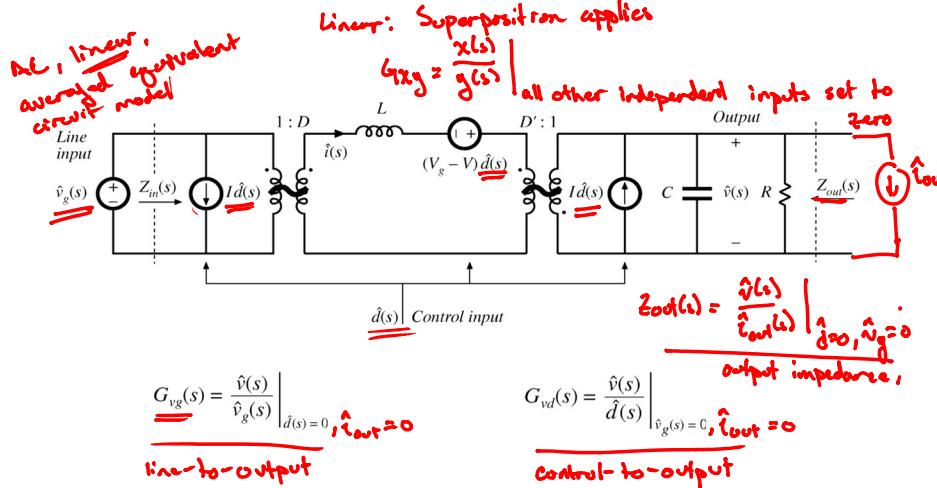


Buck Boost Model



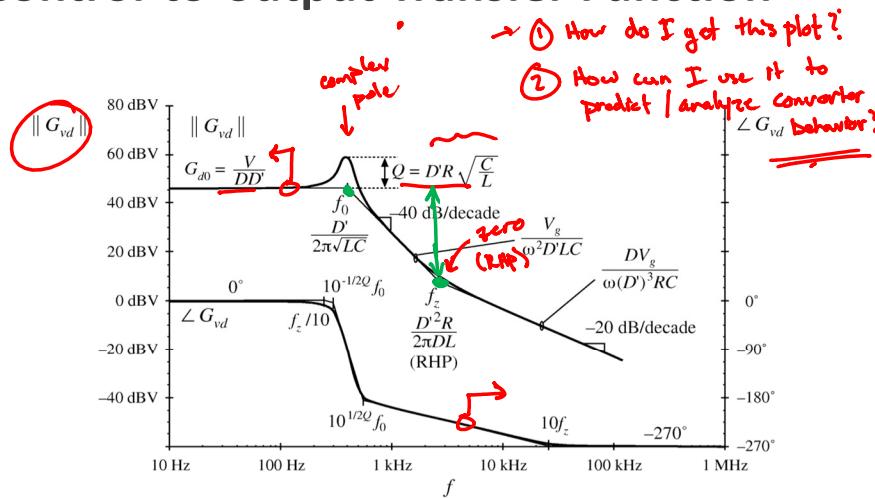
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Control-to-output Transfer Function



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Design-Oriented Analysis

Prof. Middlebrook

How to approach a real (and hence, complicated) system

Problems:

Complicated derivations

Long equations

Algebra mistakes

Huge equations result from exact analysis

→ Just simulate

Design objectives:

Obtain physical insight which leads engineer to synthesis of a good design

Obtain simple equations that can be inverted, so that element values can be chosen to obtain desired behavior. Equations that cannot be inverted are useless for design! $BW_{ac} > f(L, C, R) \rightarrow R, C, L \rightarrow f^{-1}(BW_{ac})$

Design-oriented analysis is a structured approach to analysis, which attempts to avoid the above problems

Chapter 8: Design-Oriented Analysis

- Writing transfer functions in normalized form, to directly expose salient features
- Obtaining simple analytical expressions for asymptotes, corner frequencies, and other salient features, allows element values to be selected such that a given desired behavior is obtained
- Use of inverted poles and zeroes, to refer transfer function gains to the most important asymptote
- Analytical approximation of roots of high-order polynomials ✓
- Graphical construction of Bode plots of transfer functions and polynomials, to
 - avoid algebra mistakes
 - approximate transfer functions
 - obtain insight into origins of salient features

8.1 Review of Bode Plots

Magnitude
Phase

Decibels

$$\|G\|_{dB} = 20 \log_{10}(\|G\|)$$

Decibels of quantities having units (impedance example): normalize before taking log

$$\rightarrow \|Z\|_{dB} = 20 \log_{10}\left(\frac{\|Z\|}{R_{base}}\right)$$

$\log_{10}(s) = M$

5Ω is equivalent to 14dB with respect to a base impedance of $R_{base} = 1\Omega$, also known as 14dBΩ.

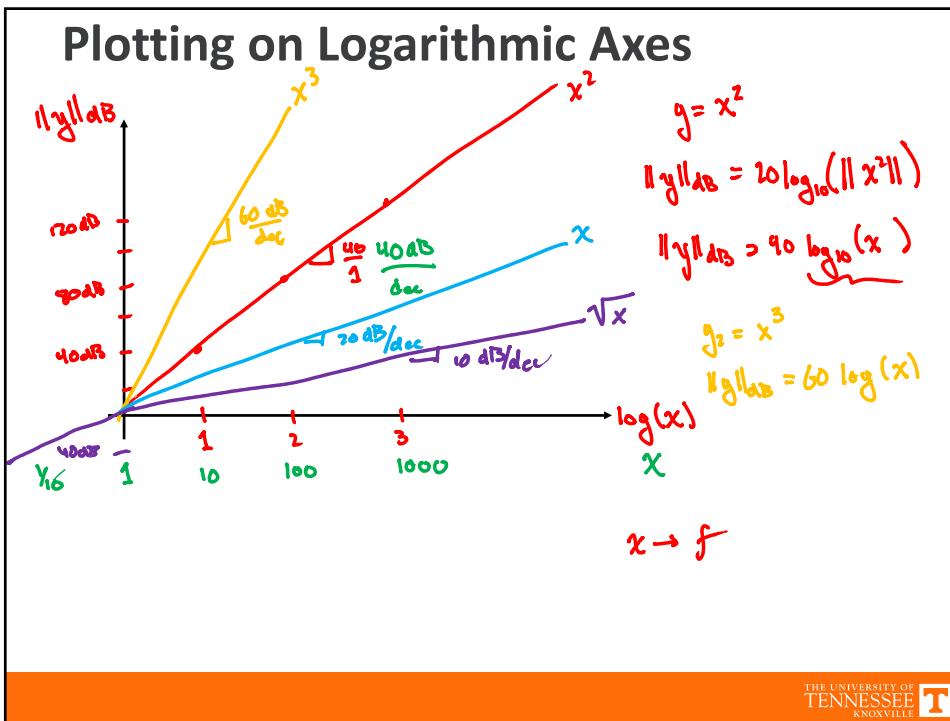
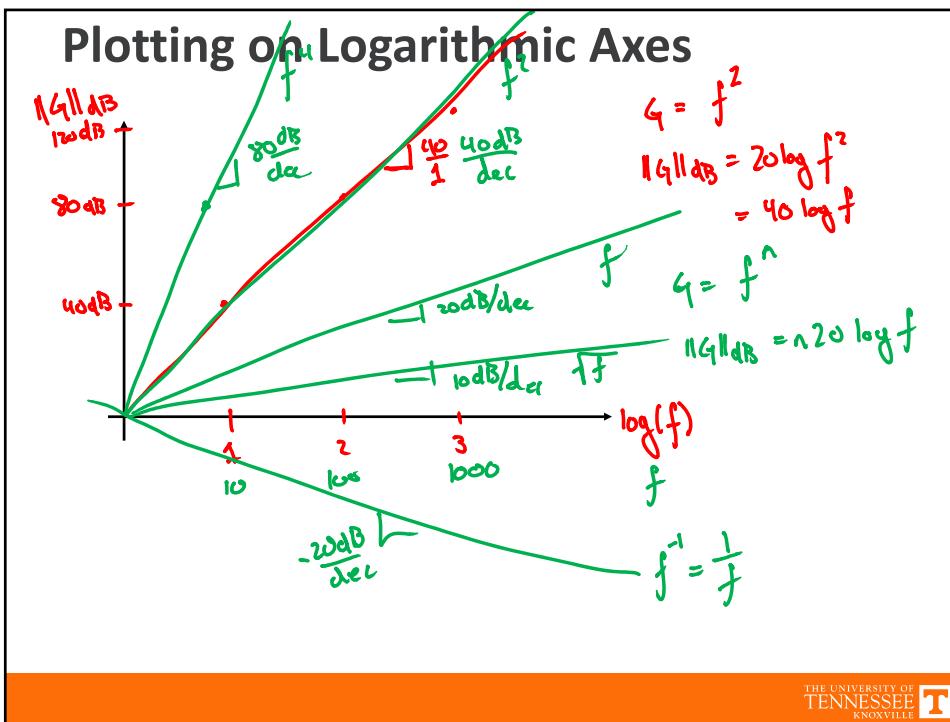
$\rightarrow 60dB\mu A$ is a current 60dB greater than a base current of $1\mu A$, or $1mA$.

$$1mA = 0dBmA = 60dBmA - 60dB$$

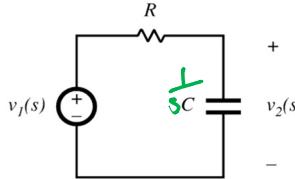
Logarithm Review

$$\log_a y = \frac{\log_a y}{\log_a x} \stackrel{eq.}{=} \frac{\ln(y)}{\ln(x)} \quad \log_{10} 5 = \frac{\ln(5)}{\ln(10)}$$

$$\left\{ \begin{array}{l} \log(x \cdot y) = \log(x) + \log(y) \\ \log(x/y) = \log(x) - \log(y) \\ \log(x^n) = n \log(x) \end{array} \right.$$



Single Pole Response



$$G(s) = \frac{v_2(s)}{v_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

Standard form : $\frac{A}{1 + \frac{s}{\omega_0}}$

$$G(s) = \frac{1}{1 + R s C} = \frac{1}{1 + \frac{s}{\omega_0}}$$

$$\omega_0 = \frac{1}{RC}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}$$

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Magnitude of Single Pole Response

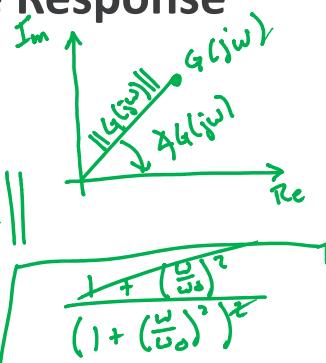
$$G(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

$$\|G(s=j\omega)\| = \left\| \frac{1}{1 + \frac{j\omega}{\omega_0}} \right\|$$

$$= \left\| \frac{1}{1 + \frac{j\omega}{\omega_0}} \cdot \frac{1 - \frac{j\omega}{\omega_0}}{1 - \frac{j\omega}{\omega_0}} \right\| = \left\| \frac{1 - \frac{j\omega}{\omega_0}}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \right\|$$

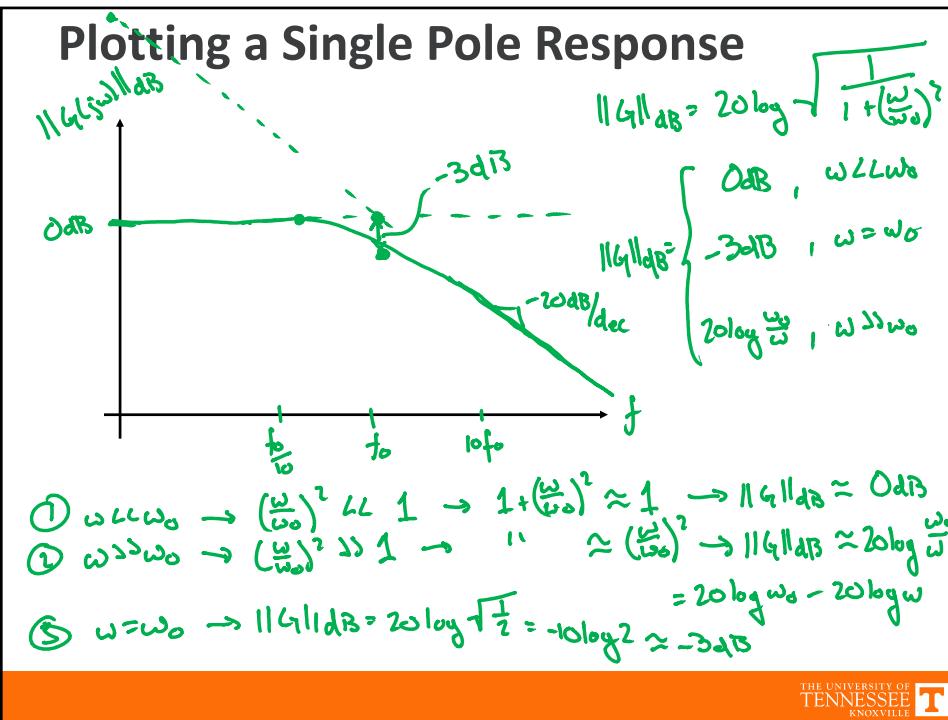
$$= \sqrt{\left[\frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \right]^2 + \left[\frac{-\frac{\omega}{\omega_0}}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \right]^2}$$

$$\|G(j\omega)\| = \sqrt{\frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

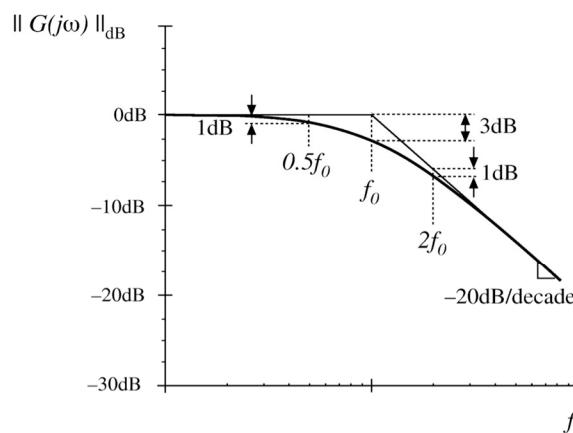


$$\|G(j\omega)\|_{dB} = 20 \log \|G(j\omega)\|$$

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Summary: Single Pole Magnitude



Phase of Single Pole

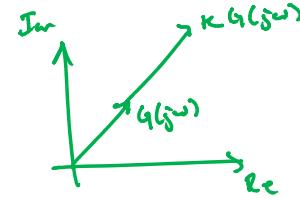
$$\Delta G(j\omega) = \tan^{-1} \left(\frac{\text{Im}(G)}{\text{Re}(G)} \right)$$

Phase operator identities

$$\textcircled{1} \quad \Delta k G(j\omega) = \Delta G(j\omega) \quad k = \text{const}$$

$$\textcircled{2} \quad \Delta G_1(j\omega) + G_2(j\omega) = \Delta G_1(j\omega) + \Delta G_2(j\omega)$$

$$\Delta G(j\omega) = \tan^{-1} \left(-\frac{\omega}{1} \right)$$



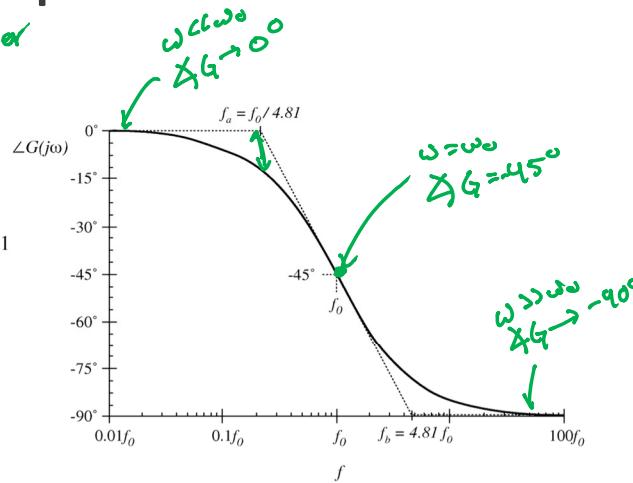
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Phase Asymptotes

Hard to remember

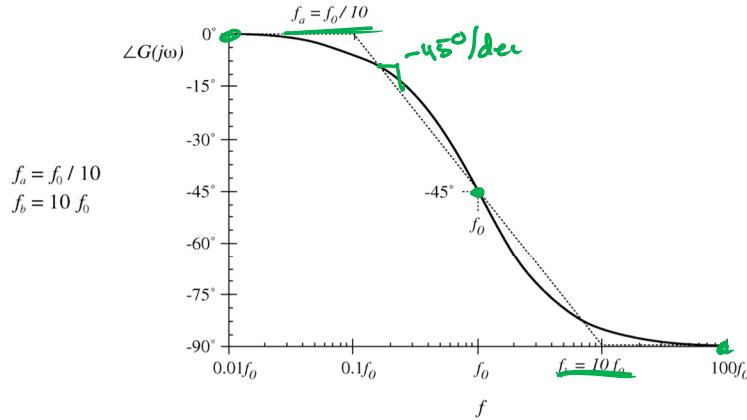
$$f_a = f_0 e^{-\pi/2} \approx f_0 / 4.81$$

$$f_b = f_0 e^{\pi/2} \approx 4.81 f_0$$



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Phase Asymptotes: A Simpler Choice



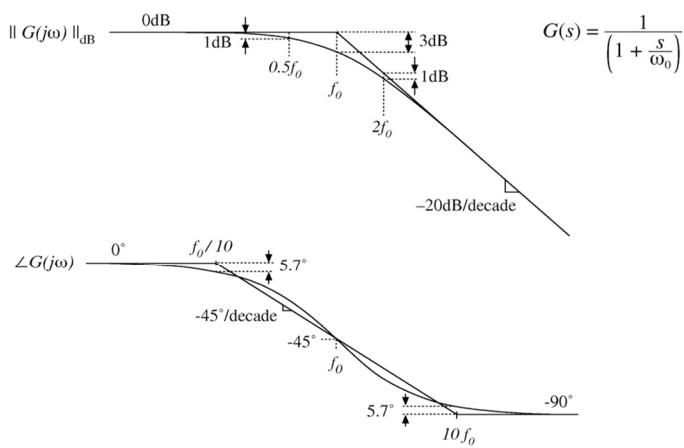
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Summary: Single Real Pole



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