

Single Pole Response

$$G(s) = \frac{v_2(s)}{v_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

Standard form : $\frac{A}{1 + \frac{s}{\omega_0}}$

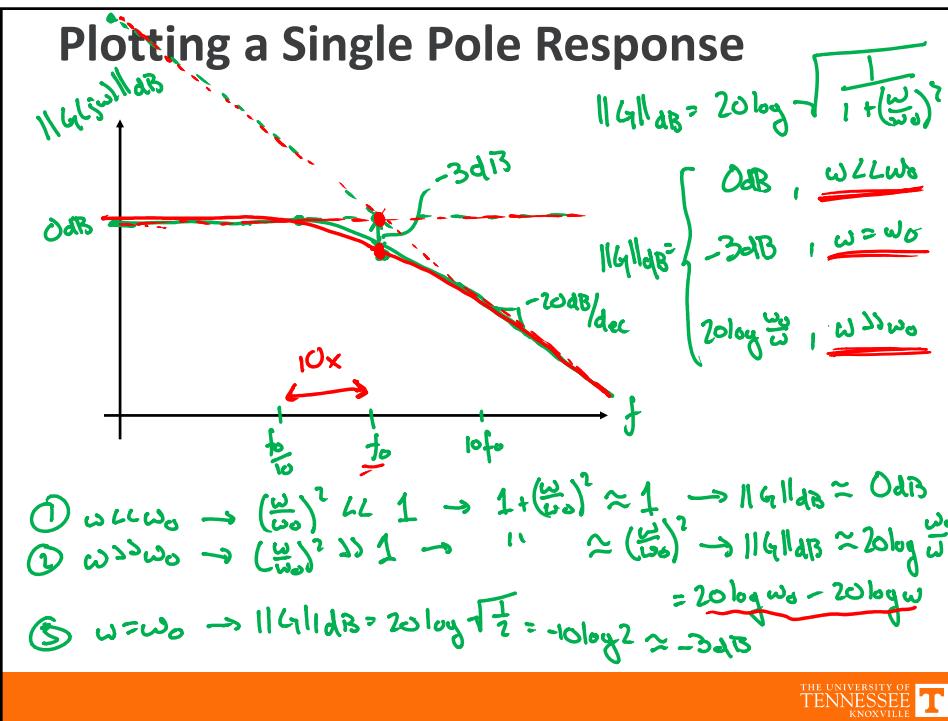
$$A=1$$

$$G(s) = \frac{1}{1 + R s C} = \frac{1}{1 + \frac{s}{\omega_0}}$$

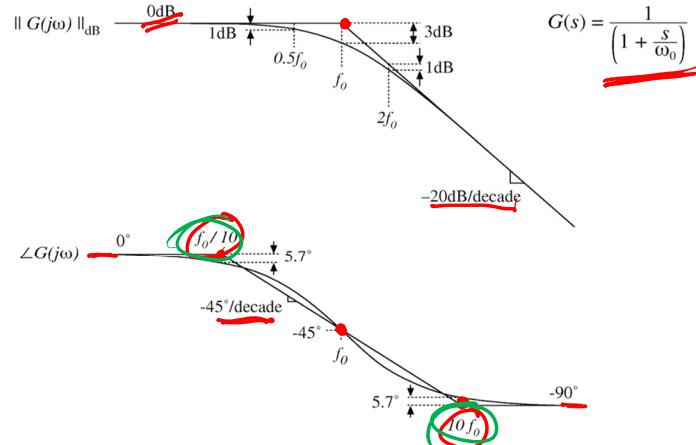
$$\omega_0 = \frac{1}{RC}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}$$

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Summary: Single Real Pole



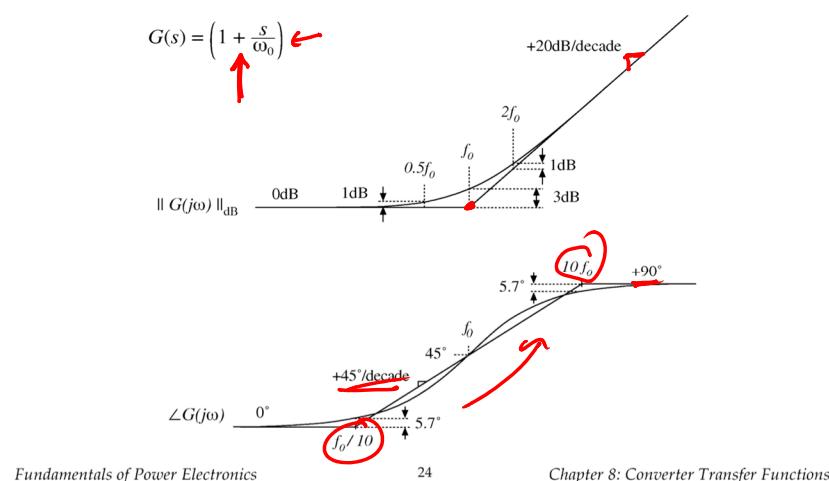
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Bode Plot: Real Zero



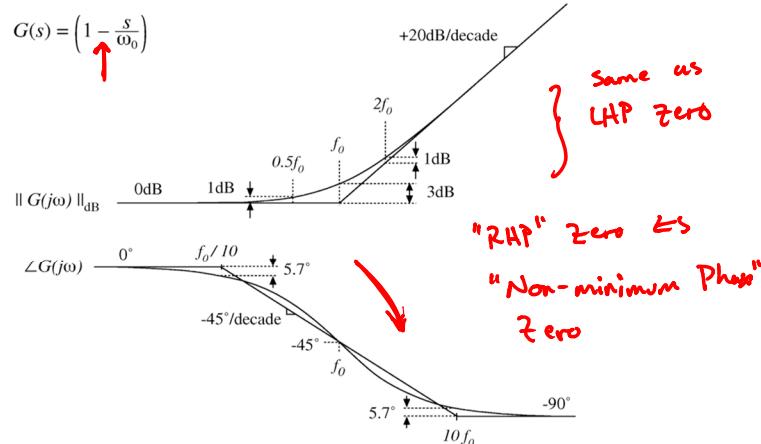
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RHP Zero



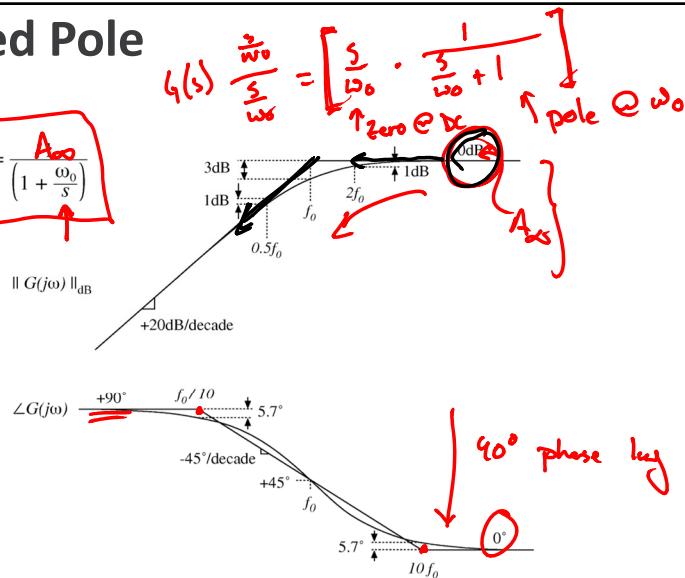
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Inverted Pole



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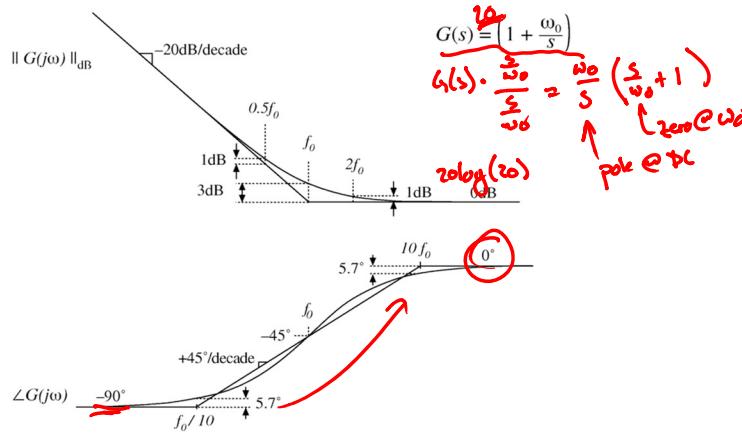
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Inverted Zero

$$A_p = 0 \text{dB} = 1$$



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Multiplying Transfer Functions

$$\begin{aligned}
 G_1(s) &= A_1 e^{s\theta_1} \\
 G_2(s) &= A_2 e^{s\theta_2} \\
 G_1(s) \cdot G_2(s) &= A_1 e^{s\theta_1} \cdot A_2 e^{s\theta_2} \\
 &= A_1 A_2 e^{s(\theta_1 + \theta_2)}
 \end{aligned}$$

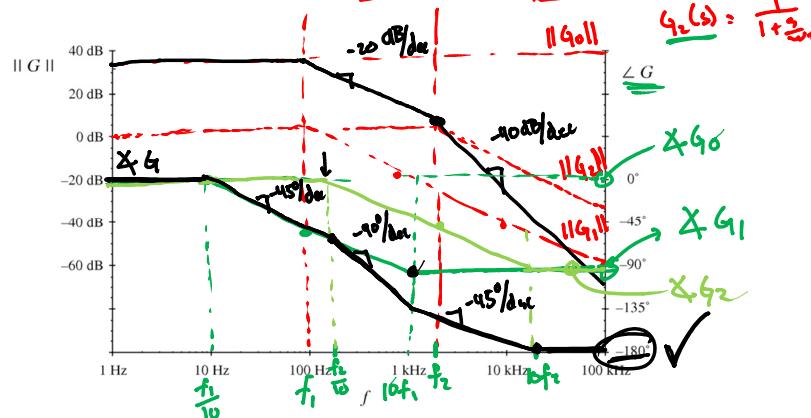
- Multiply two TFs
 - Phases will add
 - Magnitudes will multiply
(Add in log domain)

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Example 1

$$\underline{G(s)} = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} = G_0 \cdot \left(\frac{1}{1 + \frac{s}{\omega_1}}\right) \cdot \left(\frac{1}{1 + \frac{s}{\omega_2}}\right)$$

with $G_0 = 40 \Rightarrow 32 \text{ dB}$, $f_1 = \omega_1/2\pi = 100 \text{ Hz}$, $f_2 = \omega_2/2\pi = 2 \text{ kHz}$



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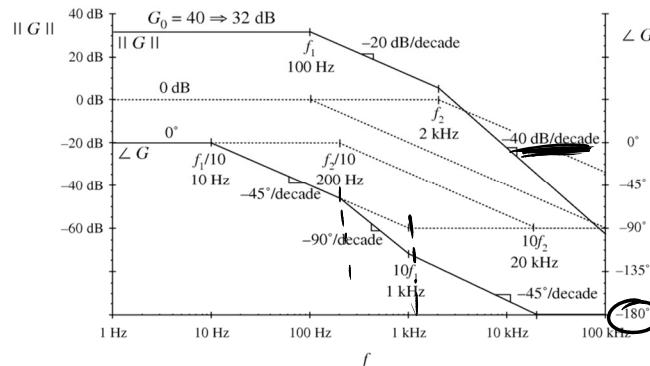
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Example 1

$$\underline{G(s)} = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

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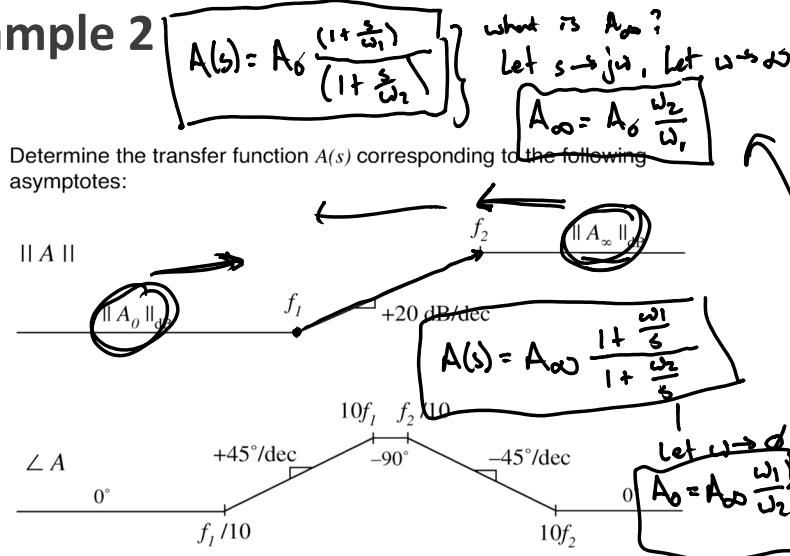


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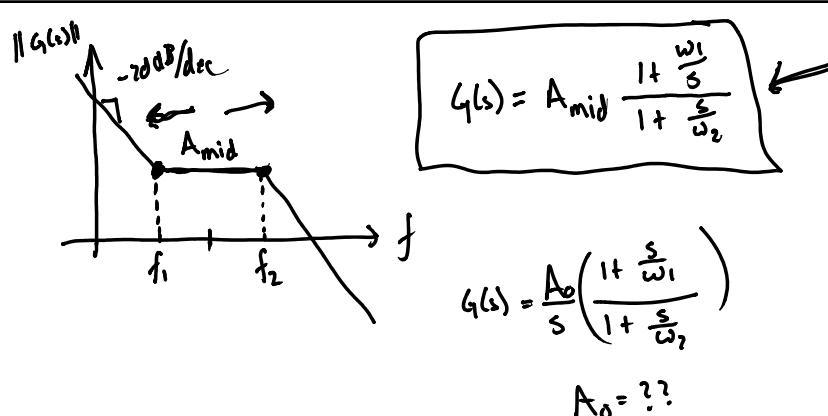


Example 2

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8.1.6 Resonant Poles

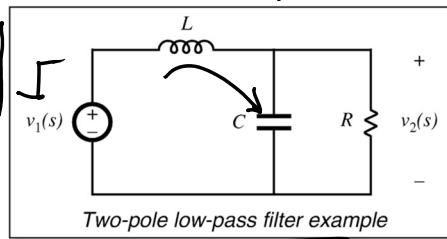
Example

$$G(s) = \frac{v_o(s)}{v_i(s)} = \frac{1}{1 + s\frac{R}{Q\omega_0} + s^2LC}$$

Second-order denominator, of the form

$$G(s) = \frac{1}{1 + a_1s + a_2s^2}$$

with $a_1 = L/R$ and $a_2 = LC$



Apply quadratic formula

How should we construct the Bode diagram?

Standard Form for Complex Poles

$$G(s) = \frac{1}{1 + 2\xi \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

or

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

- When the coefficients of s are real and positive, then the parameters ξ , ω_0 , and Q are also real and positive
- The parameters ξ , ω_0 , and Q are found by equating the coefficients of s
- The parameter ω_0 is the angular corner frequency, and we can define $f_0 = \omega_0/2\pi$
- The parameter ξ is called the damping factor. ξ controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $\xi < 1$.
- In the alternative form, the parameter Q is called the quality factor. Q also controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $Q > 0.5$.

The Q Factor

In a second-order system, ζ and Q are related according to

$$Q = \frac{1}{2\zeta}$$

Q is a measure of the dissipation in the system. A more general definition of Q , for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that Q has a simple interpretation in the Bode diagrams of second-order transfer functions.

Magnitude Asymptotes

In the form $G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$

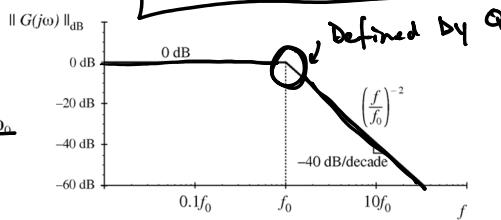
let $s = j\omega$ and find magnitude:

$$\|G(j\omega)\| = \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}$$

Asymptotes are

$$\|G\| \rightarrow 1 \quad \text{for } \omega \ll \omega_0$$

$$\|G\| \rightarrow \left(\frac{f}{f_0}\right)^{-2} \quad \text{for } \omega \gg \omega_0$$



Exact Magnitude Curve

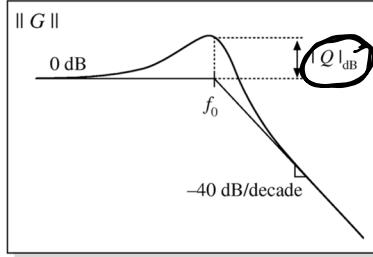
for $Q \gg \frac{1}{2}$

$$\|G(j\omega)\| = \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}$$

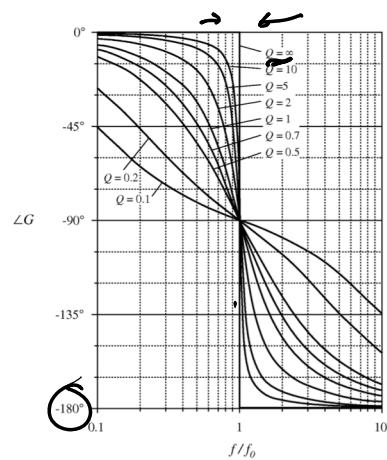
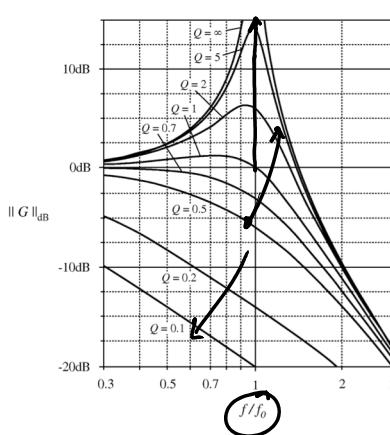
At $\omega = \omega_0$, the exact magnitude is

$$\|G(j\omega_0)\| = Q \quad \text{or, in dB:} \quad \|G(j\omega_0)\|_{\text{dB}} = |Q|_{\text{dB}}$$

The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $|Q|_{\text{dB}}$

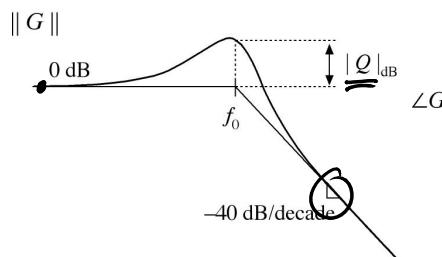


Curves for Varying Q

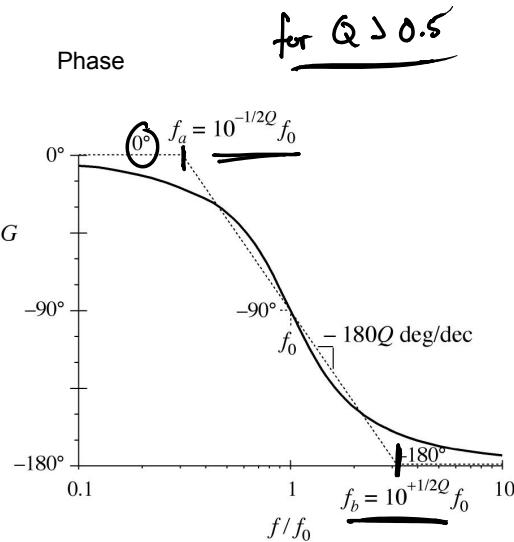


Summary: Asymptotes for Complex Poles

Magnitude

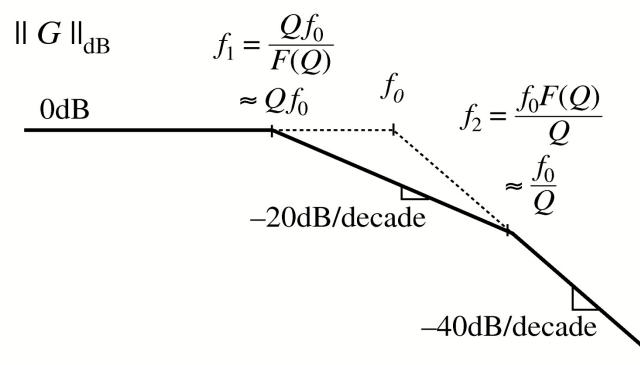


Phase



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The Low-Q Approximation



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