Lecture 4: Steady-State Averaged Modeling

ECE 481: Power Electronics

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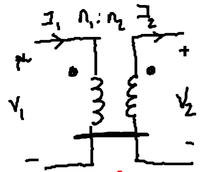
Chapter 3. Steady-State Equivalent Circuit Modeling, Losses, and Efficiency

Ideal Transformer Model



$1 : \frac{n_2}{n_1} = n$

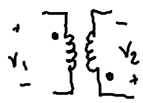
① $\frac{V_1}{n_1} = \frac{V_2}{n_2} \left(= \frac{V_3}{n_3} = \frac{V_4}{n_4} = \dots \right)$

② $I_1 n_1 + I_2 n_2 = 0$
$(+ I_3 n_3 + I_4 n_4 \dots)$

"Dot Notation"
- Pos. term of voltage
- Current "into" one dot comes "out" the other

① $V_2 = n V_1$

② $I_2 = \frac{1}{n} I_1$



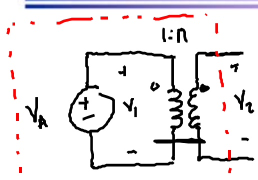
$P_1 = V_1 I_1$

$P_2 = V_2 I_2$

$P_2 = \cancel{n} V_1 \cancel{\tfrac{1}{n}} I_1$

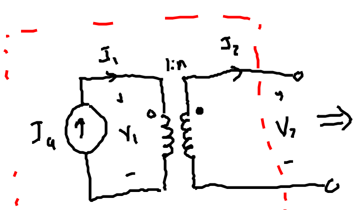
$P_2 = V_1 I_1 = P_1$

Simplifying Circuits With Ideal XF

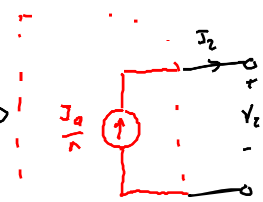


$\rightarrow V_2 = nV_1$

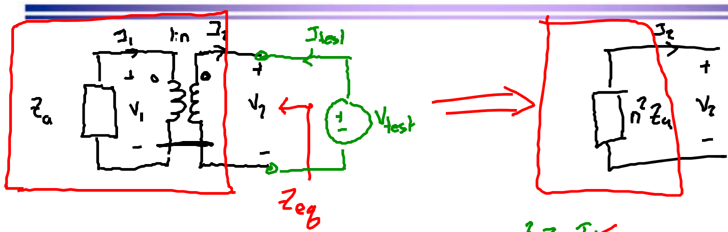
"Pushing" or "reflecting" through a XF



$\Rightarrow I_2 = \frac{1}{n} I_1 \Rightarrow$



Simplifying Circuits With Ideal XF



$Z_{eq} = \frac{V_{test}}{I_{test}}$ → $Z_{eq} = \frac{n^2 Z_a \cancel{I_{test}}}{\cancel{I_{test}}}$

$\boxed{Z_{eq} = n^2 Z_a}$

$I_2 = -I_{test}$

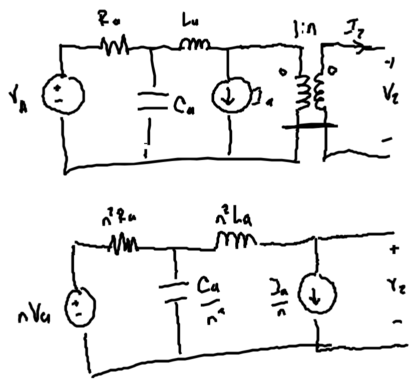
$I_1 = n I_2 = -n I_{test}$

$V_1 = -I_1 Z_a = Z_a n I_{test}$

$V_2 = n V_1 = \underline{n^2 Z_a I_{test}}$

$V_2 = V_{test}$

Simplifying Circuits With Ideal XF



$Z_c = \frac{1}{sC_a}$

$Z_c' = \frac{n^2}{sC_a} \Rightarrow \frac{C_a}{n^2}$

3.1. The dc transformer model

Basic equations of an ideal dc-dc converter:

$P_{in} = P_{out}$ $\quad (\eta = 100\%)$

$V_g I_g = V I$

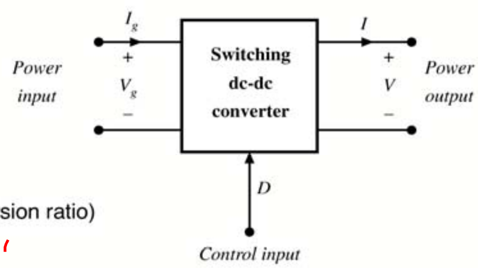
$$\boxed{\begin{aligned} V &= M(D)\, V_g \\ I_g &= M(D)\, I \end{aligned}}$$

(ideal conversion ratio)

$V_2 = n V_1$

$I_2 = \frac{1}{n} I_1$

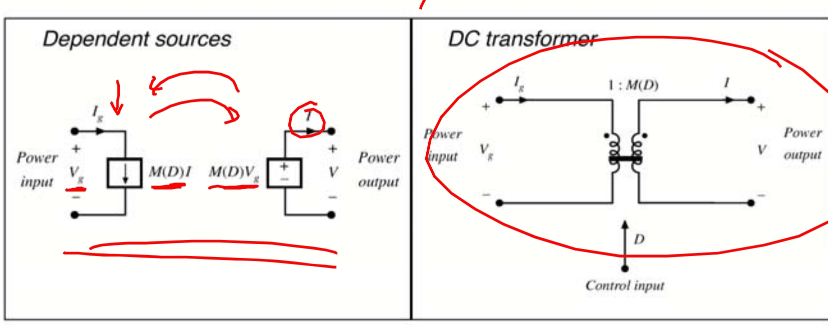
$I_1 = n I_2$



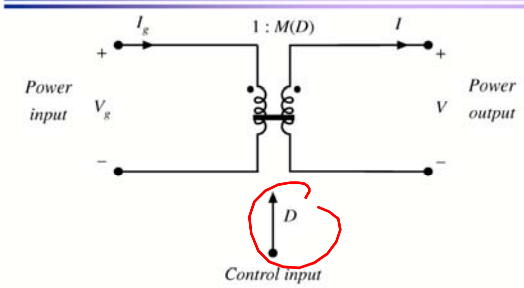
These equations are valid in steady-state. During transients, energy storage within filter elements may cause $P_{in} \neq P_{out}$

Equivalent circuits corresponding to ideal dc-dc converter equations

$P_{in} = P_{out} \qquad V_g I_g = V I \qquad \boxed{V = M(D)\, V_g \qquad I_g = M(D)\, I}$



The DC transformer model



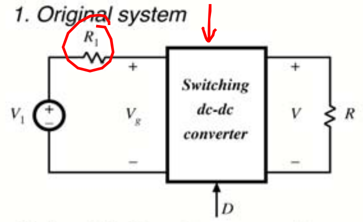
Models basic properties of ideal dc-dc converter:

- conversion of dc voltages and currents, ideally with 100% efficiency
- conversion ratio M = turns ratio controllable via duty cycle

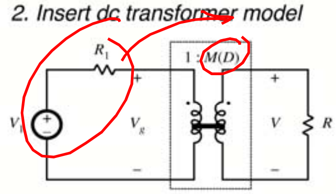
- Solid line denotes ideal transformer model, capable of passing dc voltages and currents
- Time-invariant model (no switching) which can be solved to find dc components of converter waveforms

Example: use of the DC transformer model

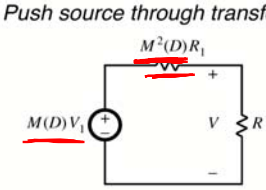
1. Original system



2. Insert dc transformer model



3. Push source through transformer



4. Solve circuit

$$V = M(D)\, V_1 \frac{R}{R + M^2(D)\, R_1}$$

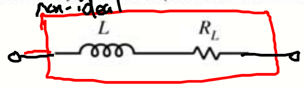
3.2. Inclusion of inductor copper loss

Dc transformer model can be extended, to include converter nonidealities.

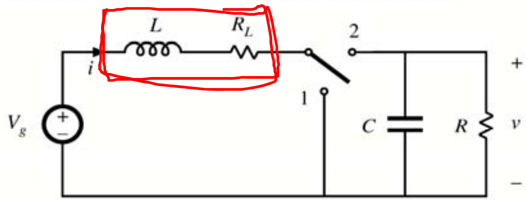
Example: inductor copper loss (resistance of winding):

Ideal

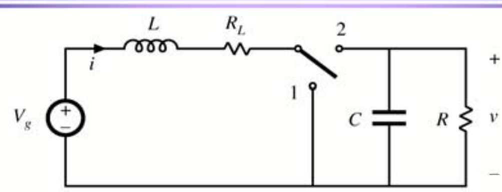
non-ideal



Insert this inductor model into boost converter circuit:

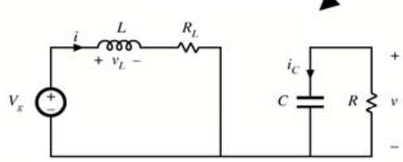


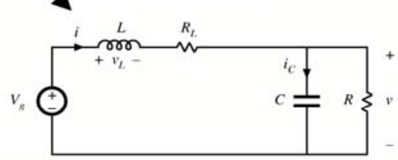
Analysis of nonideal boost converter



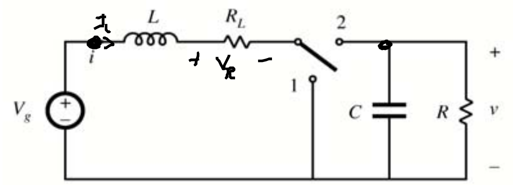
switch in position 1

switch in position 2





Nonideal Boost Converter



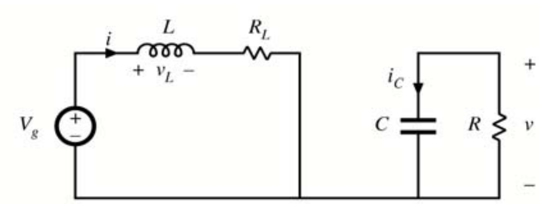
$$\langle v_L \rangle = 0 = V_g - I_L R_L - D' V$$

$$\langle i_C \rangle = 0 = D' I_L - \frac{V}{R}$$

Circuit equations, switch in position 1

Inductor current and capacitor voltage:

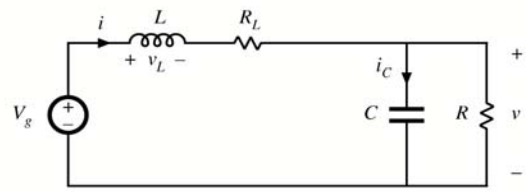
$$v_L(t) = V_g - i(t)\, R_L$$
$$i_C(t) = -v(t) \,/\, R$$



Small ripple approximation:

$$v_L(t) = V_g - I\, R_L$$
$$i_C(t) = -V \,/\, R$$

Circuit equations, switch in position 2



$$v_L(t) = V_g - i(t)\,R_L - v(t) \approx V_g - I\,R_L - V$$
$$i_C(t) = i(t) - v(t)\,/\,R \approx I - V\,/\,R$$

Inductor voltage and capacitor current waveforms

Average inductor voltage:

$$\left\langle v_L(t) \right\rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt$$
$$= D(V_g - I\,R_L) + D'(V_g - I\,R_L - V)$$

Inductor volt-second balance:

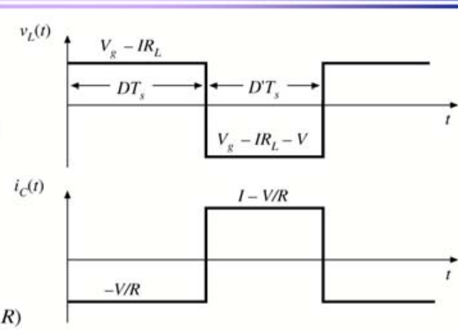
$$0 = V_g - I\,R_L - D'V$$

Average capacitor current:

$$\left\langle i_C(t) \right\rangle = D\,(-\,V\,/\,R) + D'\,(I - V\,/\,R)$$

Capacitor charge balance:

$$0 = D'I - V\,/\,R$$



Solution for output voltage

We now have two equations and two unknowns:

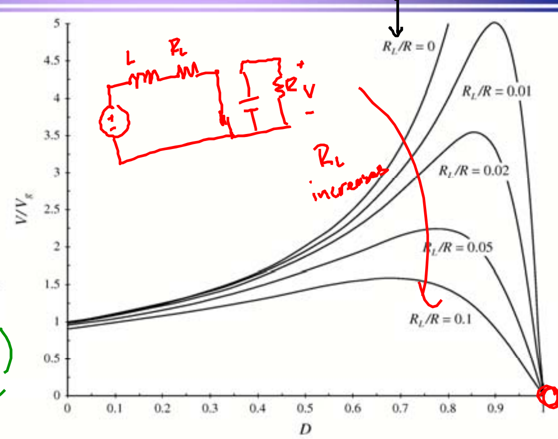
$$0 = V_g - I R_L - D'V$$

$$0 = D'I - V / R$$

Eliminate I and solve for V:

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} \frac{1}{(1 + R_L / D'^2 R)}$$

$$M = M_{ideal} \underbrace{\left(\frac{1}{1 + \frac{R_L}{D'^2 R}} \right)}_{= \eta}$$



$I = I_L$ $V_C = V$

3.3. Construction of equivalent circuit model

Results of previous section (derived via inductor volt-sec balance and capacitor charge balance):

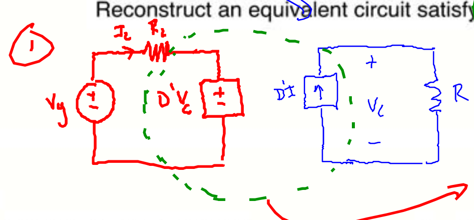
① $\langle v_L \rangle = 0 = V_g - I R_L - D'V$ → KVL around I_L loop

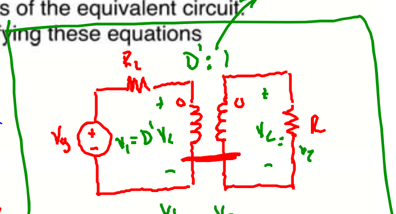
② $\langle i_C \rangle = 0 = D'I - V_c / R$ → KCL at node V_c

View these as loop and node equations of the equivalent circuit. Reconstruct an equivalent circuit satisfying these equations

1: M(D)

1: $\frac{1}{D'}$



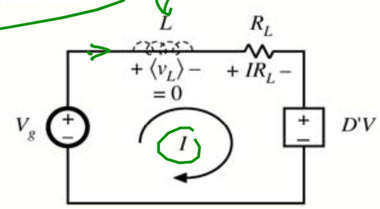


$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

$n_1 = D'$ $\frac{\cancel{D'} V_c}{\cancel{D'}} = \frac{V_2}{\cancel{n_2}}$ $n_2 = 1$

Inductor voltage equation

$$\langle v_L \rangle = 0 = V_g - I\,R_L - D'V$$

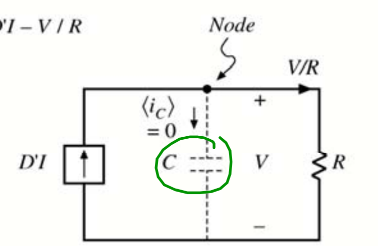


- Derived via Kirchhoff's voltage law, to find the inductor voltage during each subinterval
- Average inductor voltage then set to zero
- This is a loop equation: the dc components of voltage around a loop containing the inductor sum to zero

- IR_L term: voltage across resistor of value R_L having current I
- $D'V$ term: for now, leave as dependent source

Capacitor current equation

$$\langle i_C \rangle = 0 = D'I - V / R$$

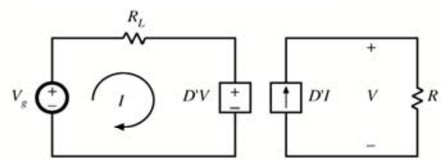


- Derived via Kirchoff's current law, to find the capacitor current during each subinterval
- Average capacitor current then set to zero
- This is a node equation: the dc components of current flowing into a node connected to the capacitor sum to zero

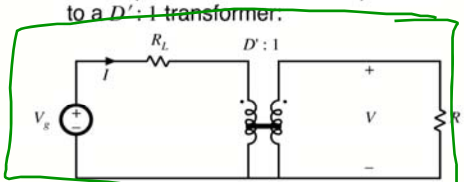
- V/R term: current through load resistor of value R having voltage V
- $D'I$ term: for now, leave as dependent source

Complete equivalent circuit

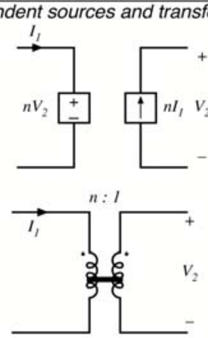
The two circuits, drawn together:



The dependent sources are equivalent to a D' : 1 transformer:



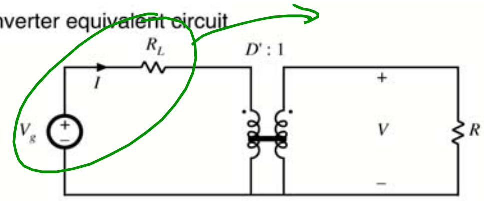
Dependent sources and transformers



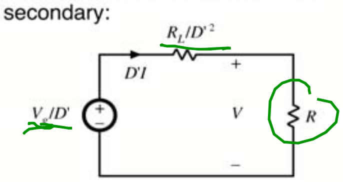
- *sources have same coefficient*
- *reciprocal voltage/current dependence*

Solution of equivalent circuit

Converter equivalent circuit



Refer all elements to transformer secondary:

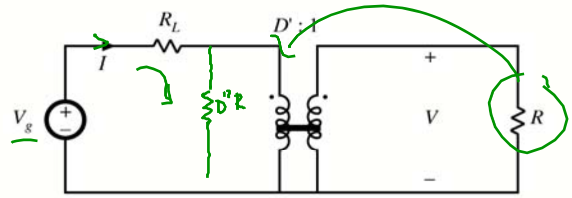


Solution for output voltage using voltage divider formula:

$$V = \frac{V_g}{D'} \frac{R}{R + \frac{R_L}{D'^2}} = \boxed{\frac{V_g}{D'} \frac{1}{1 + \frac{R_L}{D'^2 R}}}$$

$$M = \frac{1}{D'} \cdot \frac{1}{1 + \frac{R_L}{D'^2 R}} = M_{ideal} \, ?$$

Solution for input (inductor) current



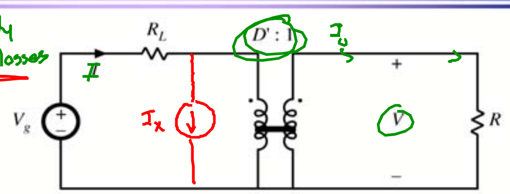
$$I = \frac{V_g}{D'^2 R + R_L} = \frac{V_g}{D'^2} \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

Solution for converter efficiency

If we have only cond. losses

$$P_{in} = (V_g)\,(I)$$

$$P_{out} = (V)\,(D'I)$$



$I_o = I_g D'$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{(V)\,(D'\cancel{I})}{(V_g)\,(\cancel{I})} = \boxed{\frac{V}{V_g} D'} = M \frac{1}{M_{ideal}}$$

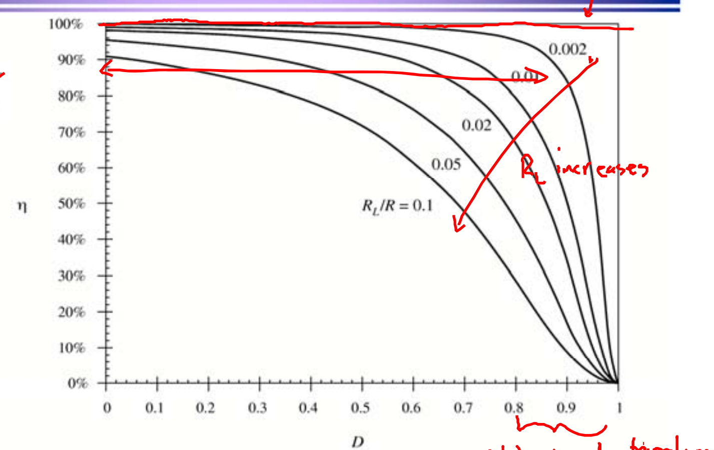
$$\eta = \frac{1}{1 + \frac{R_L}{D'^2 R}}$$

→ $M = M_{ideal}$?

Efficiency, for various values of R_L

$\frac{R_L}{R} = \emptyset$

$$\eta = \frac{1}{1 + \frac{R_L}{\cancel{D^2} R}}$$

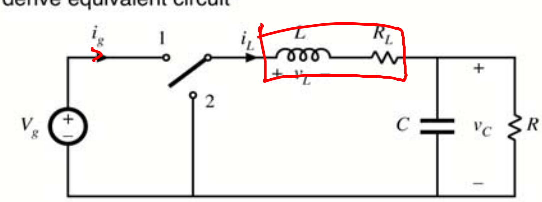


3.4. How to obtain the input port of the model

Buck converter example —use procedure of previous section to derive equivalent circuit

① $i_g = i_L$

② $i_g = \emptyset$

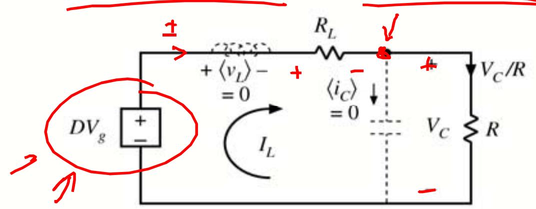


Average inductor voltage and capacitor current:

$$\langle v_L \rangle = 0 = DV_g - I_L R_L - V_C \qquad \langle i_C \rangle = 0 = I_L - V_C/R$$

Construct equivalent circuit as usual

$\langle v_L \rangle = 0 = DV_g - I_L R_L - V_C$ $\qquad$ $\langle i_C \rangle = 0 = I_L - V_C/R$



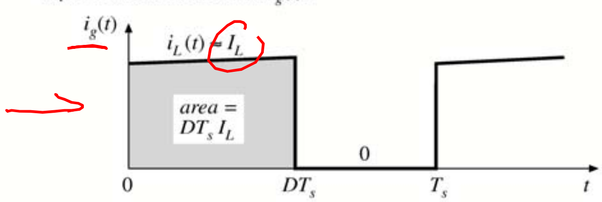
What happened to the transformer?

- Need another equation

Add 1 eq: Averaging of input port

Modeling the converter input port

Input current waveform $i_g(t)$:



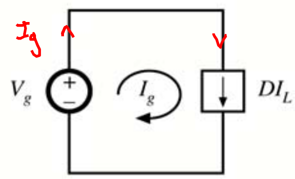
Dc component (average value) of $i_g(t)$ is

$$I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)\,dt = DI_L \rightarrow \boxed{I_g = D I_L}$$

Input port equivalent circuit

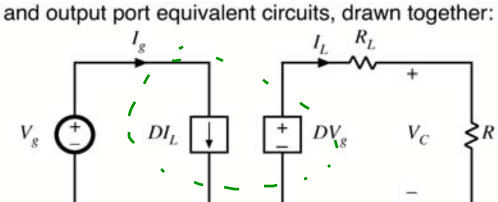
$$I_g = \frac{1}{T_s}\int_0^{T_s} i_g(t)\,dt = DI_L$$

$I_g = DI_L$



Complete equivalent circuit, buck converter

Input and output port equivalent circuits, drawn together:

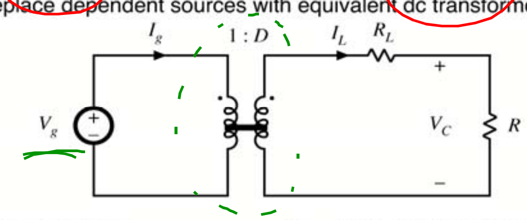


input port averaging

volt-sec balance

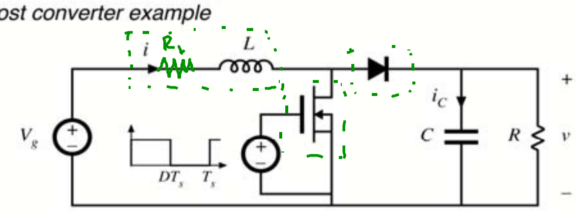
cap-q balance

Replace dependent sources with equivalent dc transformer:



3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

Boost converter example



Models of on-state semiconductor devices:

MOSFET: on-resistance R_{on}

Diode: constant forward voltage V_D plus on-resistance R_D

Insert these models into subinterval circuits

Modeling of component conduction losses