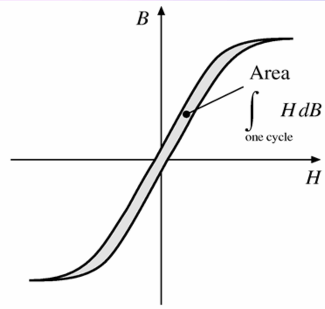
Substitute into integral:

$$W = \int_{one\ cycle} \left(nA_c \frac{dB(t)}{dt}\right)\left(\frac{H(t)\ell_m}{n}\right)dt$$

$$= \left(A_c\ell_m\right) \int_{one\ cycle} H\,dB$$

Core loss: Hysteresis loss

$$W = \left(A_c \ell_m\right) \int_{one\ cycle} H\, dB$$



The term $A_c \ell_m$ is the volume of the core, while the integral is the area of the B–H loop.

(energy lost per cycle) = (core volume) (area of B–H loop)

$$P_H = \left(f\right)\left(A_c \ell_m\right) \int_{one\ cycle} H\, dB$$

Hysteresis loss is directly proportional to applied frequency

Modeling hysteresis loss

- Hysteresis loss varies directly with applied frequency
- Dependence on maximum flux density: how does area of B–H loop depend on maximum flux density (and on applied waveforms)? Empirical equation (Steinmetz equation):

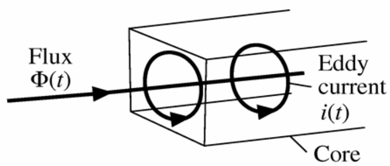
  $$P_H = K_H f B_{\max}^{\alpha} (core\ volume)$$

  The parameters K_H and α are determined experimentally.

  Dependence of P_H on B_{max} is predicted by the theory of magnetic domains.

Core loss: eddy current loss

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.



Eddy current loss $i^2(t)R$

Modeling eddy current loss

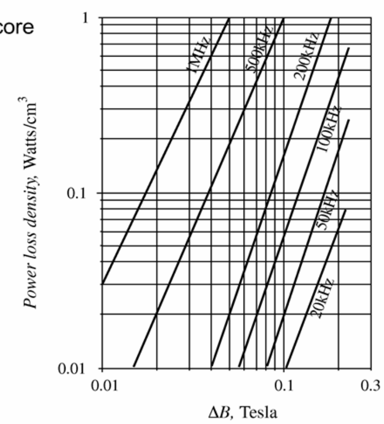
- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency f.
- Classical Steinmetz equation for eddy current loss:

$$P_E = K_E f^2 B_{\max}^2 (core\ volume)$$

- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as f^4.

Total core loss: manufacturer's data

Ferrite core material



Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe} (\Delta B)^{\beta} A_c \ell_m$$

Core materials

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

13.3.2 Low-frequency copper loss

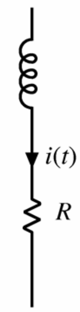
DC resistance of wire

$$R = \rho \frac{\ell_b}{A_w}$$

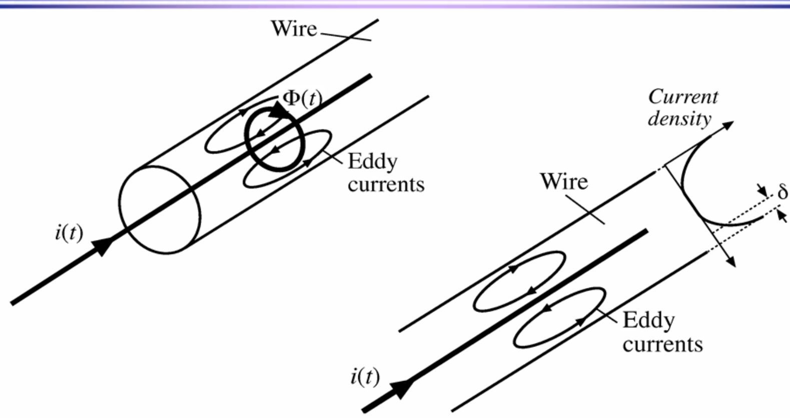
where A_w is the wire bare cross-sectional area, and ℓ_b is the length of the wire. The resistivity ρ is equal to $1.724 \cdot 10^{-6}\ \Omega\ \mathrm{cm}$ for soft-annealed copper at room temperature. This resistivity increases to $2.3 \cdot 10^{-6}\ \Omega\ \mathrm{cm}$ at $100^\circ\mathrm{C}$.

The wire resistance leads to a power loss of

$$P_{cu} = I_{rms}^2 R$$



13.4 Eddy currents in winding conductors
13.4.1 Intro to the skin and proximity effects



Penetration depth δ

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length δ known as the *penetration depth* or *skin depth*.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$