Lecture 23: Design of Power Magnetics

ECE 481: Power Electronics

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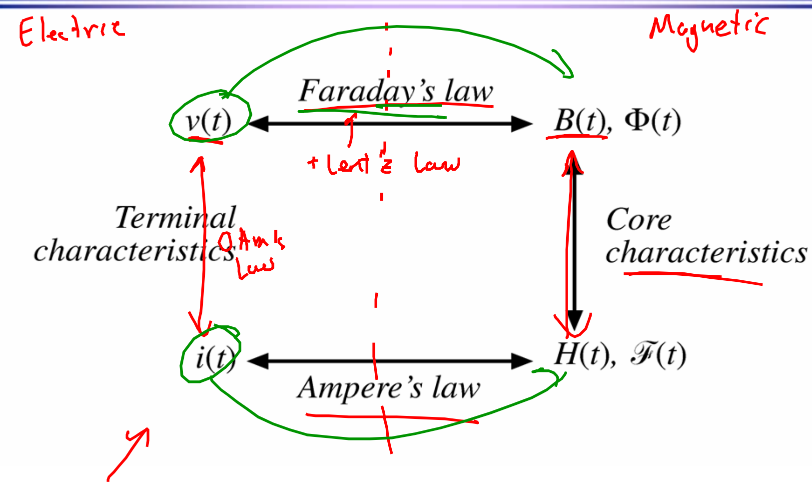
Announcements

- Homework #9 Due Tuesday
- ECE 482: Power Electronic Circuits
- http://oira.tennessee.edu/sais/

13.1 Review of Basic Magnetics

13.1.1 Basic relationships

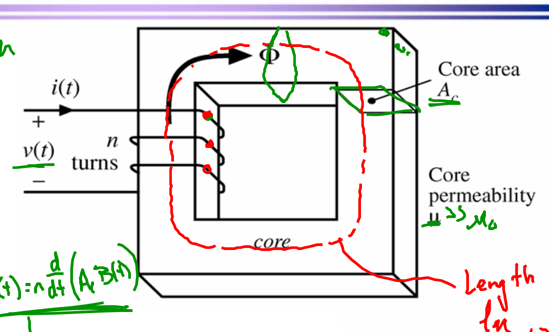


Example: a simple inductor

Because $\mu \gg \mu_0$

→ Assume Φ flows through the core

→ Assume B & H are uniform throughout the core material



Faraday's: $v_{turn}(t) = \frac{d\Phi(t)}{dt}$

$$v(t) = n v_{turn}(t) = n \frac{d\Phi(t)}{dt}$$

$$\Phi(t) = A_c B(t)$$

$$\left.\right\} v(t) = n \frac{d}{dt}(A_c B(t))$$

Core Material

① $B = \mu H$ if not saturated → $v(t) = n \frac{d}{dt}(A_c \mu H(t))$

Ampere's: $H(t) l_m = n i(t)$

$v_L(t) = L \frac{di_L(t)}{dt}$

② $B = B_{sat}$ if saturated

$v(t) = \phi \quad (L = \phi)$

$$v(t) = n \frac{d}{dt}\left(A_c \mu \frac{n}{l_m} i(t)\right) = \frac{n^2 \mu A_c}{l_m} \frac{di(t)}{dt}$$

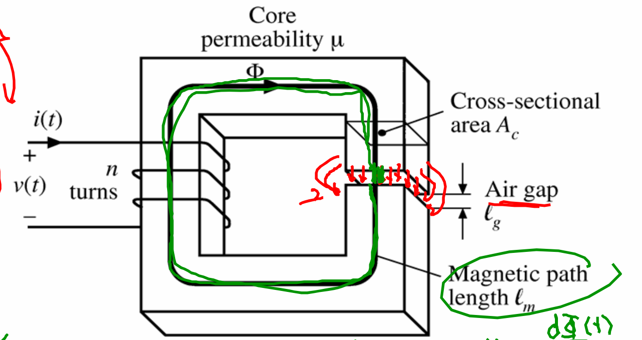
$$\boxed{L = \frac{n^2 \mu A_c}{l_m}}$$

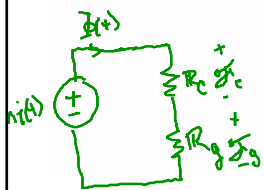
Example: inductor with air gap

$$L = \frac{n^2}{\frac{l_m}{\mu A_c} + \frac{l_g}{\mu_0 A_c}}$$

if $\mu \gg \mu_0$

$$\boxed{L \propto \frac{\mu_0 A_c n^2}{l_g}}$$





$$\Phi(t) = \frac{ni(t)}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\mathcal{R}_c = \frac{l_m - l_g}{\mu A_c}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_c}$$

$$v(t) = n\frac{d\Phi(t)}{dt}$$

$$v(t) = \boxed{\frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}} \frac{di(t)}{dt}$$

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

Effect of air gap

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

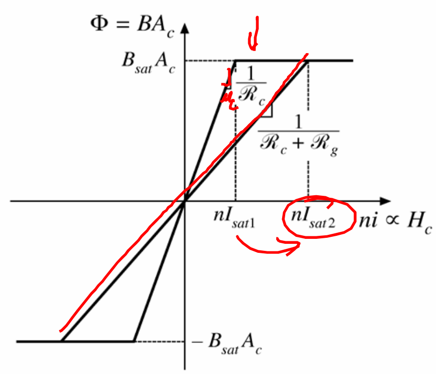
$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\Phi_{sat} = B_{sat} A_c$$

$$I_{sat} = \frac{B_{sat} A_c}{n}\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

Effect of air gap:

- decrease inductance
- increase saturation current
- inductance is less dependent on core permeability (μ) → μ is a function of Temp, core magnetization.



13.2 Transformer modeling

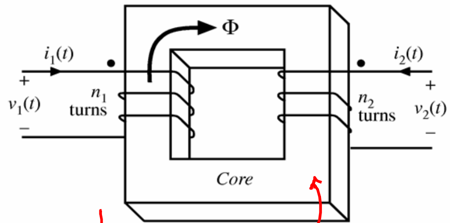
Two windings, no air gap:

$$\mathcal{R} = \frac{\ell_m}{\mu A_c}$$

$$\mathcal{F}_c = n_1 i_1 + n_2 i_2$$

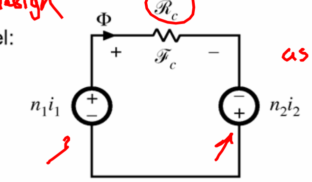
$$\Phi \mathcal{R} = n_1 i_1 + n_2 i_2$$

goes to zero for ideal XF design



Large μ

Magnetic circuit model:



as $\mathcal{F}_c \to 0$

$n_1 i_1 = -n_2 i_2$

ideal XF

13.2.1 The ideal transformer

In the ideal transformer, the core reluctance $\mathcal{R}$ approaches zero.

MMF $\mathcal{F}_c = \Phi \mathcal{R}$ also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday's law,

$$v_1 = n_1 \frac{d\Phi}{dt}$$

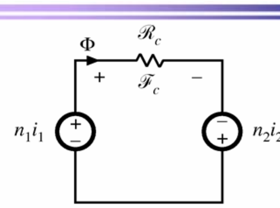
$$v_2 = n_2 \frac{d\Phi}{dt}$$

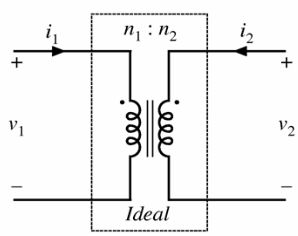
Eliminate Φ :

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\boxed{\frac{v_1}{n_1} = \frac{v_2}{n_2}} \quad \text{and} \quad \boxed{n_1 i_1 + n_2 i_2 = 0}$$

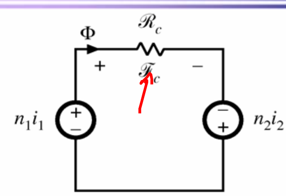




13.2.2 The magnetizing inductance

For nonzero core reluctance, we obtain

$$\Phi \mathscr{R} = n_1 i_1 + n_2 i_2 \quad \text{with} \quad v_1 = n_1 \frac{d\Phi}{dt}$$



Eliminate Φ:

$$v_1 = \frac{n_1^2}{\mathscr{R}} \frac{d}{dt}\left[i_1 + \frac{n_2}{n_1} i_2 \right]$$

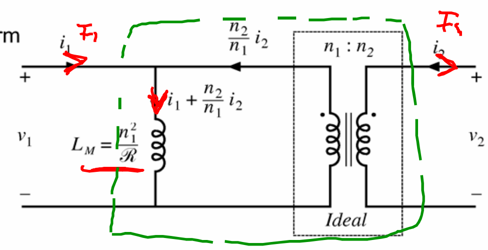
This equation is of the form

$$v_1 = L_M \frac{di_M}{dt}$$

with

$$L_M = \frac{n_1^2}{\mathscr{R}}$$

$$i_M = i_1 + \frac{n_2}{n_1} i_2$$



Magnetizing inductance: discussion

- Models magnetization of core material
- A real, physical inductor, that exhibits saturation and hysteresis
- If the secondary winding is disconnected:

  we are left with the primary winding on the core

  primary winding then behaves as an inductor

  the resulting inductor is the magnetizing inductance, referred to the primary winding
- Magnetizing current causes the ratio of winding currents to differ from the turns ratio

Transformer saturation

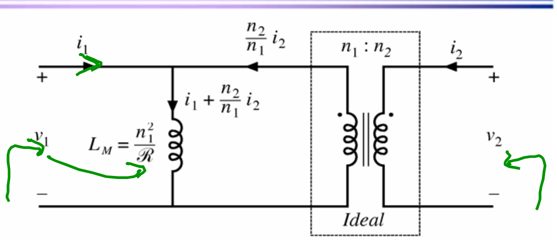
- Saturation occurs when core flux density $B(t)$ exceeds saturation flux density B_{sat}.
- When core saturates, the magnetizing current becomes large, the impedance of the magnetizing inductance becomes small, and the windings are effectively shorted out.
- Large winding currents $i_1(t)$ and $i_2(t)$ ***do not*** necessarily lead to saturation. If

$$0 = n_1 i_1 + n_2 i_2$$

then the magnetizing current is zero, and there is no net magnetization of the core.

- Saturation is caused by excessive applied volt-seconds

Saturation vs. applied volt-seconds



Magnetizing current depends on the integral of the applied winding voltage:

$$i_M(t) = \frac{1}{L_M} \int v_1(t)\,dt$$

Flux density is proportional:

$$B(t) = \frac{1}{n_1 A_c} \int v_1(t)\,dt$$

$$\rightarrow \frac{1}{n_1 A_c} \int v_1(t)\,dt < B_{sat}$$

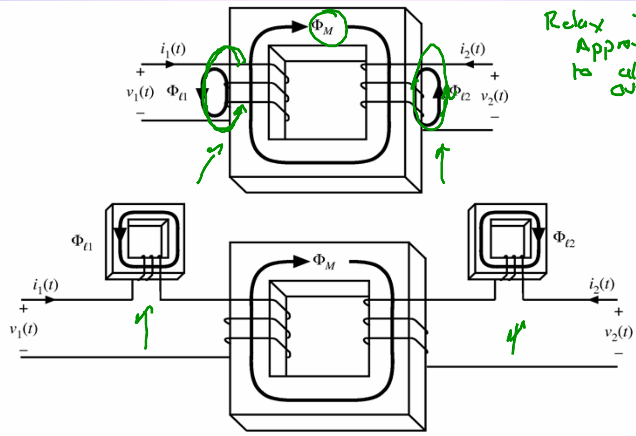
$$\frac{1}{n_2 A_c} \int v_2(t)\,dt < B_{sat}$$

Flux density becomes large, and core saturates, when the applied volt-seconds λ_1 are too large, where

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)\,dt$$

limits of integration chosen to coincide with positive portion of applied voltage waveform

13.2.3 Leakage inductances

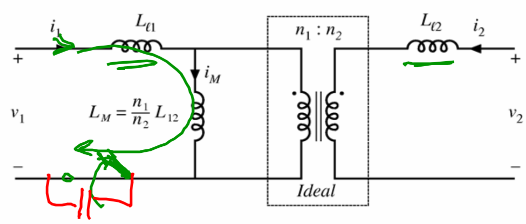


Relax the $\mu \gg \mu_0$ Approximation to allow some flux outside core

Transformer model, including leakage inductance

Terminal equations can be written in the form

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$



mutual inductance:

$$L_{12} = \frac{n_1 n_2}{\mathcal{R}} = \frac{n_2}{n_1} L_M$$

primary and secondary self-inductances:

$$L_{11} = L_{\ell 1} + \frac{n_1}{n_2} L_{12}$$

$$L_{22} = L_{\ell 2} + \frac{n_2}{n_1} L_{12}$$

effective turns ratio $n_e = \sqrt{\frac{L_{22}}{L_{11}}}$

coupling coefficient $k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}}$

$k \to 1$, $k \geq 0.99$ is reasonable for a real design

$0 < k < 1$

13.3 Loss mechanisms in magnetic devices

Core | Copper

Core → LF: Hysteresis; HF: Eddy currents

Copper → LF: DC winding resistance; HF: skin effect, proximity effect, "AC resistance"

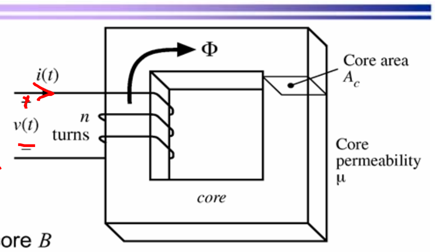
- Low-frequency losses:
  - Dc copper loss
  - Core loss: hysteresis loss
- High-frequency losses: the skin effect
  - Core loss: classical eddy current losses
  - Eddy current losses in ferrite cores
- High frequency copper loss: the proximity effect
  - Proximity effect: high frequency limit
  - MMF diagrams, losses in a layer, and losses in basic multilayer windings
  - Effect of PWM waveform harmonics

13.3.1 Core loss

Energy per cycle W flowing into n-turn winding of an inductor, excited by periodic waveforms of frequency f:

$$W = \int_{one\ cycle} \underbrace{v(t)i(t)}_{p(t)}\, dt$$

assumes no copper loss



Relate winding voltage and current to core B and H via Faraday's law and Ampere's law:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\ell_m = ni(t)$$

Substitute into integral:

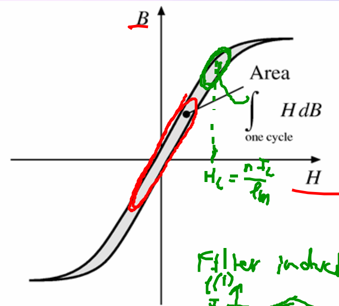
$$W = \int_{one\ cycle} \left(nA_c \frac{dB(t)}{dt} \right) \left(\frac{H(t)\ell_m}{n} \right) dt$$

$$= \left(A_c \ell_m \right) \int_{one\ cycle} H\, dB$$

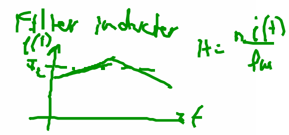
V_e or effective volume

Core loss: Hysteresis loss

$$W = \left(A_c \ell_m\right) \int_{\text{one cycle}} H\, dB$$



The term $A_c \ell_m$ is the volume of the core, while the integral is the area of the B–H loop.



(energy lost per cycle) = (core volume) (area of B–H loop)

$$P_H = \left(f\right)\left(A_c \ell_m\right) \int_{\text{one cycle}} H\, dB$$

Hysteresis loss is directly proportional to applied frequency

Modeling hysteresis loss

- Hysteresis loss varies directly with applied frequency
- Dependence on maximum flux density: how does area of B–H loop depend on maximum flux density (and on applied waveforms)? Empirical equation (Steinmetz equation):

$$P_H = K_H f B_{\max}^{\alpha} (\textit{core volume})$$

The parameters K_H and α are determined experimentally.

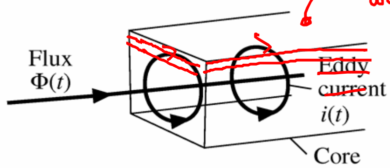
Dependence of P_H on B_{max} is predicted by the theory of magnetic domains.

#2

Core loss: eddy current loss

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.

core material would like small σ_c ($\uparrow \rho_c$)



Eddy current loss $i^2(t)R$

Modeling eddy current loss

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency f.
- Classical Steinmetz equation for eddy current loss:

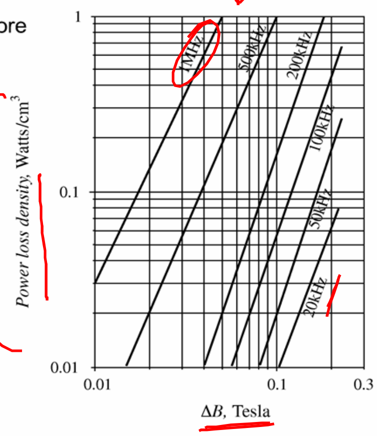
$$P_E = K_E f^2 B_{max}^2 (core\ volume)$$

- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as f^4.

Total core loss: manufacturer's data

Ferrite core material

taken for purely sinusoidal waveforms



Empirical equation, at a fixed frequency:

$$\boxed{P_{fe} = K_{fe} (\Delta B)^{\beta} A_c \ell_m}$$

$$B(t) = \frac{1}{n_1 A_c} \int_{\text{pos. half-cycle}} v_1(t)\, dt$$

K_{fe}, β, α also vary with frequency

nonlinear (can't take a fourier expansion & treat each f independently)

Core materials

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

13.3.2 Low-frequency copper loss

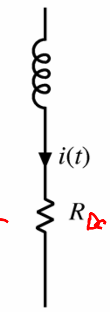
DC resistance of wire

$$R = \rho \frac{\ell_b}{A_w}$$

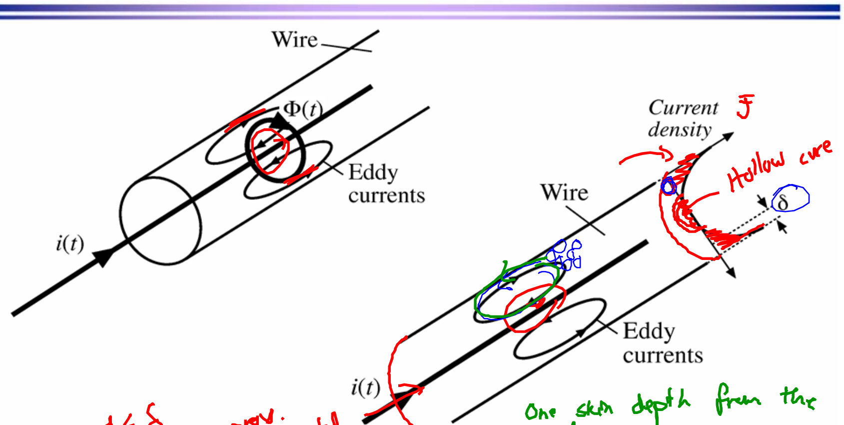
where A_w is the wire bare cross-sectional area, and ℓ_b is the length of the wire. The resistivity ρ is equal to $1.724\cdot10^{-6}\ \Omega$ cm for soft-annealed copper at room temperature. This resistivity increases to $2.3\cdot10^{-6}\ \Omega$ cm at 100°C.

The wire resistance leads to a power loss of

$$P_{cu} = I_{rms}^2 R$$



13.4 Eddy currents in winding conductors
13.4.1 Intro to the skin and proximity effects



$d < \delta$
↳ J is approx. evenly distributed

One skin depth from the surface $\frac{1}{e}$ or 0.37 of surface current density is present

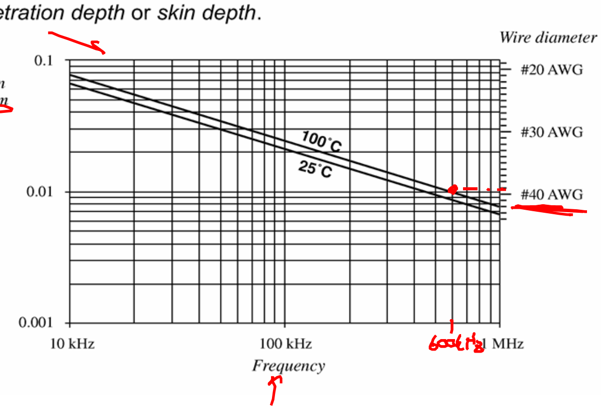
Penetration depth δ

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length δ known as the *penetration depth* or *skin depth*.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

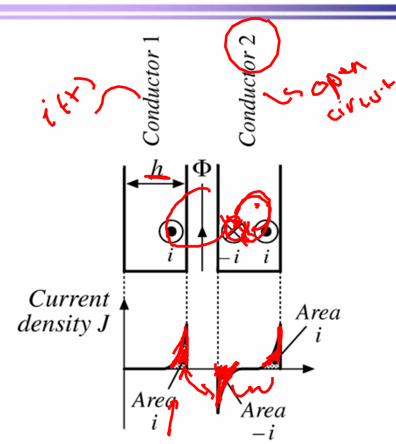
$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$



The proximity effect

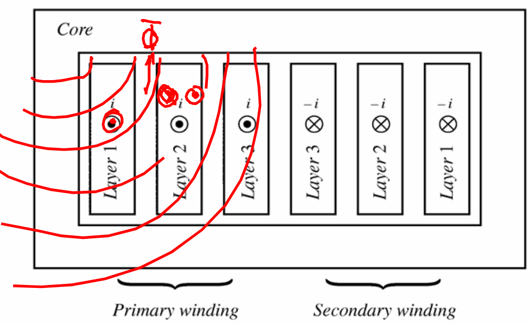
Ac current in a conductor induces eddy currents in adjacent conductors by a process called the *proximity effect*. This causes significant power loss in the windings of high-frequency transformers and ac inductors.

A multi-layer foil winding, with $h \gg \delta$. Each layer carries net current $i(t)$.



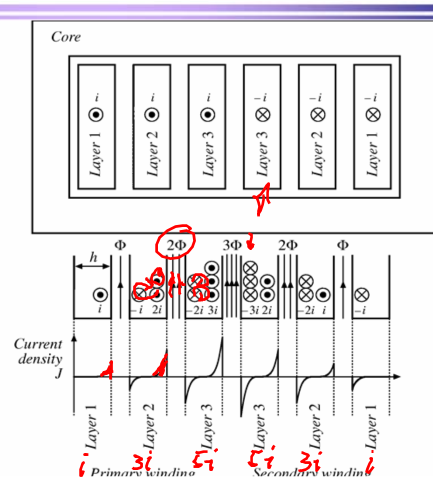
Example: a two-winding transformer

Cross-sectional view of two-winding transformer example. Primary turns are wound in three layers. For this example, let's assume that each layer is one turn of a flat foil conductor. The secondary is a similar three-layer winding. Each layer carries net current $i(t)$. Portions of the windings that lie outside of the core window are not illustrated. Each layer has thickness $h \gg \delta$.



Distribution of currents on surfaces of conductors: two-winding example

- Skin effect causes currents to concentrate on surfaces of conductors
- Surface current induces equal and opposite current on adjacent conductor
- This induced current returns on opposite side of conductor
- Net conductor current is equal to $i(t)$ for each layer, since layers are connected in series
- Circulating currents within layers increase with the numbers of layers



Estimating proximity loss: high-frequency limit

The current $i(t)$ having rms value I is confined to thickness d on the surface of layer 1. Hence the effective "ac" resistance of layer 1 is:

$$R_{ac} = \frac{h}{\delta} R_{dc}$$

This induces copper loss P_1 in layer 1:

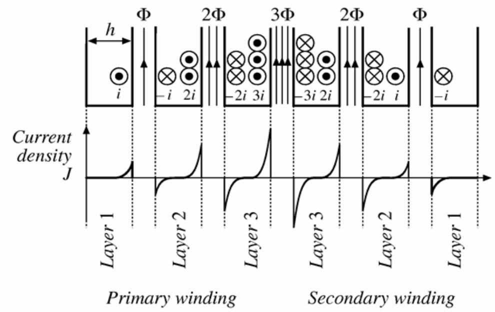
$$\underline{P_1 = I^2 R_{ac}}$$

Power loss P_2 in layer 2 is:

$$P_2 = P_1 + 4P_1 = \boxed{5P_1}$$

Power loss P_3 in layer 3 is:

$$\underline{P_3 = \left(2^2 + 3^2\right) P_1 = 13P_1}$$



Power loss P_m in layer m is:

$$P_m = I^2 \left[(m-1)^2 + m^2 \right] \underbrace{\left(\frac{h}{\delta} R_{dc} \right)}_{R_{ac}}$$

M total layer

Total loss in M-layer winding: high-frequency limit

Add up losses in each layer:

$$P = I^2 \left(\frac{h}{\delta} R_{dc} \right) \sum_{m=1}^{M} \left[(m-1)^2 + m^2 \right]$$
$$= I^2 \left(\frac{h}{\delta} R_{dc} \right) \frac{M}{3} \left(2M^2 + 1 \right)$$

Compare with dc copper loss:

If foil thickness were $H = \delta$, then at dc each layer would produce copper loss P_1. The copper loss of the M-layer winding would be

$$P_{dc} = I^2 M R_{dc}$$

So the proximity effect increases the copper loss by a factor of

$$\underline{F_R} = \frac{P}{P_{dc}} = \frac{1}{3} \left(\frac{h}{\delta} \right) \underline{\left(2M^2 + 1 \right)}$$

13.4.2 Leakage flux in windings

A simple two-winding transformer example: core and winding geometry

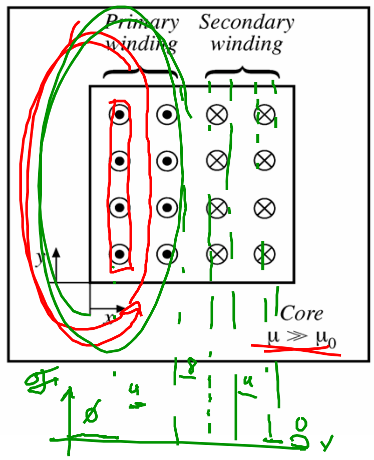
Each turn carries net current $i(t)$ in direction shown

Ampere's Law:

$F_{end} = \int H \cdot dl = \mathcal{F}$

$4i = H \cdot dl = \mathcal{F}$

$\mathcal{F} = 8i$

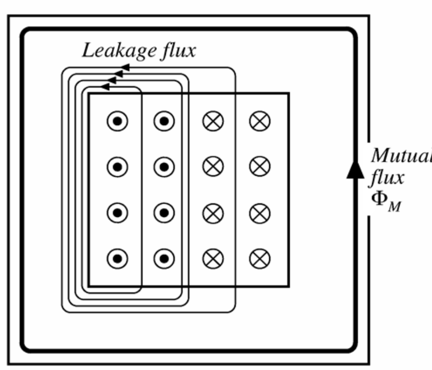


Flux distribution

Mutual flux Φ_M is large and is mostly confined to the core

Leakage flux is present, which does not completely link both windings

Because of symmetry of winding geometry, leakage flux runs approximately vertically through the windings

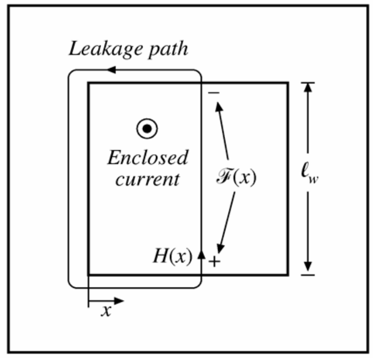


Analysis of leakage flux using Ampere's law

Ampere's law, for the closed path taken by the leakage flux line illustrated:

$$\text{Enclosed current} = \mathscr{F}(x) = H(x)\ell_w$$

(note that MMF around core is small compared to MMF through the air inside the winding, because of high permeability of core)



Ampere's law for the transformer example

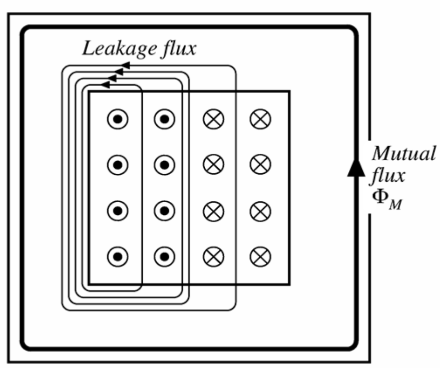
For the innermost leakage path, enclosing the first layer of the primary:

This path encloses four turns, so the total enclosed current is $4i(t)$.

For the next leakage path, enclosing both layers of the primary:

This path encloses eight turns, so the total enclosed current is $8i(t)$.

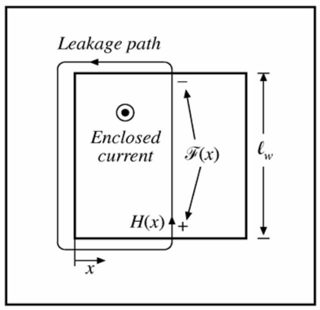
The next leakage path encloses the primary plus four turns of the secondary. The total enclosed current is $8i(t) - 4i(t) = 4i(t)$.

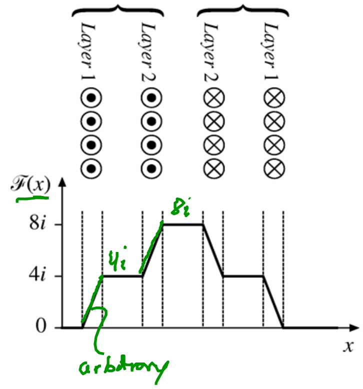


MMF diagram, transformer example

MMF $\mathscr{F}(x)$ across the core window, as a function of position x

$$\text{Enclosed current} = \mathscr{F}(x) = H(x)\ell_w$$





Two-winding transformer example

Winding layout

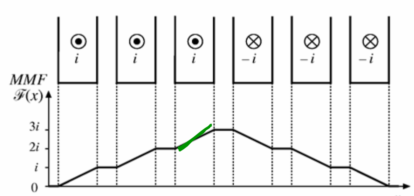
MMF diagram

Use Ampere's law around a closed path taken by a leakage flux line:

$$\left(m_p - m_s\right) i = \mathscr{F}(x)$$

m_p = number of primary layers enclosed by path

m_s = number of secondary layers enclosed by path



Two-winding transformer example
with proximity effect

Flux does not penetrate conductors

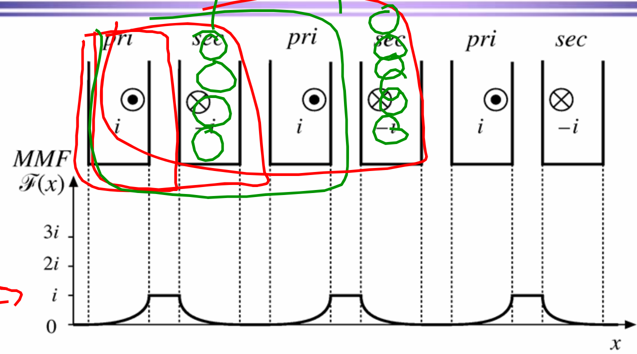
Surface currents cause net current enclosed by leakage path to be zero when path runs down interior of a conductor

Magnetic field strength $H(x)$ within the winding is given by

$$H(x) = \frac{\mathscr{F}(x)}{\ell_w}$$

if $h \gg \delta$

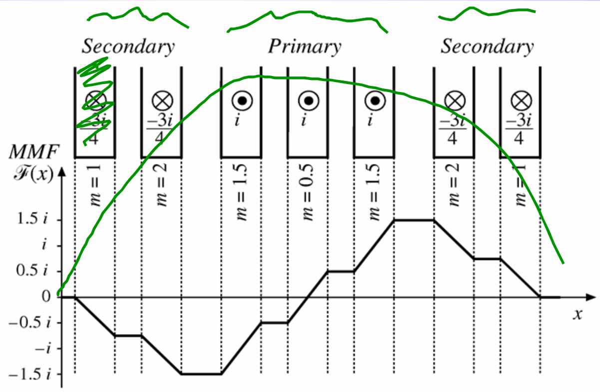
Interleaving the windings: MMF diagram



Greatly reduces the peak MMF, leakage flux, and proximity losses

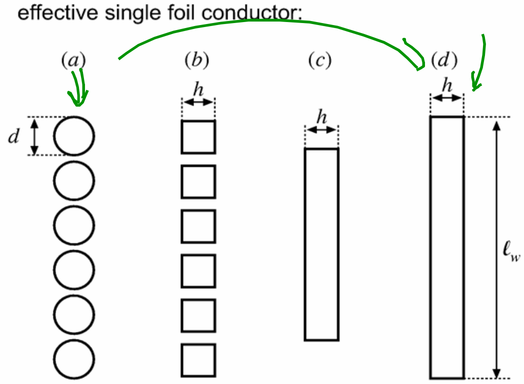
A partially-interleaved transformer

For this example, there are three primary layers and four secondary layers. The MMF diagram contains fractional values.



13.4.3 Foil windings and layers

Approximating a layer of round conductors as an effective single foil conductor:



Square conductors (b) have same cross-sectional area as round conductors (a) if

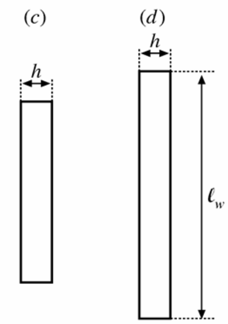
$$h = \sqrt{\frac{\pi}{4}}\, d$$

Eliminate space between square conductors: push together into a single foil turn (c)

(d) Stretch foil so its width is ℓ_w. The adjust conductivity so its dc resistance is unchanged

Winding porosity η

Stretching the conductor increases its area. Compensate by increasing the effective resistivity ρ, to maintain the same dc resistance. Define *winding porosity* η as the ratio of cross-sectional areas. If layer of width ℓ_w contains n_ℓ turns of round wire having diameter d, then the porosity is



$$\eta = \sqrt{\frac{\pi}{4}}\, d\, \frac{n_\ell}{\ell_w}$$

Typical η for full-width round conductors is $\eta = 0.8$. The increased effective resistivity increases the effective skin depth:

$$\delta' = \frac{\delta}{\sqrt{\eta}}$$

Define $\varphi = h/d$. The effective value for a layer of round conductors is

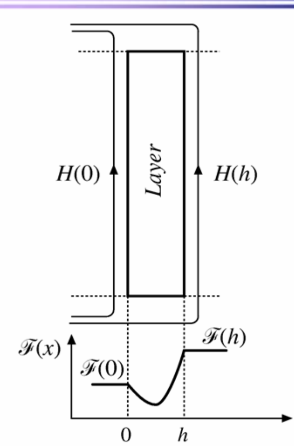
$$\varphi = \frac{h}{\delta'} = \sqrt{\eta}\,\sqrt{\frac{\pi}{4}}\,\frac{d}{\delta}$$

13.4.4 Power loss in a layer

Approximate computation of copper loss in one layer

Assume uniform magnetic fields at surfaces of layer, of strengths $H(0)$ and $H(h)$. Assume that these fields are parallel to layer surface (i.e., neglect fringing and assume field normal component is zero).

The magnetic fields $H(0)$ and $H(h)$ are driven by the MMFs $\mathscr{F}(0)$ and $\mathscr{F}(h)$. Sinusoidal waveforms are assumed, and rms values are used. It is assumed that $H(0)$ and $H(h)$ are in phase.



Solution for layer copper loss P

Solve Maxwell's equations to find current density distribution within layer. Then integrate to find total copper loss P in layer. Result is

$$P = R_{dc} \frac{\varphi}{n_\ell^2} \left[\left(\mathscr{F}^2(h) + \mathscr{F}^2(0) \right) G_1(\varphi) - 4\, \mathscr{F}(h) \mathscr{F}(0) G_2(\varphi) \right]$$

where

$$R_{dc} = \rho \frac{\ell_b}{A_w} = \rho \frac{(MLT) n_l^3}{\eta \ell_w^2}$$

n_l = number of turns in layer,
R_{dc} = dc resistance of layer,

(MLT) = mean-length-per-turn, or circumference, of layer.

$$G_1(\varphi) = \frac{\sinh(2\varphi) + \sin(2\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}$$

$$G_2(\varphi) = \frac{\sinh(\varphi)\cos(\varphi) + \cosh(\varphi)\sin(\varphi)}{\cosh(2\varphi) - \cos(2\varphi)}$$

$$\varphi = \frac{h}{\delta'} = \sqrt{\eta}\sqrt{\frac{\pi}{4}}\frac{d}{\delta} \qquad \eta = \sqrt{\frac{\pi}{4}}\, d\, \frac{n_\ell}{\ell_w}$$

Winding carrying current I, with n_l turns per layer

If winding carries current of rms magnitude I, then

$$\mathscr{F}(h) - \mathscr{F}(0) = n_\ell I$$

Express $\mathscr{F}(h)$ in terms of the winding current I, as

$$\mathscr{F}(h) = m n_\ell I$$

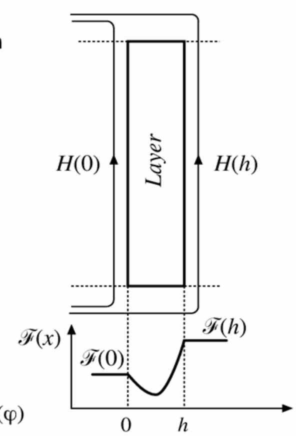
The quantity m is the ratio of the MMF $\mathscr{F}(h)$ to the layer ampere-turns $n_\ell I$. Then,

$$\frac{\mathscr{F}(0)}{\mathscr{F}(h)} = \frac{m-1}{m}$$

Power dissipated in the layer can now be written

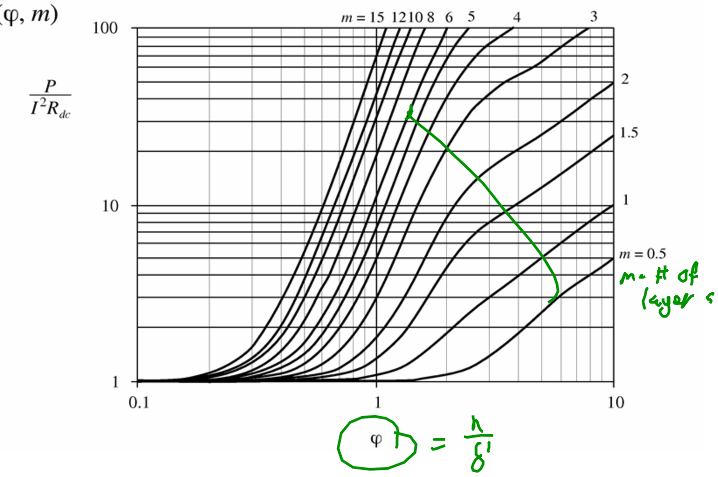
$$P = I^2 R_{dc} \varphi Q'(\varphi, m)$$

$$Q'(\varphi, m) = \left(2m^2 - 2m + 1\right) G_1(\varphi) - 4m\left(m-1\right) G_2(\varphi)$$



Increased copper loss in layer

$P = I^2 R_{dc} \varphi Q'(\varphi, m)$



Layer copper loss vs. layer thickness

$\frac{P}{P_{dc}\big|_{\varphi = 1}} = Q'(\varphi, m)$

Relative to copper loss when $h = \delta$