Lecture 3: Steady-State Converter Analysis

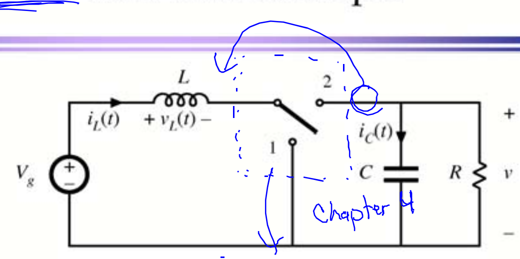
ECE 481: Power Electronics

Prof. Daniel Costinett
Department of Electrical Engineering and Computer Science
University of Tennessee Knoxville
Fall 2013

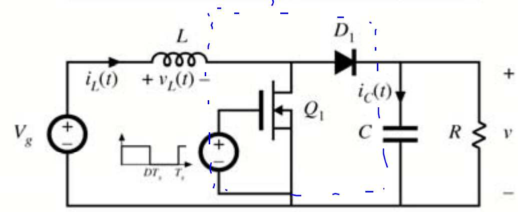
$V > V_g$

2.3 Boost converter example

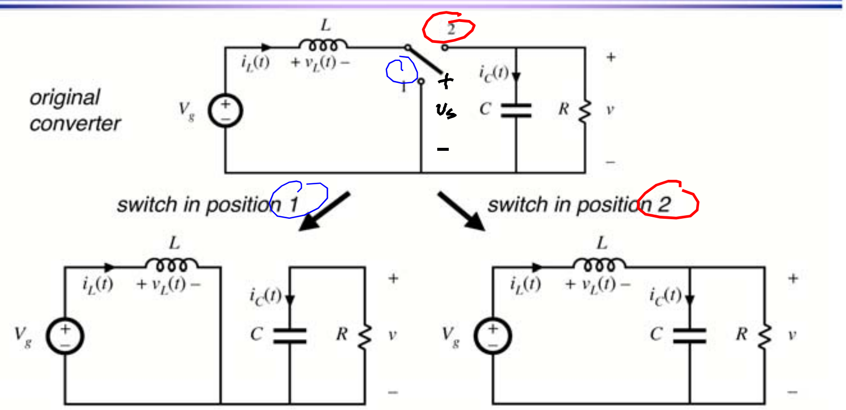
Boost converter with ideal switch

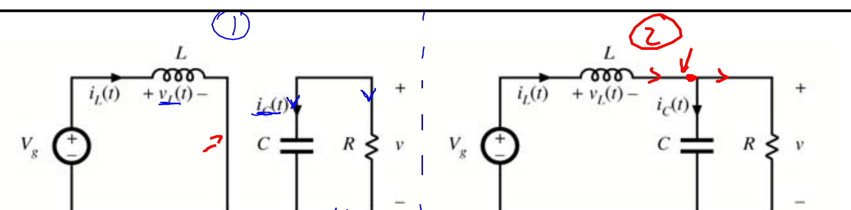


Realization using power MOSFET and diode



Boost converter analysis





(1): $v_L(t) = V_g$ $\qquad$ $i_C(t) = -\frac{v(t)}{R}$

small ripple: $v_L = V_g$ $\qquad$ $i_C = \frac{-V}{R}$

Apply to cap voltages & ind currents

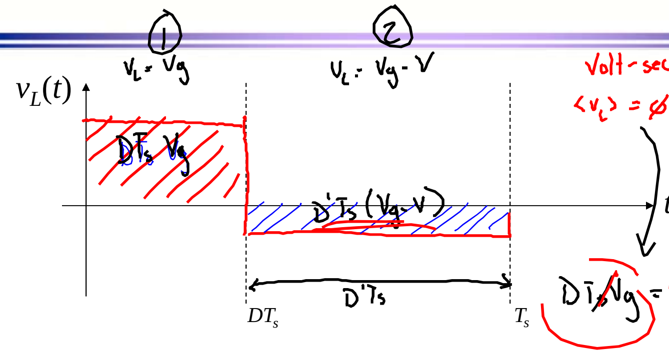
(2): $v_L(t) = V_g - v(t)$ $\qquad$ $i_C(t) = i_L(t) - \frac{v(t)}{R}$

$v_L = V_g - V$ $\qquad$ $i_C = I_L - \frac{V}{R}$

$\langle v_L \rangle = 0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt$

$\langle i_C \rangle = 0$

Inductor voltage and capacitor current waveforms



Volt-sec balance

$\langle v_L \rangle = 0$

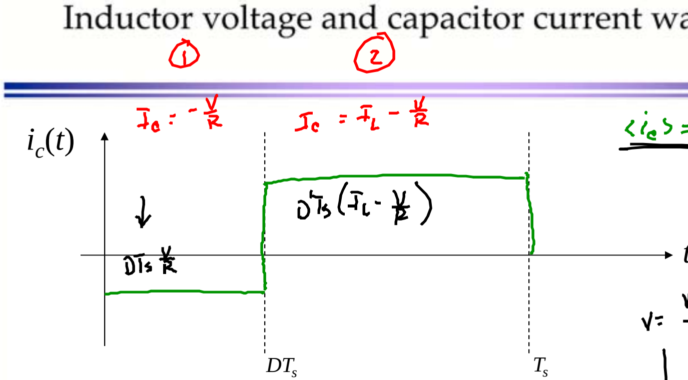
$$D \cancel{T_s} V_g = -D' \cancel{T_s} (V_g - V)$$

$$V_g \underbrace{(D + D')}_{=1} = D'(+V)$$

$$\boxed{V_g = D' V}$$

$$\frac{V}{V_g} = \frac{1}{D'} = M(D)$$

Inductor voltage and capacitor current waveforms



$\langle i_c \rangle = 0$

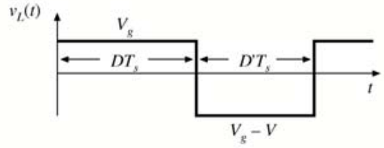
$$V = \frac{V_g}{D'}$$

$$D' \cancel{T_s} \left(I_L - \frac{V}{R}\right) - D \cancel{T_s} \frac{V}{R} = 0$$

$$D' I_L - \frac{V}{R} = 0 \quad \rightarrow \quad \boxed{I_L = \frac{V}{R D'} = \frac{V_g}{R (D')^2}}$$

Inductor volt-second balance

Net volt-seconds applied to inductor over one switching period:



$$\int_0^{T_s} v_L(t)\,dt = (V_g)\,DT_s + (V_g - V)\,D'T_s$$

Equate to zero and collect terms:

$$V_g\,(D + D') - V\,D' = 0$$

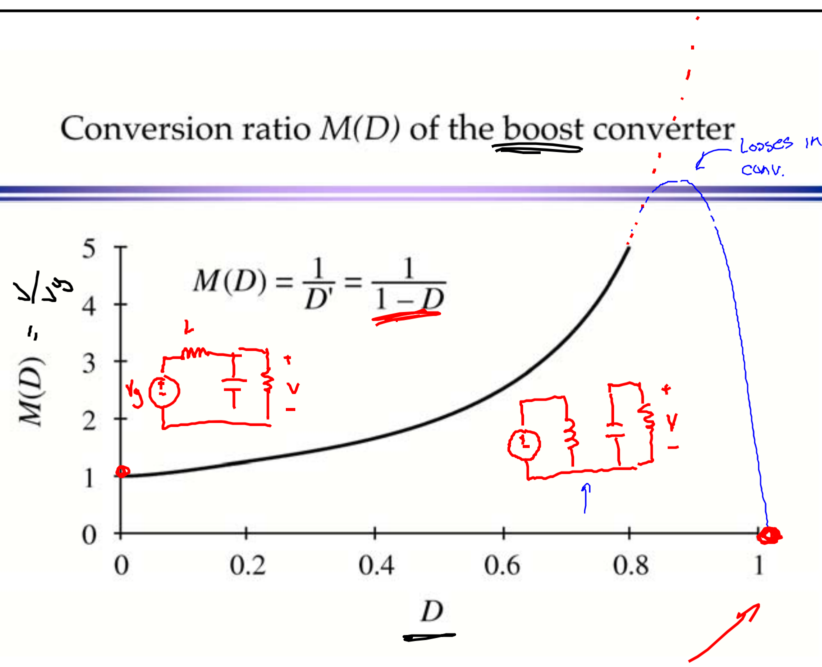
Solve for V:

$$V = \frac{V_g}{D'}$$

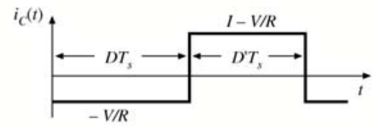
The voltage conversion ratio is therefore

$$\boxed{M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1 - D}}$$

Conversion ratio $M(D)$ of the boost converter



Determination of inductor current dc component



Capacitor charge balance:

$$\int_0^{T_s} i_C(t)\,dt = (-\frac{V}{R})\,DT_s + (I - \frac{V}{R})\,D'T_s$$

Collect terms and equate to zero:

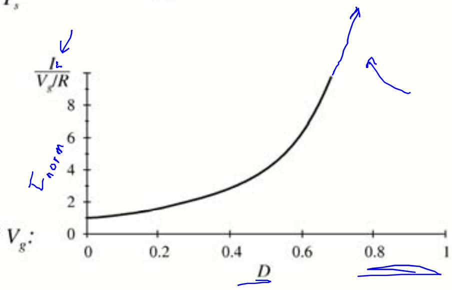
$$-\frac{V}{R}(D + D') + I\,D' = 0$$

Solve for I:

$$I = \frac{V}{D'\,R}$$

Eliminate V to express in terms of V_g:

$$I = \frac{V_g}{D'^2\,R}$$



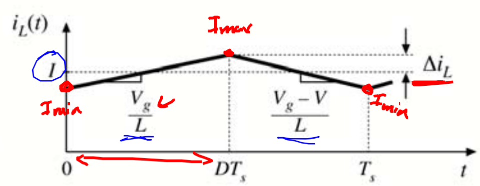
Determination of inductor current ripple

Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$



$I_{max} - I_{min} = 2\Delta i_L = \frac{V_g}{L} DT_s$

Change in inductor current during subinterval 1 is *(slope) (length of subinterval):*

$$2\Delta i_L = \frac{V_g}{L}\,DT_s$$

Solve for peak ripple:

$$\Delta i_L = \frac{V_g}{2L}\,DT_s$$

- Choose L such that desired ripple magnitude is obtained

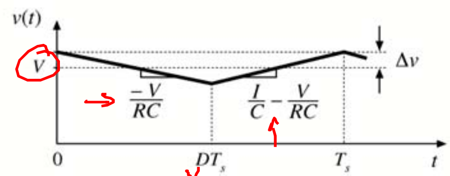
Determination of capacitor voltage ripple

Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



$\Delta v = \frac{1}{2} DT_s \frac{V}{RC}$

Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval)*:

$$-2\Delta v = \frac{-V}{RC} DT_s$$

Solve for peak ripple:

$$\Delta v = \frac{V}{2RC} DT_s$$

- Choose C such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple

"ESR"

ESR

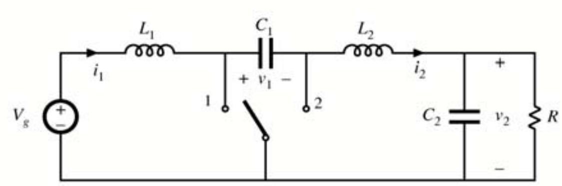
C

Slobodan Cuk → Professor at Caltech

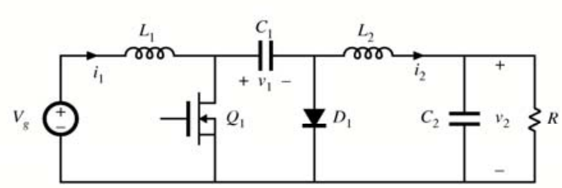
↳ Dragan Maksimovic → Professor at Colorado

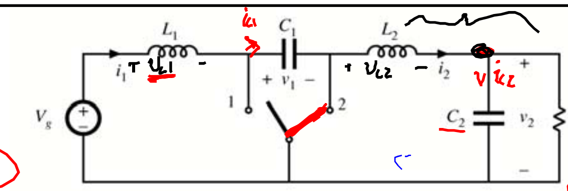
2.4 Ćuk converter example

Cuk converter, with ideal switch



Cuk converter: practical realization using MOSFET and diode





(1)

$v_{L1} = V_g$

$v_{L2} = -v_1(t) - v_2(t)$

$i_{C1} = i_{L2}(t)$

$i_{C2} = i_{L2}(t) - \frac{v_2(t)}{R}$

(2)

$v_{L1} = V_g - v_1(t)$

$v_{L2} = \emptyset - v_2(t)$

$i_{C1} = i_{L1}(t)$

$i_{C2} = i_{L2}(t) - \frac{v_2(t)}{R}$

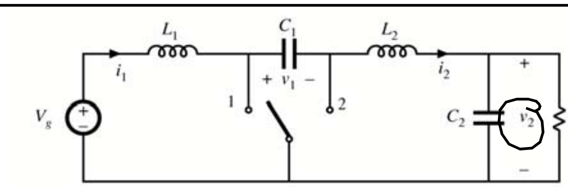
Small ripple approx

$v_1(t) \approx V_1$

$v_2(t) \approx V_2$

$i_1(t) \approx I_1$

$i_2(t) \approx I_2$



Volt-sec balance

$L_1: \langle v_{L1} \rangle = \emptyset = DV_g + D'(V_g - V_1) = V_g - D'V_1 \rightarrow \boxed{V_1 = \frac{V_g}{D'}}$

$L_2: \langle v_{L2} \rangle = \emptyset = DV_1 - V_2 \rightarrow \boxed{V_2 = DV_1}$

$\boxed{V_2 = \frac{D}{D'} V_g}$

$M(D) = \frac{D}{D'}$

cap-charge balance

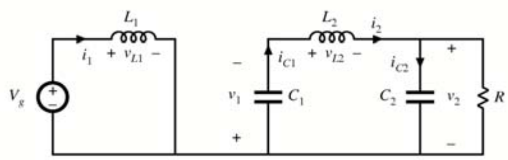
$C_1: \langle i_{C1} \rangle = \emptyset = D I_{L2} + D' I_{L1} \rightarrow \boxed{I_{L2} = -\frac{D'}{D} I_{L1}}$

$C_2: \langle i_{C2} \rangle = \emptyset = I_{L2} - \frac{V_2}{R} \rightarrow \boxed{I_{L2} = \frac{V_2}{R}}$

Cuk converter circuit
with switch in positions 1 and 2

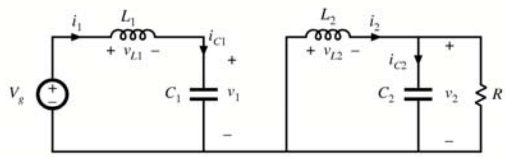
Switch in position 1: MOSFET conducts

Capacitor C_1 releases energy to output



Switch in position 2: diode conducts

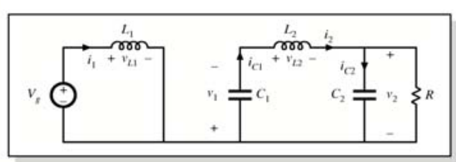
Capacitor C_1 is charged from input



Waveforms during subinterval 1
MOSFET conduction interval

Inductor voltages and capacitor currents:

$$v_{L1} = V_g$$
$$v_{L2} = -v_1 - v_2$$
$$i_{C1} = i_2$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$

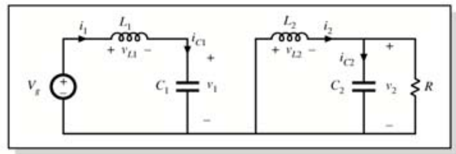


Small ripple approximation for subinterval 1:

$$v_{L1} = V_g$$
$$v_{L2} = -V_1 - V_2$$
$$i_{C1} = I_2$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Waveforms during subinterval 2
Diode conduction interval

Inductor voltages and capacitor currents:



$$v_{L1} = V_g - v_1$$
$$v_{L2} = -v_2$$
$$i_{C1} = i_1$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$

Small ripple approximation for subinterval 2:

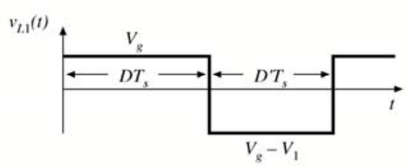
$$v_{L1} = V_g - V_1$$
$$v_{L2} = -V_2$$
$$i_{C1} = I_1$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Equate average values to zero

The principles of inductor volt-second and capacitor charge balance state that the average values of the periodic inductor voltage and capacitor current waveforms are zero, when the converter operates in steady state. Hence, to determine the steady-state conditions in the converter, let us sketch the inductor voltage and capacitor current waveforms, and equate their average values to zero.

Waveforms:

Inductor voltage $v_{L1}(t)$

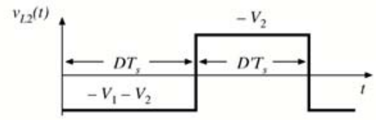


Volt-second balance on L_1:

$$\langle v_{L1} \rangle = DV_g + D'(V_g - V_1) = 0$$

Equate average values to zero

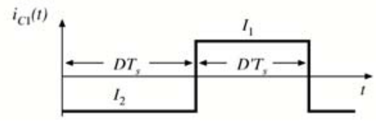
Inductor L_2 voltage



Average the waveforms:

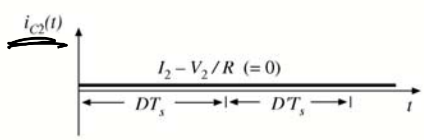
$$\langle v_{L2} \rangle = D(-V_1 - V_2) + D'(-V_2) = 0$$
$$\langle i_{C1} \rangle = DI_2 + D'I_1 = 0$$

Capacitor C_1 current



Equate average values to zero

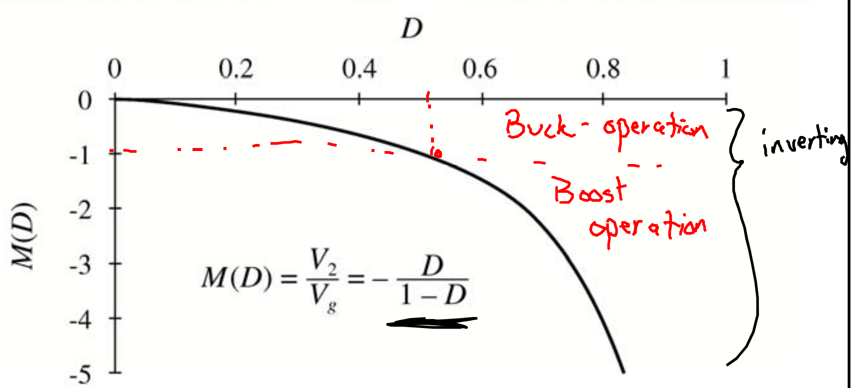
Capacitor current $i_{C2}(t)$ waveform



$$\langle i_{C2} \rangle = I_2 - \frac{V_2}{R} = 0$$

Note: during both subintervals, the capacitor current i_{C2} is equal to the difference between the inductor current i_2 and the load current V_2/R. When ripple is neglected, i_{C2} is constant and equal to zero.

Cuk converter conversion ratio $M = V / V_g$



Inductor current waveforms

Interval 1 slopes, using small ripple approximation:

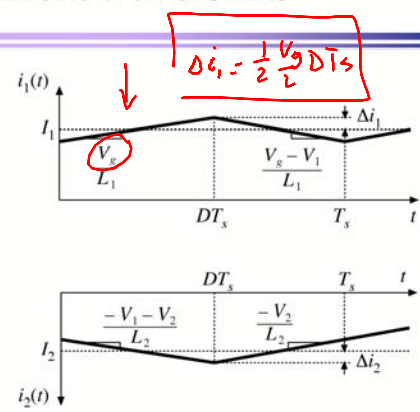
$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g}{L_1}$$

$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_1 - V_2}{L_2}$$

Interval 2 slopes:

$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g - V_1}{L_1}$$

$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_2}{L_2}$$



Capacitor C_1 waveform

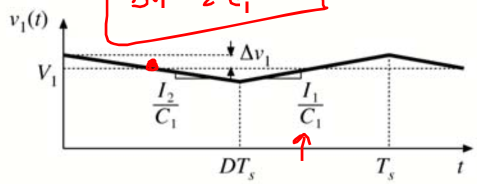
$\Delta v_1 = -\frac{1}{2}\frac{I_2}{C_1} DT_s$

Subinterval 1:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_2}{C_1}$$

Subinterval 2:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_1}{C_1}$$



Ripple magnitudes

Analysis results

$$\Delta i_1 = \frac{V_g DT_s}{2L_1}$$

$$\Delta i_2 = \frac{V_1 + V_2}{2L_2} DT_s$$

$$\Delta v_1 = \frac{-I_2 DT_s}{2C_1}$$

Use dc converter solution to simplify:

$$\Delta i_1 = \frac{V_g DT_s}{2L_1}$$

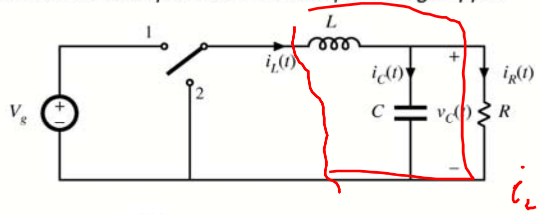
$$\Delta i_2 = \frac{V_g DT_s}{2L_2}$$

$$\Delta v_1 = \frac{V_g D^2 T_s}{2D'RC_1}$$

Q: How large is the output voltage ripple?

2.5 Estimating ripple in converters containing two-pole low-pass filters

Buck converter example: Determine output voltage ripple

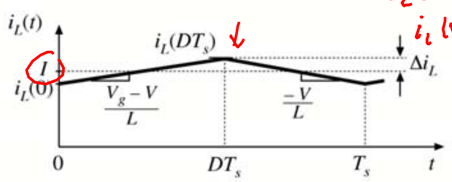


$i_L(t) \approx I_L$

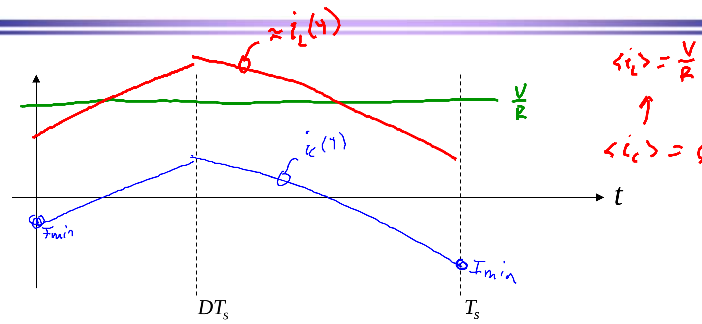
$i_L(t) \approx I_L$ + "linear ripple"

Inductor current waveform.

What is the capacitor current?



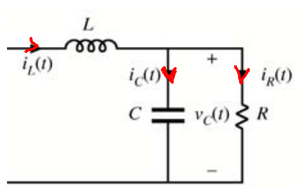
Relaxing Small Ripple Approximation



$\langle i_L \rangle = \frac{V}{R}$

↑

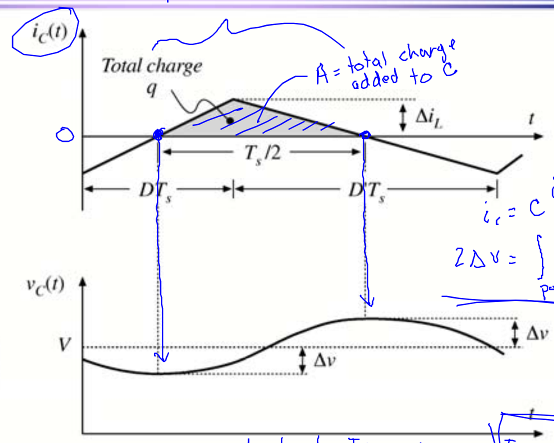
$\langle i_C \rangle = \emptyset$



Capacitor current and voltage, buck example

pos $i_c(t)$

Must not neglect inductor current ripple!



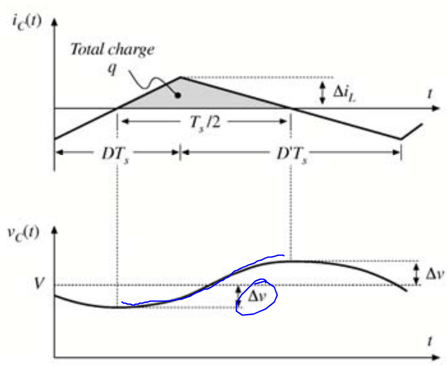
If the capacitor voltage ripple is small, then essentially all of the ac component of inductor current flows through the capacitor.

$i_c = C \frac{dv_c}{dt}$

$2\Delta v = \int_{pos} \frac{1}{C} i_c$

$\Delta v = \frac{1}{2}\frac{1}{C}\frac{1}{2}\frac{T_s}{2}\Delta i_L = \boxed{\frac{T_s}{8C}\Delta i_L}$

Estimating capacitor voltage ripple Δv

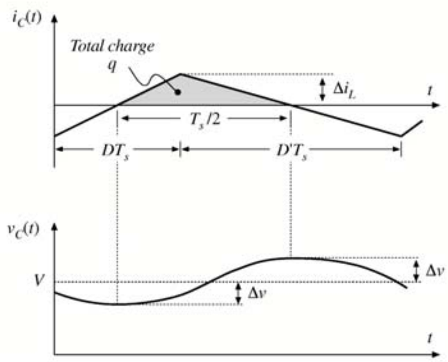


Current $i_C(t)$ is positive for half of the switching period. This positive current causes the capacitor voltage $v_C(t)$ to increase between its minimum and maximum extrema. During this time, the total charge q is deposited on the capacitor plates, where

$$q = C\,(2\Delta v)$$

(*change in charge*) =
C (*change in voltage*)

Estimating capacitor voltage ripple Δv



The total charge q is the area of the triangle, as shown:

$$q = \frac{1}{2}\Delta i_L \frac{T_s}{2}$$

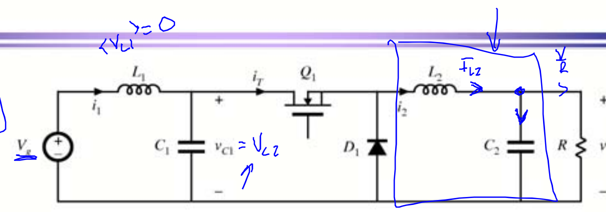
Eliminate q and solve for Δv:

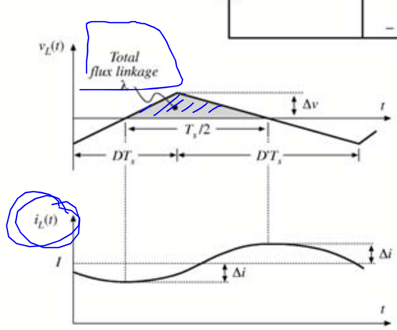
$$\Delta v = \frac{\Delta i_L\, T_s}{8\, C}$$

Note: in practice, capacitor equivalent series resistance (esr) further increases Δv.

Inductor current ripple in two-pole filters

Example: problem 2.9





can use similar arguments, with
$\lambda = L\,(2\Delta i)$

λ = inductor flux linkages

= inductor volt-seconds

Chapter 2: Summary

- Converter steady-state (DC) solution obtained by averaging over a switching period
- Complete steady state solution involves:
  - Small ripple approximation (capacitor voltage and inductor currents)
  - Volt-second and cap-charge balance
  - Inductor current and capacitor voltage ripples
    - Based on (approx) constant slopes, except in 2nd order filters

Chapter 3. Steady-State Equivalent Circuit Modeling, Losses, and Efficiency

3.1. The dc transformer model

3.2. Inclusion of inductor copper loss

3.3. Construction of equivalent circuit model

3.4. How to obtain the input port of the model

3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

3.6. Summary of key points