Lecture 14: Discontinuous Conduction Mode Examples

ECE 481: Power Electronics

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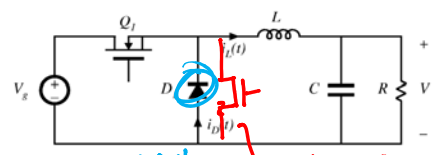
Buck Converter Example



synchronous implementation (No DCM)

$V = DV_g$

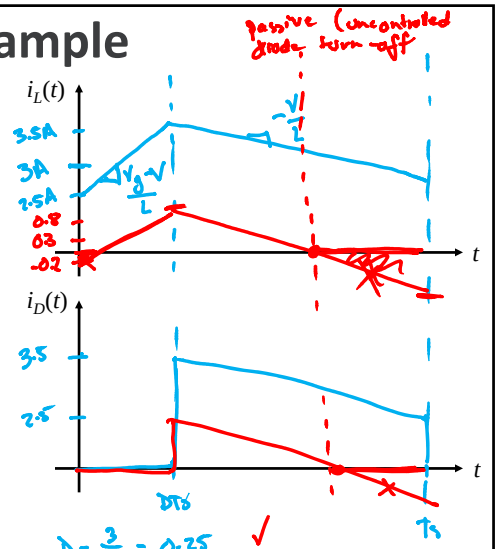
$\rightarrow I_L = \frac{V}{R}$

$\rightarrow 2\Delta i_L = \frac{V_g - V}{L} DT_s$

Ex/ $V_g = 12$ $V = 3$ $L = 100\mu H$ $T_s = 50\mu s$

first, $R = 1\Omega$

Now $R \rightarrow 10\Omega$



$D = \frac{3}{12} = 0.25$ ✓

$I_L = \frac{3V}{1\Omega} = 3A \rightarrow 0.3A$ *

$(\Delta i_L > I_L ?)$

$\Delta i_L = 0.56A$ ✓

K_{crit} and R_{crit} $(V = DV_g)$

Buck converter:

$I_L = \frac{V}{R} = \frac{DV_g}{R}$ $\quad \Delta i_L = \frac{V_g - V}{2L} D T_s = \frac{V_g D'}{2L} D T_s$

converter parameters

- L
- T_s } Δi_L
- $R \rightarrow I_L$

In CCM when $I_L > \Delta i_L$

Go into DCM when $I_L < \Delta i_L$

for CCM:

$$\frac{\cancel{D} V_g}{R} > \frac{V_g D'}{2L} \cancel{D} T_s \qquad \underbrace{\frac{2L}{D' T_s}}_{R_{crit}} > R$$

$$K = \boxed{\frac{2L}{R T_s} > D'} = K_{crit}(D)$$

$K > K_{crit}(D)$ for CCM

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DCM Mode Boundary: Summary

fixed $\quad$ fixed

$K > K_{crit}(D)$ or $R < R_{crit}(D)$ for CCM

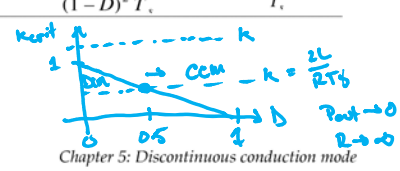
$K < K_{crit}(D)$ or $R > R_{crit}(D)$ for DCM

when as a function of D, does DCM occur?

Table 5.1 CCM-DCM mode boundaries for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	$\max_{0 \le D \le 1}(K_{crit})$	$R_{crit}(D)$	$\min_{0 \le D \le 1}(R_{crit})$
Buck	$(1-D)$	1	$\frac{2L}{(1-D)T_s}$	$2\frac{L}{T_s}$
Boost	$D(1-D)^2$	$\frac{4}{27}$	$\frac{2L}{D(1-D)^2 T_s}$	$\frac{27}{2}\frac{L}{T_s}$
Buck-boost	$(1-D)^2$	1	$\frac{2L}{(1-D)^2 T_s}$	$2\frac{L}{T_s}$

$K = \frac{2L}{RT_s}$



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Finding the Conversion Ratio M(D,K)

$\frac{V}{V_g} = M$

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance ✓

$$\langle v_L \rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = 0$$

Capacitor charge balance ✓

$$\langle i_C \rangle = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = 0$$

Only assumption was steady-state operation

$i_L(T_s) = i_L(0)$

$v_C(T_s) = v_C(0)$

Small ripple approximation sometimes applies:

$v(t) \approx V$ because $\Delta v << V$

$i(t) \approx I$ is a poor approximation when $\Delta i > I$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

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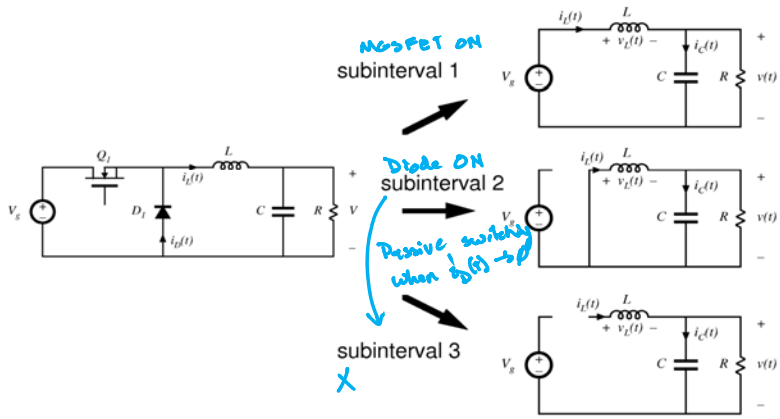
Buck Converter in DCM



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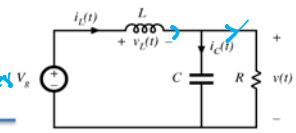
Subinterval Analysis

$v_L = V_g - v(t) \approx V_g - V$ ← SRA still applies

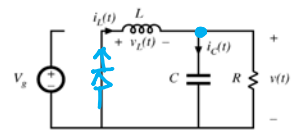
① $i_c = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R}$

↑ SRA no longer applies



$v_L = 0 - v(t) \approx -V$

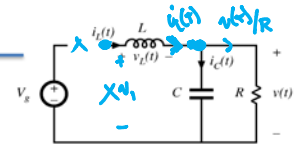
② $i_c = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R}$



$v_L = \phi$

③ $i_c = i_L(t) - \frac{v(t)}{R} = i_L(t) - \frac{V}{R}$

↓ has gone to zero



$v_1 = v(t)$

Waveforms in DCM

$\langle v_L \rangle = \phi = D_1(V_g - V) + D_2(-V) + D_3(\phi)$

$\phi = D_1 V_g - V(D_1 + D_2)$

$$\boxed{\frac{V}{V_g} = \frac{D_1}{D_1 + D_2}}$$

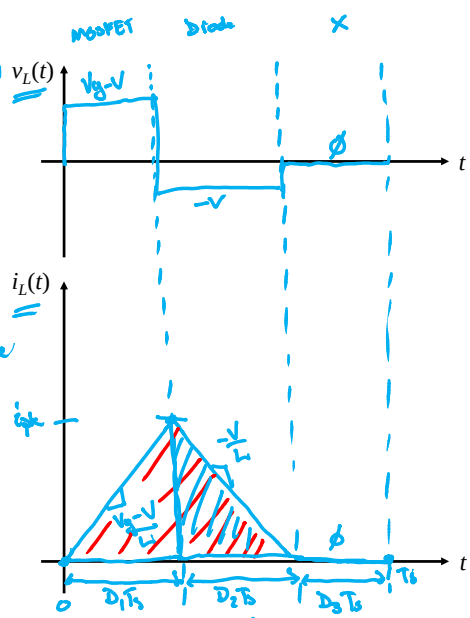
$\langle i_c \rangle = \phi = \frac{1}{T_s}\int_0^{T_s} i_L(t) - \frac{V}{R}\,dt$

↑ constant in time

$\langle i_c \rangle = \phi = \underbrace{\frac{1}{T_s}\int_0^{T_s} i_L(t)\,dt}_{} - \frac{V}{R}$

$\frac{V}{R} = \langle i_L \rangle$

$\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2} D_1 T_s i_{pk} + \frac{1}{2} D_2 T_s i_{pk}\right]$



$(D_1 + D_2 + D_3) = 1$

$$i_{pk} = \frac{V_g - V}{L} D_1 T_s$$

$$\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2} D_1 T_s \, i_{pk} + \frac{1}{2} D_2 T_s \, i_{pk}\right]$$

$$\boxed{\frac{V}{R} = \frac{1}{2}(D_1 + D_2)\frac{(V_g - V)}{L} D_1 T_s}$$

Solving *M*(*D*,*K*)

Two equations and two unknowns (V and D_2):

$$\begin{cases} V = V_g \dfrac{D_1}{D_1 + D_2} & \text{(from inductor volt-second balance)} \\[2ex] \dfrac{V}{R} = \dfrac{D_1 T_s}{2L}(D_1 + D_2)(V_g - V) & \text{(from capacitor charge balance)} \end{cases}$$

Eliminate D_2, solve for V:

$$M(D_1, K) \qquad \frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}$

Controlled duty cycle

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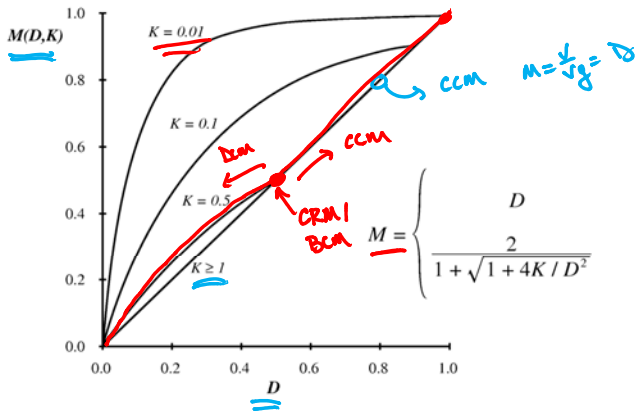
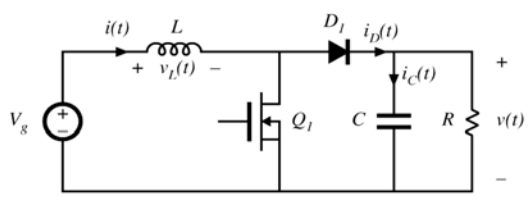
Buck Converter *M*(*D*,*K*)



$$M = \begin{cases} D & \text{for } K > K_{crit} \rightarrow \text{CCM} \\ \dfrac{2}{1+\sqrt{1+4K/D^2}} & \text{for } K < K_{crit} \rightarrow \text{DCM} \end{cases}$$

Boost Converter in DCM



$$\frac{V_g}{D'^2 R} > \frac{V_g}{2L} D T_s$$

$$\boxed{\frac{2L}{R T_s} > D D'^2}$$

K $K_{crit}(D)$

Mode boundary:

$I > \Delta i_L$ for CCM
$I < \Delta i_L$ for DCM

Previous CCM soln:

$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} D T_s$$

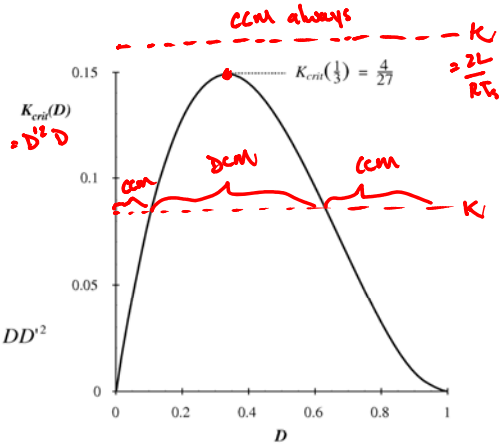
Boost DCM Boundary

$$\frac{V_g}{D'^2 R} > \frac{DT_s V_g}{2L} \quad \text{for CCM}$$

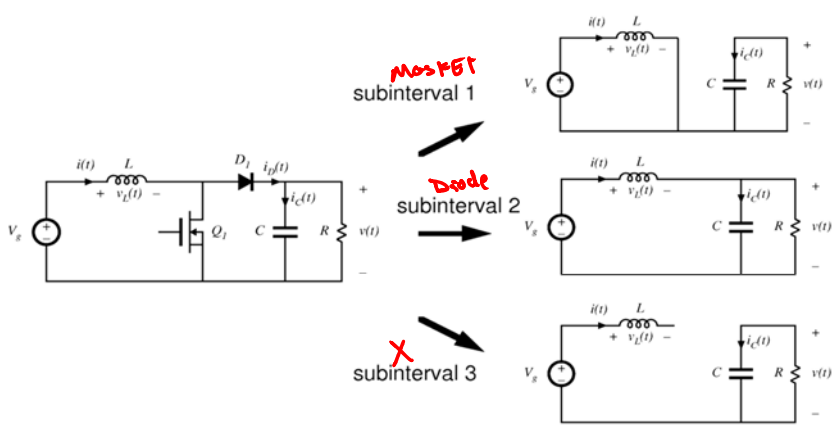
$$\frac{2L}{RT_s} > DD'^2 \quad \text{for CCM}$$

$$K > K_{crit}(D) \quad \text{for CCM}$$
$$K < K_{crit}(D) \quad \text{for DCM}$$

where $K = \frac{2L}{RT_s}$ and $K_{crit}(D) = DD'^2$



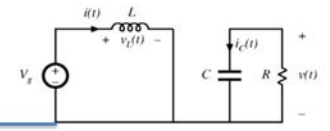
Boost Converter Subintervals



Boost Conversion Ratio

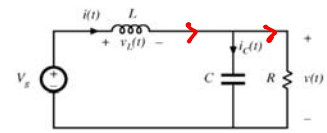
$v_L = V_g$

① $i_c = -\frac{v(t)}{R} \approx \frac{-V}{R}$



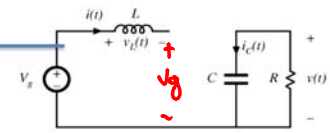
$v_L = V_g - v(t) \approx V_g - V$

② $i_c = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R}$



$v_L = \phi$

③ $i_c = -\frac{v(t)}{R} \approx -\frac{V}{R}$



Boost Waveforms in DCM

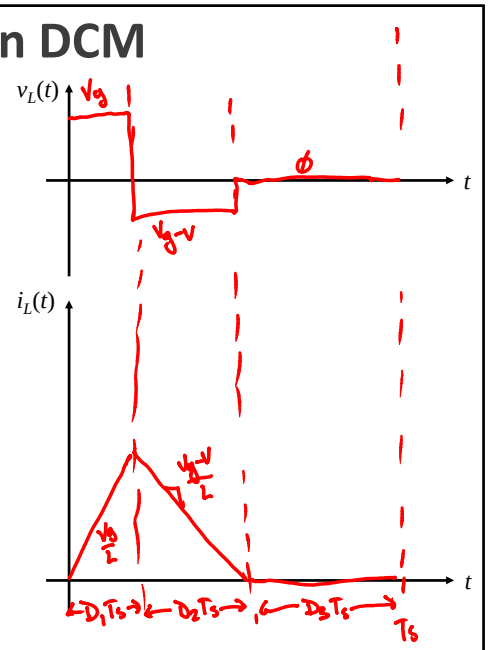
$\langle v_L \rangle = \phi = D_1 V_g + D_2 (V_g - V) + D_3 \phi$

$\phi = V_g (D_1 + D_2) + D_2 (-V)$

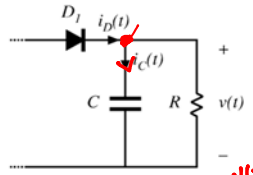
$$\boxed{\frac{V}{V_g} = \frac{D_1 + D_2}{D_2}}$$

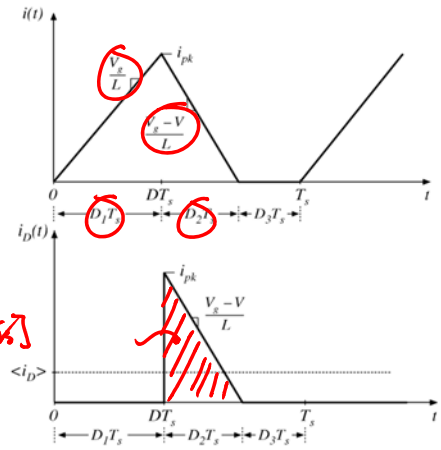
$\langle i_c \rangle = -\frac{V}{R} + \frac{1}{T_s} \int_{D_1 T_s}^{(D_1+D_2)T_s} i_L(t)\, dt$

$= -\frac{V}{R} + \langle i_D(t) \rangle$



Boost Cap-Charge Balance





$$\phi = \langle i_C \rangle = \langle i_D \rangle - \left\langle \frac{v(t)}{R} \right\rangle$$

$$\langle i_C \rangle = \phi = -\frac{V}{R} + \left[\frac{1}{2} D_2 T_s \, i_{pk}\right] \frac{1}{T_s}$$

$$i_{pk} = \frac{V_g}{L} D_1 T_s$$

$$\langle i_C \rangle = \phi = -\frac{V}{R} + \frac{1}{T_s}\left[\frac{1}{2} D_2 T_s \frac{V_g}{L} D_1 T_s\right]$$

$$\boxed{\frac{V}{R} = \frac{D_1 D_2 V_g T_s}{2L}}$$

Boost DCM Conversion Ratio

$$V^2 - V V_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive V, while other leads to negative V. Select positive root:

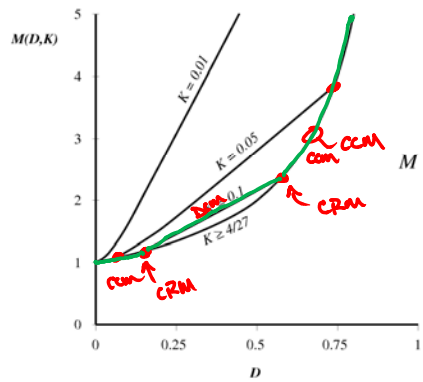
$$\frac{V}{V_g} = M(D_1, K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

Boost Conversation Ratio



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\ \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Summary of DCM Characteristics

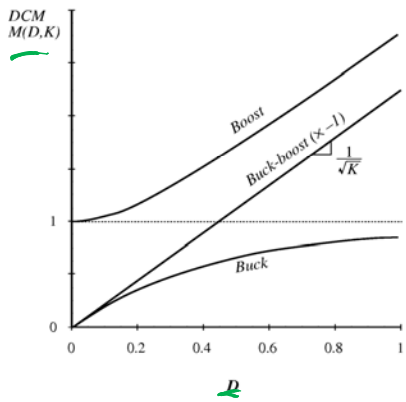
Table 5.2 Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	*DCM* $M(D,K)$	*DCM* $D_2(D,K)$	*CCM* $M(D)$
Buck	$(1-D)$	$\dfrac{2}{1+\sqrt{1+4K/D^2}}$	$\dfrac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\dfrac{1+\sqrt{1+4D^2/K}}{2}$	$\dfrac{K}{D}M(D,K)$	$\dfrac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\dfrac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\dfrac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.

$D_1 \rightarrow$ control
$D_2 \rightarrow$ from table
$D_3 = 1 - D_1 - D_2$

Summary of DCM Characteristics



- DCM buck and boost characteristics are asymptotic to $M = 1$ and to the DCM buck-boost characteristic
- DCM buck-boost characteristic is linear
- CCM and DCM characteristics intersect at mode boundary. Actual M follows characteristic having larger magnitude
- DCM boost characteristic is nearly linear

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Chapter 5 Summary

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.
2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.
3. The dc conversion ratio M of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.
4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.
5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.

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