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Lecture 2: Volt-Second and Capacitor Charge Balance

ECE 481: Power Electronics
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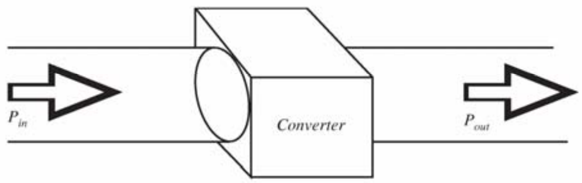
Chapter 2
Principles of Steady-State Converter Analysis

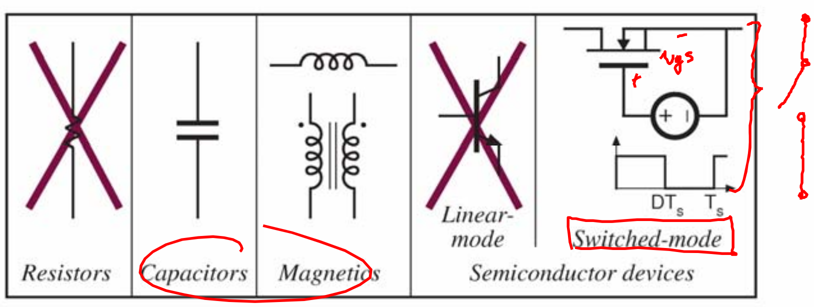
A high-efficiency converter



A goal of current converter technology is to construct converters of small size and weight, which process substantial power at high efficiency

Cost, "performance"

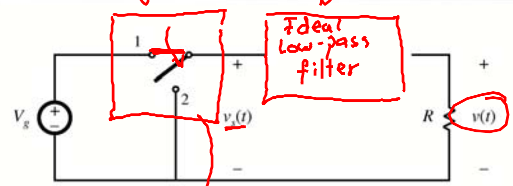
Devices available to the circuit designer



Power processing: avoid lossy elements

2.1 Introduction
Buck converter

SPDT switch changes dc component

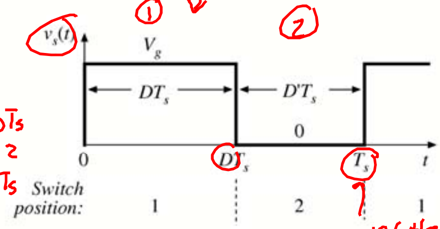


Switch output voltage waveform

switch in pos 1
$0 < t < DT_s$
.. in pos 2
$DT_s < t < T_s$

Duty cycle D:
$\underline{0 \le D \le 1}$

complement D':
$D' = 1 - D$

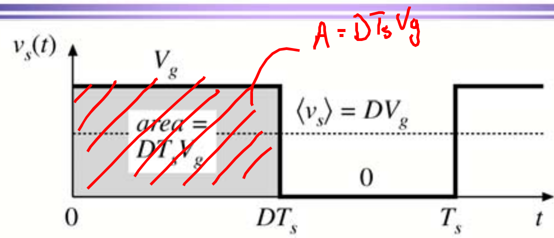


$T_s = \frac{1}{f_s}$

f_s 10kHz, 1MHz

Dc component of switch output voltage

$A = DT_s V_g$



Fourier analysis: Dc component = average value

$$\langle v_s \rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt$$

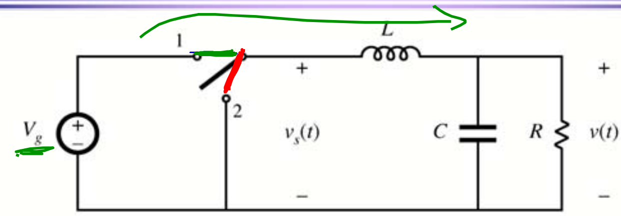
$$\langle v_s \rangle = \frac{1}{T_s}(DT_sV_g) = DV_g$$

$v(t) = \langle v_s \rangle$

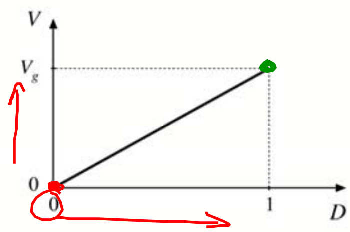
$v(t) \approx \boxed{V = DV_g}$

"Pulse Width Modulation" PWM

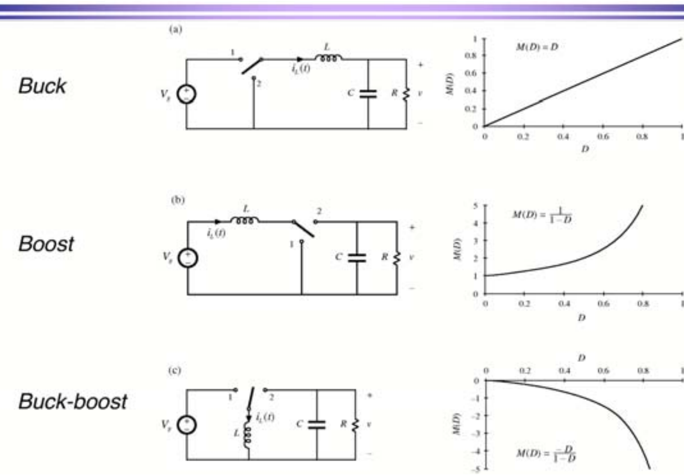
Insertion of low-pass filter to remove switching harmonics and pass only dc component



$$v \approx \langle v_s \rangle = DV_g$$



Three basic dc-dc converters



$M(D)$ = "Conversion Ratio"

$M(D) = \frac{V}{V_g}$

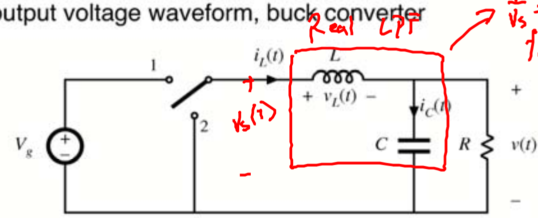
Objectives of this chapter

- Develop techniques for easily determining output voltage of an arbitrary converter circuit
- Derive the principles of *inductor volt-second balance* and *capacitor charge (amp-second) balance*
- Introduce the key *small ripple approximation*
- Develop simple methods for selecting filter element values
- Illustrate via examples

2.2. Inductor volt-second balance, capacitor charge balance, and the small ripple approximation

Actual output voltage waveform, buck converter

Buck converter containing practical low-pass filter



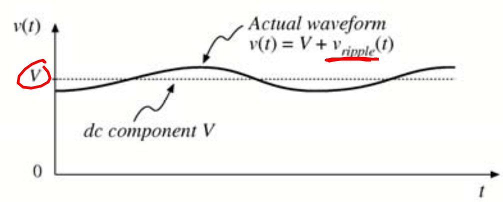
$\frac{V}{V_s} = \frac{1}{s^2LC+1}$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$f_0 \ll f_s$

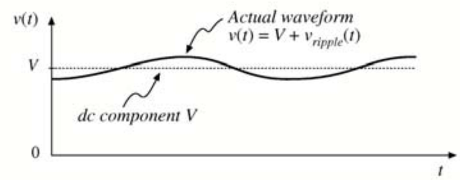
Actual output voltage waveform

$v(t) = V + v_{ripple}(t)$



The small ripple approximation

$$v(t) = V + v_{ripple}(t)$$



In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

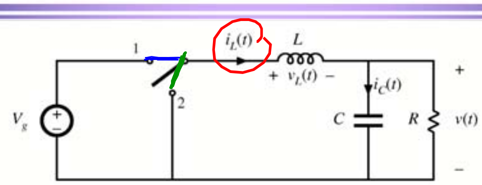
$$\left\| v_{ripple} \right\| \ll V$$

$$\boxed{v(t) \approx V}$$

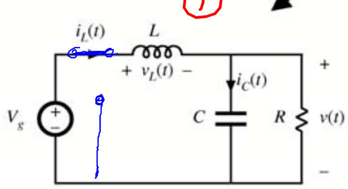
$\|v_{ripple}\| \neq \emptyset$ → neglect relative to V

Buck converter analysis: inductor current waveform

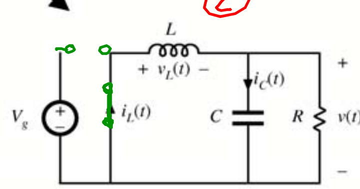
original converter



switch in position 1



switch in position 2



Inductor voltage and current
Subinterval 1: switch in position 1

$0 < t < DT_s$

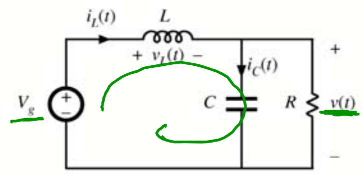
Inductor voltage

$V_{L1} = V_g - v(t)$

Small ripple approximation:

$v(t) \approx V$

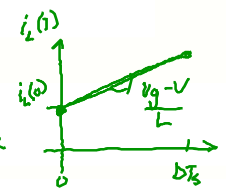
$V_{L1} = V_g - V$



$v_L = L \dfrac{di_L}{dt}$

Solve for the slope:

$\dfrac{di_L}{dt} = \dfrac{V_g - V}{L} \rightarrow \dfrac{V_g - V}{L} > 0$ $\quad V_g > V$ for buck



Inductor voltage and current
Subinterval 2: switch in position 2

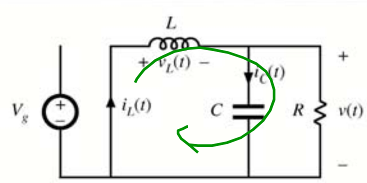
Inductor voltage

$V_{L2} = 0 - v(t)$

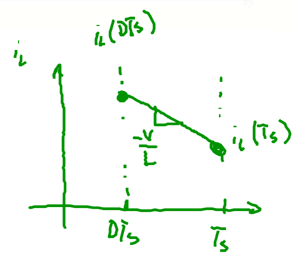
Small ripple approximation:

$v(t) \approx V$

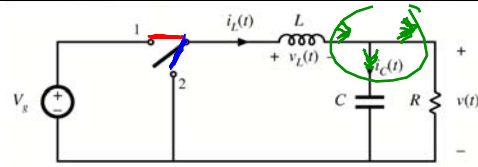
$V_{L2} = -V$

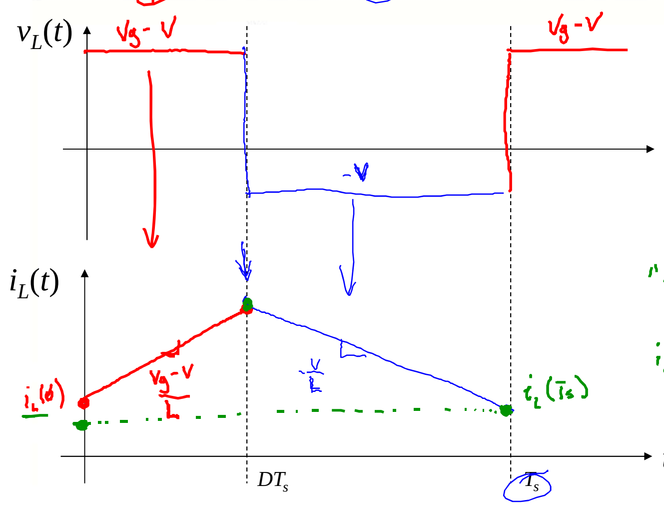


$\dfrac{di_L}{dt} = \dfrac{-V}{L} < 0$



Inductor Voltage and Current Waveforms





$V_L = L \frac{di_L}{dt}$

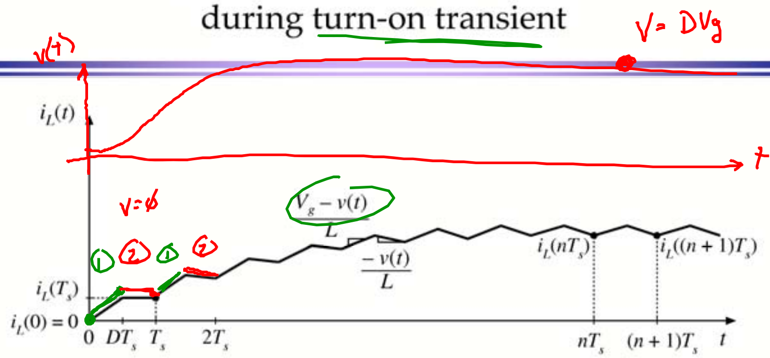
"Steady state" need

$i_L(T_s) = i_L(0)$

"Buck" Converter

Inductor current waveform during turn-on transient

$V = DV_g$

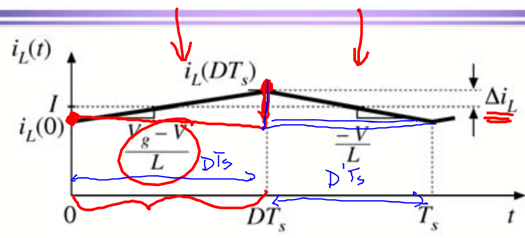


When the converter operates in equilibrium:

$$i_L((n+1)T_s) = i_L(nT_s)$$

'Steady state'

Determination of inductor current ripple magnitude



$2\Delta i_L$ $\quad$ DT_s

(change in i_L) = (slope)(length of subinterval)

$$2\Delta i_L = \frac{V_g - V}{L}(DT_s)$$

$$\Delta i_L = \frac{V_g - V}{2L} DT_s$$

$$\boxed{L = \frac{V_g - V}{2\Delta i_L} DT_s}$$

$$2\Delta i_L = \frac{V_g - DV_g}{L} DT_s = \frac{V_g(1-D)}{L} DT_s$$

$$-2\Delta i_L = -\frac{V}{L}(T_s - DT_s)$$

$$D' = 1 - D$$

$$2\Delta i_L = \frac{V}{L} D' T_s$$

$$2\Delta i_L = \frac{DV_g}{L}(1-D) T_s$$

The principle of inductor volt-second balance: Derivation

Inductor defining relation:

$$v_L(t) = L \frac{di_L(t)}{dt}$$

steady state
$i_L(T_s) = i_L(0)$

Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)\, dt$$

In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)\, dt$$

Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

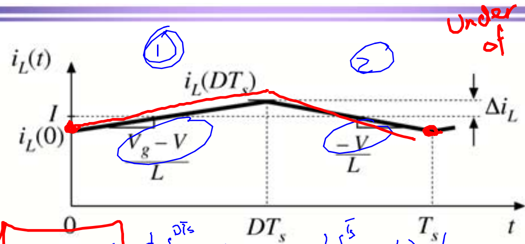
An equivalent form:

$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\, dt = \langle v_L \rangle$$

The average inductor voltage is zero in steady state.

Buck Volt-Second Balance

Under Assumption of steady state



$$\boxed{\langle v_L \rangle = 0} = \frac{1}{T_s}\int_0^{DT_s} \underbrace{v_L(t)}\,dt + \frac{1}{T_s}\int_{DT_s}^{T_s} \underbrace{v_L(t)}\,dt$$

Recall $1-D = D'$

$$0 = \frac{1}{T_s}\left[(V_g - V)DT_s + -V(1-D)T_s\right]$$

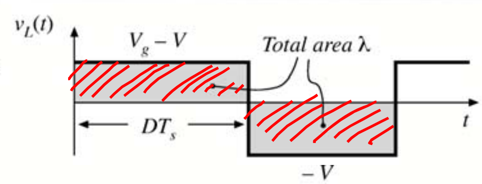
$$0 = \frac{1}{\cancel{T_s}}\left[V_g D\cancel{T_s} - VD\cancel{T_s} - VD\cancel{T_s}\right]$$

$$0 = V_g D - V(\underbrace{D + D'}_{=1})$$

$$0 = V_g D - V \rightarrow \boxed{V = DV_g}$$

Inductor volt-second balance: Buck converter example

Inductor voltage waveform, previously derived:



volt-sec

Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for V:

$$0 = DV_g - (D + D')V = DV_g - V \quad \Rightarrow \quad V = DV_g$$

"Conversion Ratio"

$$\frac{V}{V_g} = D = M(D)$$

The principle of capacitor charge balance: Derivation

Capacitor defining relation:

$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$

Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

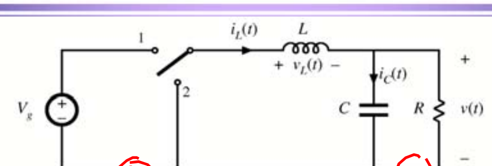
In periodic steady state, the net change in capacitor voltage is zero:

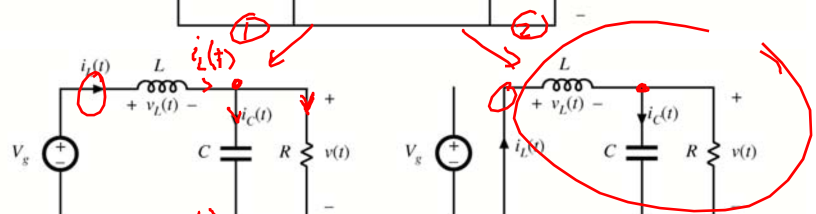
$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \langle i_C \rangle$$

Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

Buck Capacitor-Charge Balance

original converter





$i_c = i_L(t) - \frac{v(t)}{R}$

small ripple $\begin{cases} i_L(t) \approx I_L \\ v(t) \approx V \end{cases}$

$I_{c1} = I_L - \frac{V}{R}$

$I_{c1} = I_L - \frac{V}{R}$

$\langle i_c \rangle = 0 \rightarrow v(0) = v(T_s)$

$0 = \int_0^{T_s} i_c\,dt = \cancel{T_s}\left(I_L - \frac{V}{R}\right)$

$\boxed{I_L = \frac{V}{R}}$