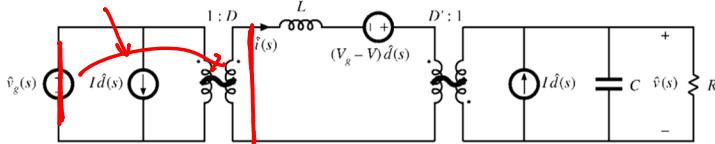
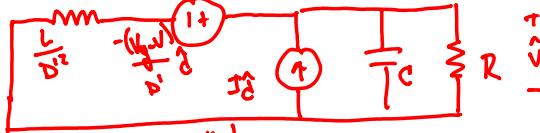


Control-to-Output TF



$$G_{vd} = \frac{\hat{v}}{\delta} \Big|_{v_g = 0}$$



Apply Superposition:

$$\hat{v} = \left(\frac{sL}{D^2} \parallel \frac{1}{sC} \parallel R \right) I_d + \left(\frac{R \parallel \frac{1}{sC}}{R \parallel \frac{1}{sC} + \frac{sL}{D^2}} \right) \frac{V - V_g}{D'} \hat{d}$$

$$\begin{aligned} G_{vd} &= \frac{\hat{v}}{\delta} = \left(\frac{sL}{D^2} \parallel \frac{R}{1+sCR} \right) I_d + \left(\frac{\frac{R}{1+sCR}}{\frac{R}{1+sCR} + \frac{sL}{D^2}} \right) \frac{V - V_g}{D'} \\ &= \left(\frac{\frac{sL}{D^2}}{1 + sL/D^2(1+sCR)} \right) I_d + \left(\frac{1}{1 + \frac{(1+sCR)}{R} \left(\frac{sL}{D^2} \right)} \right) \frac{V - V_g}{D'} \end{aligned}$$

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$$G_{vd} = \frac{\frac{sL}{D^2} I_d + \frac{V - V_g}{D'}}{1 + s \frac{L}{RD^2} + s^2 \frac{LC}{D^2}}$$

$$= \frac{V - V_g}{D'} \frac{\left(\frac{sL}{D'(V - V_g)} + 1 \right)}{1 + s \frac{L}{RD^2} + s^2 \frac{LC}{D^2}}$$

$$= G_{dc} \frac{\left(1 + \frac{s}{\omega_0} \right)}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

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Numerical Example

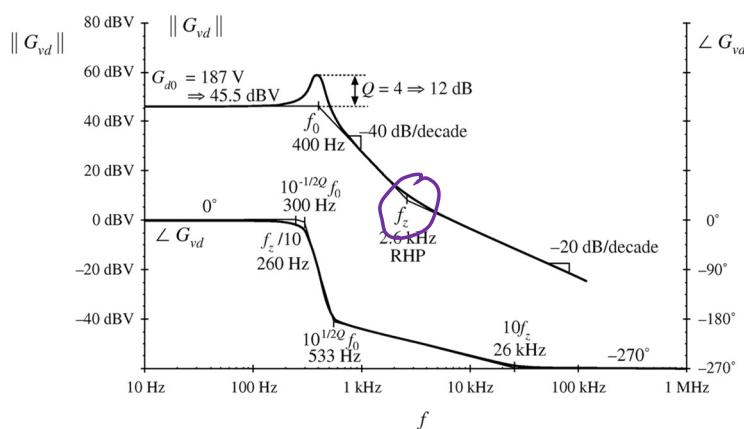
Suppose we are given the following numerical values:

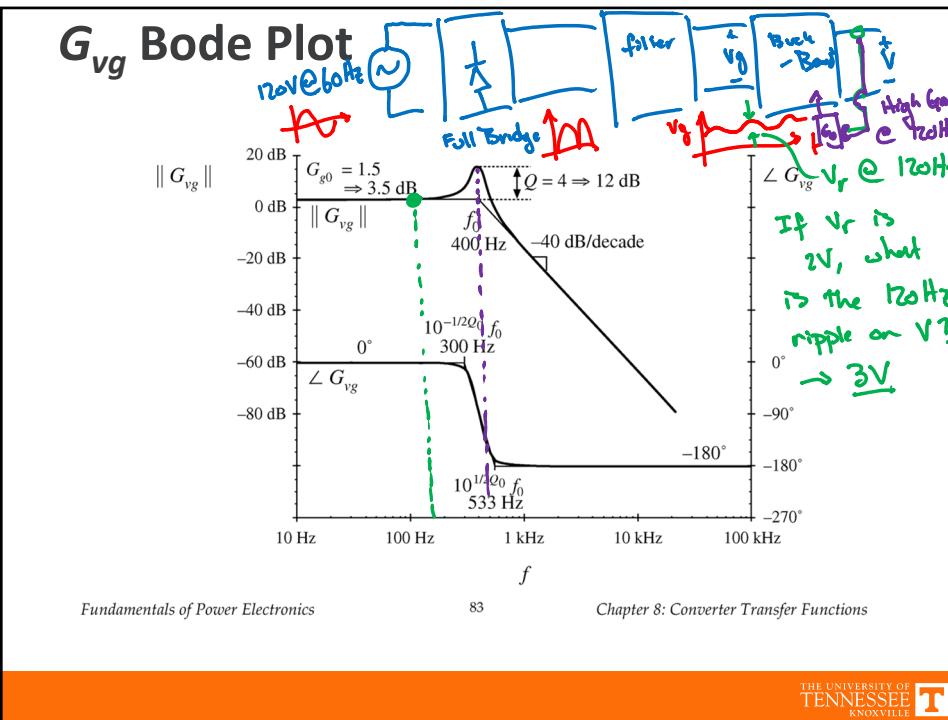
$$\left\{ \begin{array}{l} D = 0.6 \\ R = 10\Omega \\ V_g = 30V \\ L = 160\mu H \\ C = 160\mu F \end{array} \right.$$

Then the salient features have the following numerical values:

$$\begin{aligned} \rightarrow |G_{g0}| &= \frac{D}{D'} = 1.5 \Rightarrow 3.5 \text{ dB} \\ \rightarrow |G_{d0}| &= \frac{|V|}{DD'} = 187.5 \text{ V} \Rightarrow 45.5 \text{ dBV} \\ f_0 &= \frac{\omega_0}{2\pi} = \frac{D}{2\pi\sqrt{LC}} = 400 \text{ Hz} \\ Q &= D'R\sqrt{\frac{C}{L}} = 4 \Rightarrow 12 \text{ dB} \\ f_z &= \frac{\omega_z}{2\pi} = \frac{D^2 R}{2\pi D L} = 2.65 \text{ kHz} \end{aligned}$$

G_{vd} Bode Plot



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TFs of Some Basic Converters

Table 8.2. Salient features of the small-signal CCM transfer functions of some basic dc-dc converters

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\frac{V}{D}$	$\frac{1}{fLC}$	$R \sqrt{\frac{C}{L}}$	∞
boost	$\frac{1}{D}$	$\frac{V}{D}$	$\frac{D'}{fLC}$	$D'R \sqrt{\frac{C}{L}}$	$\frac{D'^2 R}{L}$
buck-boost	$-\frac{D}{D'}$	$\frac{V}{D D'}$	$\frac{D}{fLC}$	$D'R \sqrt{\frac{C}{L}}$	$\frac{D'^2 R}{D L}$

where the transfer functions are written in the standard forms

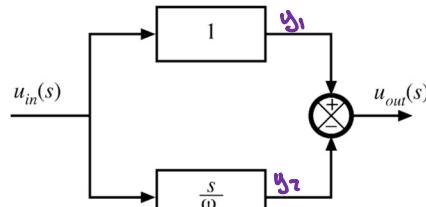
$$G_{vd}(s) = G_{d0} \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

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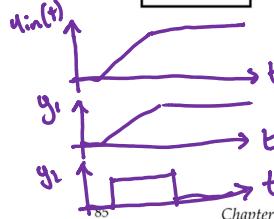
Origins of RHP Zero

$$\rightarrow G(s) = \left(1 - \frac{s}{\omega_z}\right)$$



- phase reversal at high frequency
- transient response: output initially tends in wrong direction

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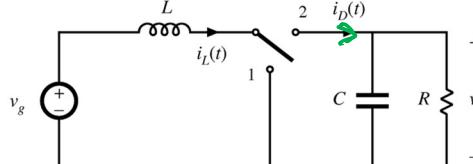
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CCM RHP Zero

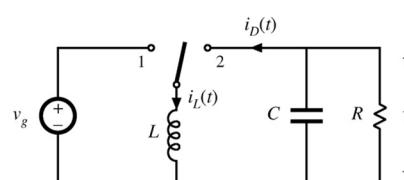
$d \uparrow$
 $d' \downarrow$

$$\langle i_D \rangle_{T_s} = d' \langle i_L \rangle_{T_s}$$

Boost



Buck-boost



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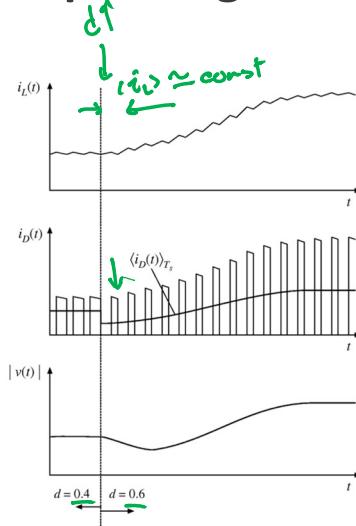
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Waveforms of Step Change in d

$$\langle i_D \rangle_{T_s} = d^r \langle i_L \rangle_{T_s}$$

- Increasing $d(t)$ causes the average diode current to initially decrease
- As inductor current increases to its new equilibrium value, average diode current eventually increases



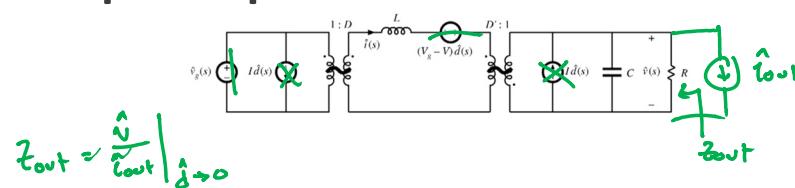
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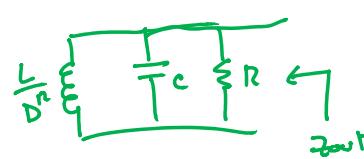
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Output Impedance



$$Z_{out} \approx \frac{\hat{v}_{out}}{\hat{i}_{out}} \Big|_{d \rightarrow 0}$$



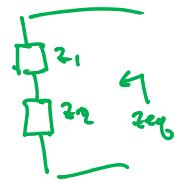
$$Z_{out} = \left(\frac{L}{D^2} \parallel \frac{1}{C} \parallel R \right)$$

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Graphical Construction of Bode Plots

Two cases:

① Series Impedances

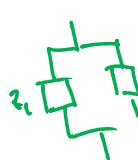


$$Z_{eq} = Z_1 + Z_2$$

$$\|Z_{eq}\|_{dB} = \|Z_1 + Z_2\| =$$

$$\begin{cases} \|Z_1\|, \|Z_1\| \gg \|Z_2\| \\ \|Z_2\|, \|Z_2\| \gg \|Z_1\| \end{cases}$$

② Parallel



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \Rightarrow \|Z_{eq}\| =$$

$$\begin{cases} \|Z_1\|, \|Z_1\| \gg \|Z_2\| \\ \|Z_2\|, \|Z_2\| \gg \|Z_1\| \end{cases}$$