Lecture 24: Modulator Modeling, Bode Plot Review

ECE 481: Power Electronics

Prof. Daniel Costinett
Department of Electrical Engineering and Computer Science
University of Tennessee Knoxville
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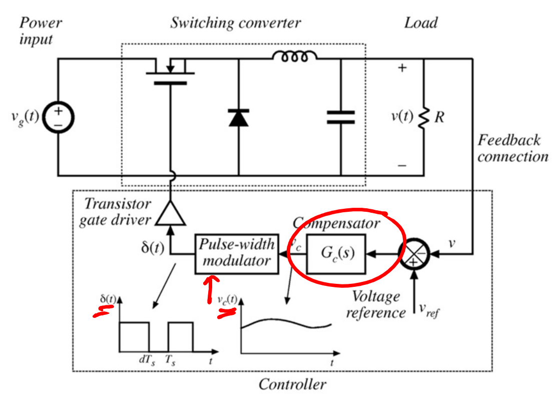
7.6: Modeling Pulse Width Modulator

Pulse-width modulator converts voltage signal $v_c(t)$ into duty cycle signal $d(t)$.

What is the relationship between $v_c(t)$ and $d(t)$?



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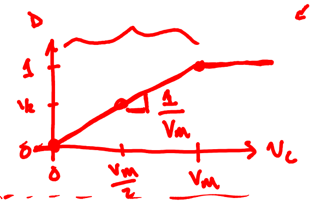
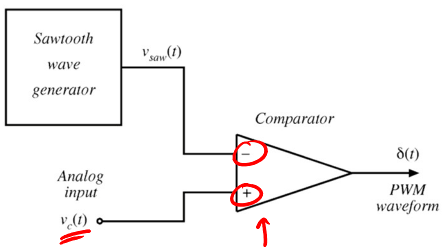
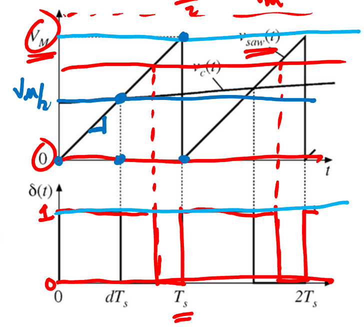
Functional PWM Model

for $0 < v_c(t) < V_m$

$$\boxed{D = \frac{1}{V_m} v_c(t)}$$





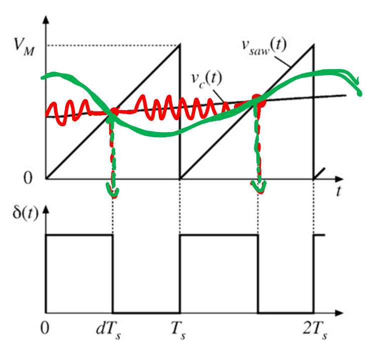


Equations of PWM Model

For a linear sawtooth waveform:

$$\boxed{d(t) = \frac{v_c(t)}{V_M} \qquad \text{for } 0 \le v_c(t) \le V_M}$$

So $d(t)$ is a linear function of $v_c(t)$.



Perturbed Behavior of PWM

$V_c \to \boxed{1/V_m} \to D = \frac{V_c}{V_m}$

$\hat{v}_c \to \boxed{1/V_m} \to \hat{d} = \frac{\hat{v}_c}{V_m}$

PWM equation:

$$d(t) = \frac{v_c(t)}{V_M} \qquad for\ 0 \le v_c(t) \le V_M$$

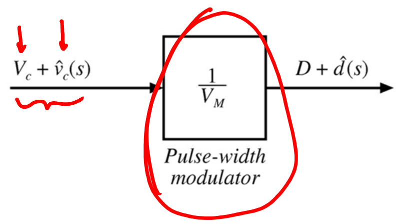
Perturb:

$$v_c(t) = V_c + \hat{v}_c(t)$$
$$d(t) = D + \hat{d}(t)$$

Result:

$$D + \hat{d}(t) = \frac{V_c + \hat{v}_c(t)}{V_M}$$

Block diagram:



Dc and ac relationships:

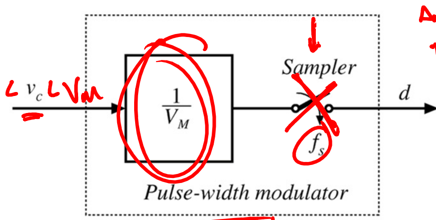
$$D = \frac{V_c}{V_M}$$
$$\hat{d}(t) = \frac{\hat{v}_c(t)}{V_M}$$

Sampling Behavior of PWM

Nonlinearities: ① saturation ✗ ② sampling

The input voltage is a continuous function of time, but there can be only one discrete value of the duty cycle for each switching period.

$0 < v_c < V_M$



Averaging over T_s → signals vary at rate $\omega_m \ll \omega_s$

$v_c(t) \to \boxed{PWM} \to d[n]_{T_s}$

Therefore, the pulse-width modulator samples the control waveform, with sampling rate equal to the switching frequency.

In practice, this limits the useful frequencies of ac variations to values much less than the switching frequency. Control system bandwidth must be sufficiently less than the Nyquist rate $f_s/2$. Models that do not account for sampling are accurate only at frequencies much less than $f_s/2$.

Chapter 7: Summary

1. The CCM converter analytical techniques of Chapters 2 and 3 can be extended to predict converter ac behavior. The key step is to average the converter waveforms over one switching period. This removes the switching harmonics, thereby exposing directly the desired dc and low-frequency ac components of the waveforms. In particular, expressions for the averaged inductor voltages, capacitor currents, and converter input current are usually found.
2. Since switching converters are nonlinear systems, it is desirable to construct small-signal linearized models. This is accomplished by perturbing and linearizing the averaged model about a quiescent (dc) operating point.
3. Ac equivalent circuits can be constructed, in the same manner used in Chapter 3 to construct dc equivalent circuits. If desired, the ac equivalent circuits may be refined to account for the effects of converter losses and other nonidealities.
4. The conventional pulse-width modulator circuit has $\boxed{\text{linear gain}}$, dependent on the slope of the sawtooth waveform, or equivalently on its peak-to-peak magnitude.

Chapter 8: Converter Transfer Functions

Converter Transfer Functions

8.3. Graphical construction of converter transfer functions

- 8.3.1. Series impedances: addition of asymptotes
- 8.3.2. Parallel impedances: inverse addition of asymptotes
- 8.3.3. Another example
- 8.3.4. Voltage divider transfer functions: division of asymptotes

8.4. Measurement of ac transfer functions and impedances (briefly)

8.5. Summary of key points

↓

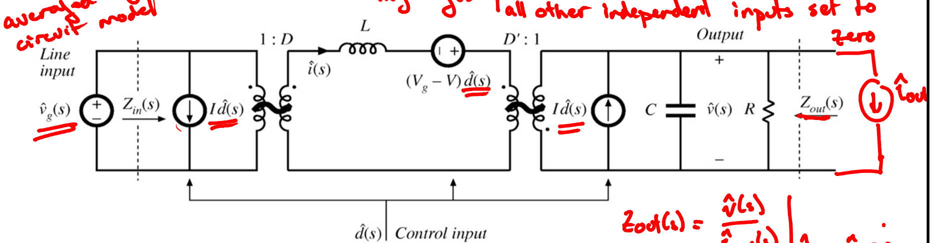
Chapter 9 : Controller design

Buck Boost Model

Linear: Superposition applies

AC, linear, averaged equivalent circuit model

$$G_{xy} = \frac{x(s)}{y(s)}\Bigg|_{\text{all other independent inputs set to zero}}$$



$$Z_{out}(s) = \frac{\hat{v}(s)}{\hat{i}_{out}(s)}\Bigg|_{\hat{d}=0,\ \hat{v}_g=0}$$

output impedance,

$$G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)}\Bigg|_{\hat{d}(s)=0,\ \hat{i}_{out}=0}$$

line-to-output

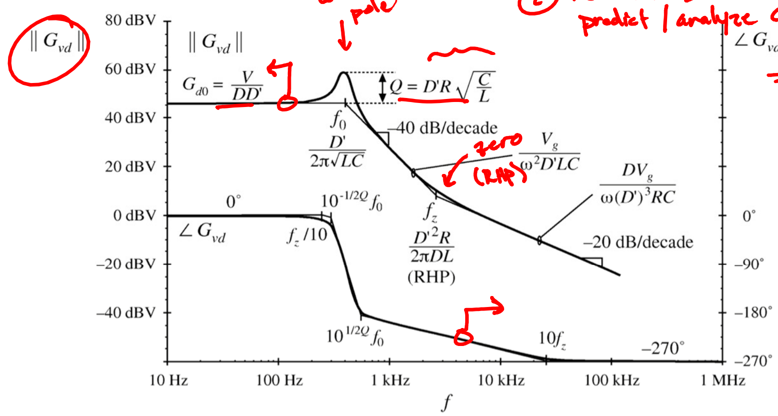
$$G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)}\Bigg|_{\hat{v}_g(s)=0,\ \hat{i}_{out}=0}$$

control-to-output

Control-to-output Transfer Function

→ ① How do I get this plot?

② How can I use it to predict / analyze converter behavior?



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Design-Oriented Analysis

Prof Middlebrook

How to approach a real (and hence, complicated) system

Problems:

- Complicated derivations
- Long equations
- Algebra mistakes

Huge equations result from exact analysis

→ Just simulate

Design objectives:

- Obtain physical insight which leads engineer to synthesis of a good design
- Obtain simple equations that can be inverted, so that element values can be chosen to obtain desired behavior. Equations that cannot be inverted are useless for design! $BW_{cl} = f(L, C, R) \rightarrow R, C, L \Rightarrow f^{-1}(BW_{cl})$

Design-oriented analysis is a structured approach to analysis, which attempts to avoid the above problems

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Chapter 8: Design-Oriented Analysis

- Writing transfer functions in normalized form, to directly expose salient features
- Obtaining simple analytical expressions for asymptotes, corner frequencies, and other salient features, allows element values to be selected such that a given desired behavior is obtained
- Use of inverted poles and zeroes, to refer transfer function gains to the most important asymptote
- Analytical approximation of roots of high-order polynomials ✓
- Graphical construction of Bode plots of transfer functions and polynomials, to
  - avoid algebra mistakes
  - approximate transfer functions
  - obtain insight into origins of salient features

8.1 Review of Bode Plots

Magnitude
Phase

Decibels

$$\| G \|_{dB} = 20 \log_{10}\left(\| G \| \right)$$

Decibels of quantities having units (impedance example): normalize before taking log

$$\| Z \|_{dB} = 20 \log_{10}\left(\frac{\| Z \|}{R_{base}} \right)$$

$20 \log_{10}(5) = 14$

Table 8.1. Expressing magnitudes in decibels

Actual magnitude	*Magnitude in dB*
1/2	– 6dB
1	0 dB
2	6 dB
5 = 10/2	20 dB – 6 dB = 14 dB
10	20dB
1000 = 10^3	3 · 20dB = 60 dB

5Ω is equivalent to 14dB with respect to a base impedance of R_{base} = 1Ω, also known as 14dBΩ.

60dBμA is a current 60dB greater than a base current of 1μA, or 1mA.

1mA = 0dBmA = 60dBμA = -60dB

Logarithm Review

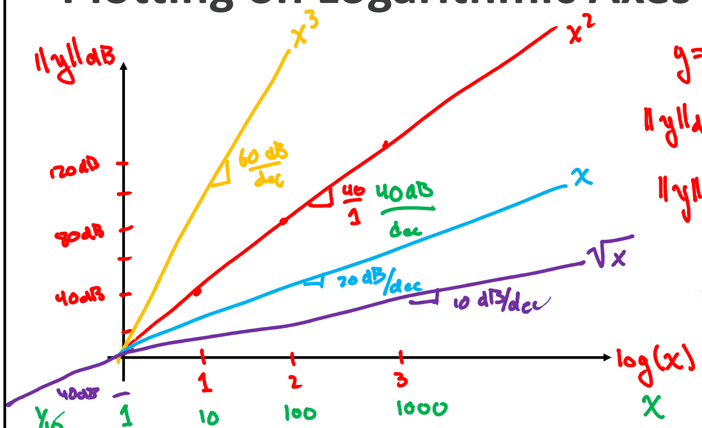
$$\log_x y = \frac{\log_a y}{\log_a x} \overset{e.g.}{=} \frac{\ln(y)}{\ln(x)} \qquad \log_{10} 5 = \frac{\ln(5)}{\ln(10)}$$

$$\log(x \cdot y) = \log(x) + \log(y)$$

$$\log(x/y) = \log(x) - \log(y)$$

$$\log(x^n) = n \log(x)$$

Plotting on Logarithmic Axes



$$y = x^2$$

$$\|y\|_{dB} = 20\log_{10}(\|x^2\|)$$

$$\|y\|_{dB} = 40 \log_{10}(x)$$

$$y_2 = x^3$$

$$\|y\|_{dB} = 60 \log(x)$$

$$x \rightarrow f$$