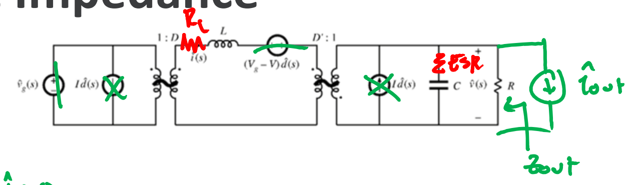
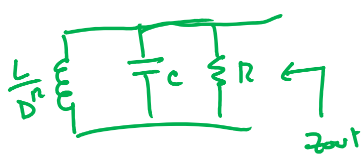
Announcements

- Remote lecture Monday and Wednesday next week

Output Impedance



$$Z_{out} = \frac{\hat{v}}{\hat{i}_{out}}\Bigg|_{\substack{\hat{d} \to 0 \\ \hat{v}_g \to 0}}$$

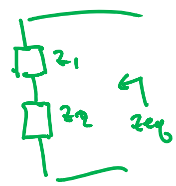


$$Z_{out} = \left(\frac{L}{D'^2} \,\Big\|\, \frac{1}{sC} \,\Big\|\, R\right)$$

Graphical Construction of Bode Plots

Two Cases:

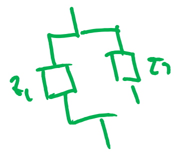
① Series Impedances



$Z_{eq} = Z_1 + Z_2$

$\|Z_{eq}\|_{dB} = \|Z_1 + Z_2\| = \begin{cases} \|Z_1\|, & \|Z_1\| >> \|Z_2\| \\ \|Z_2\|, & \|Z_2\| >> \|Z_1\| \end{cases}$

② Parallel



$Z_{eq} = \dfrac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \Rightarrow \|Z_{eq}\| = \begin{cases} \|Z_2\|, & \|Z_1\| >> \|Z_2\| \\ \|Z_1\|, & \|Z_2\| >> \|Z_1\| \end{cases}$

Graphical Construction

If $\|Z_1\| \simeq \|Z_2\|$ if one Real & one Imag

→ 3 dB of asymptotes

if both Imaginary:

$\|j\omega L\| \approx \|\frac{-j}{\omega C}\|$

$\omega = \frac{1}{\sqrt{LC}}$

$\|j \frac{1}{\sqrt{LC}} L\| = \|\frac{-j}{\frac{1}{\sqrt{LC}} C}\|$

$\|j\sqrt{\frac{L}{C}}\| = \|-j\sqrt{\frac{L}{C}}\|$

$R_0 = \sqrt{\frac{L}{C}}$

$L = 160\,\mu H$

$D = 0.6$

$R = 10\,\Omega$

$C = 160\,\mu F$

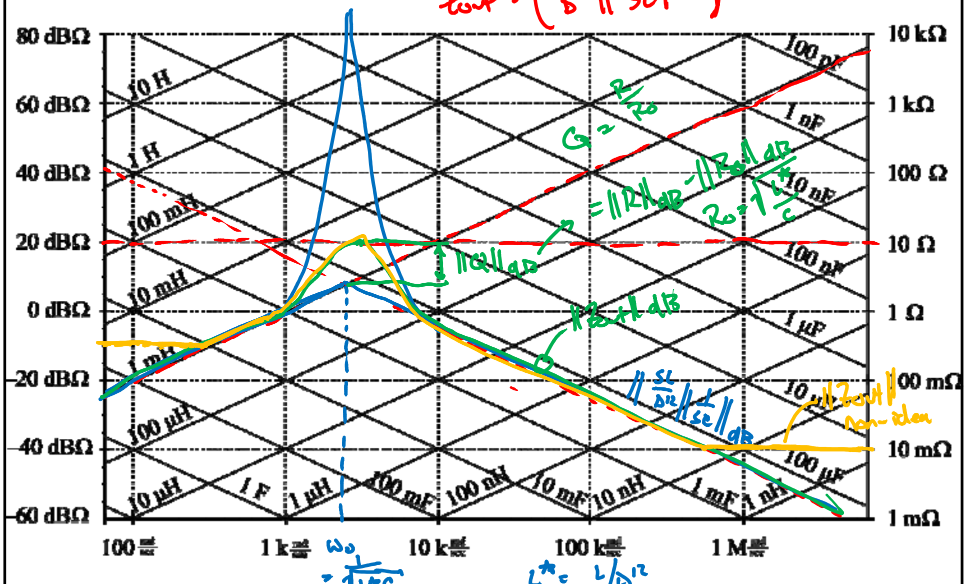
$\frac{L}{D'^2} = 1\,mH$

$R_L = 316\,m\Omega = -10\,dB\Omega$

$ESR = 10\,m\Omega = -40\,dB\Omega$

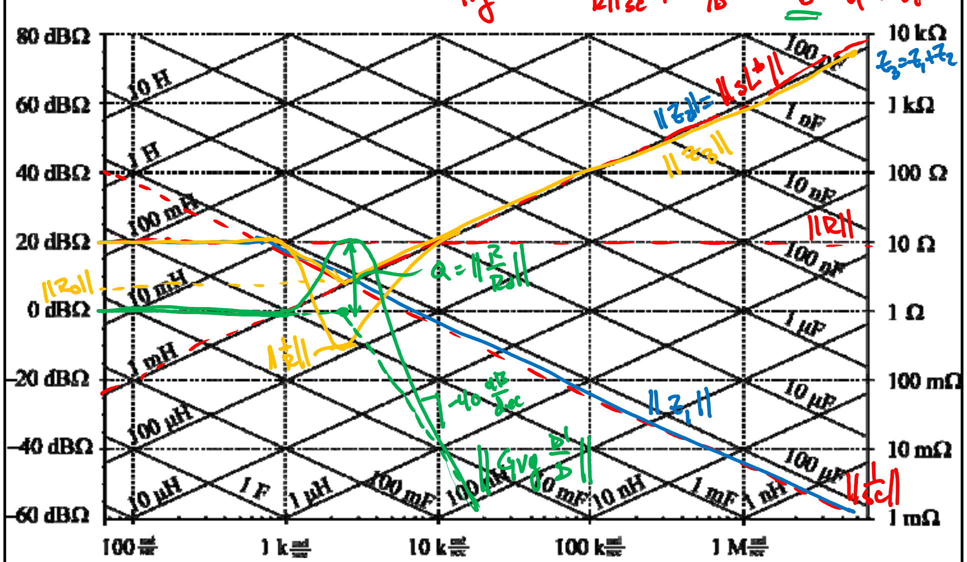
Reactance Paper

$$Z_{out} = \left(\frac{sL}{D'^2} \,\|\, \frac{1}{sC} \,\|\, R\right)$$



Reactance Paper

$$G_{vg} = -\frac{D}{D'} \frac{R \| \frac{1}{sC}}{R \| \frac{1}{sC} + sL/D'^2} = -\frac{D}{D'} \frac{Z_1}{Z_1 + Z_2}$$



Chapter 8 Summary

1. The magnitude Bode diagrams of functions which vary as $(f / f_0)n$ have slopes equal to $20n$ dB per decade, and pass through 0dB at $f = f_0$.
2. It is good practice to express transfer functions in normalized pole-zero form; this form directly exposes expressions for the salient features of the response, i.e., the corner frequencies, reference gain, etc.
3. The right half-plane zero exhibits the magnitude response of the left half-plane zero, but the phase response of the pole.
4. Poles and zeroes can be expressed in frequency-inverted form, when it is desirable to refer the gain to a high-frequency asymptote.

Chapter 8 Summary

5. A two-pole response can be written in the standard normalized form of Eq. (8-53). When $Q > 0.5$, the poles are complex conjugates. The magnitude response then exhibits peaking in the vicinity of the corner frequency, with an exact value of Q at $f = f_0$. High Q also causes the phase to change sharply near the corner frequency.
6. When the Q is less than 0.5, the two pole response can be plotted as two real poles. The low- Q approximation predicts that the two poles occur at frequencies f_0 / Q and Qf_0. These frequencies are within 10% of the exact values for $Q \leq 0.3$.
7. The low- Q approximation can be extended to find approximate roots of an arbitrary degree polynomial. Approximate analytical expressions for the salient features can be derived. Numerical values are used to justify the approximations.

Chapter 8 Summary

8. Salient features of the transfer functions of the buck, boost, and buck-boost converters are tabulated in section 8.2.2. The line-to-output transfer functions of these converters contain two poles. Their control-to-output transfer functions contain two poles, and may additionally contain a right half-pland zero.

9. Approximate magnitude asymptotes of impedances and transfer functions can be easily derived by graphical construction. This approach is a useful supplement to conventional analysis, because it yields physical insight into the circuit behavior, and because it exposes suitable approximations. Several examples, including the impedances of basic series and parallel resonant circuits and the transfer function $H_e(s)$ of the boost and buck-boost converters, are worked in section 8.3.

Chapter 9: Controller Design

9.1. Introduction

9.2. Effect of negative feedback on the network transfer functions

 9.2.1. Feedback reduces the transfer function from disturbances to the output

 9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

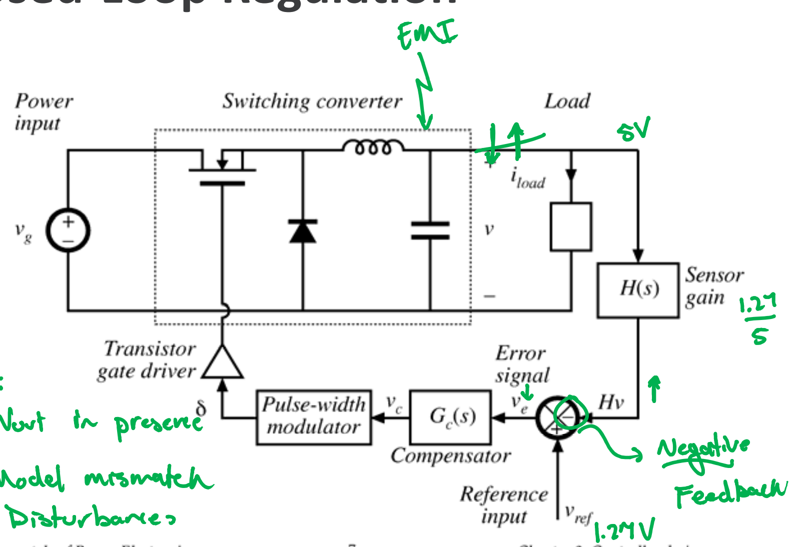
9.3. Construction of the important quantities $1/(1+T)$ and $T/(1+T)$ and the closed-loop transfer functions

Chapter 9: Controller Design

9.4. Stability
- 9.4.1. The phase margin test
- 9.4.2. The relation between phase margin and closed-loop damping factor
- 9.4.3. Transient response vs. damping factor

9.5. Regulator design
- 9.5.1. Lead (PD) compensator
- 9.5.2. Lag (PI) compensator
- 9.5.3. Combined (PID) compensator
- 9.5.4. Design example

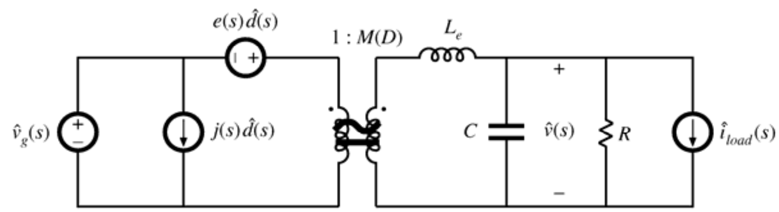
Closed-Loop Regulation



Purpose:
Regulat Vout in presence of
(1) Model mismatch
(2) Disturbances

Open-Loop Behavior

Small signal model: open-loop converter



Output voltage can be expressed as

$$\hat{v}(s) = G_{vd}(s)\,\hat{d}(s) + G_{vg}(s)\,\hat{v}_g(s) - Z_{out}(s)\,\hat{i}_{load}(s)$$ ← by superposition

where

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\substack{\hat{v}_g = 0 \\ \hat{i}_{load} = 0}} \qquad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{i}_{load} = 0}} \qquad Z_{out}(s) = -\left.\frac{\hat{v}(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d} = 0 \\ \hat{v}_g = 0}}$$

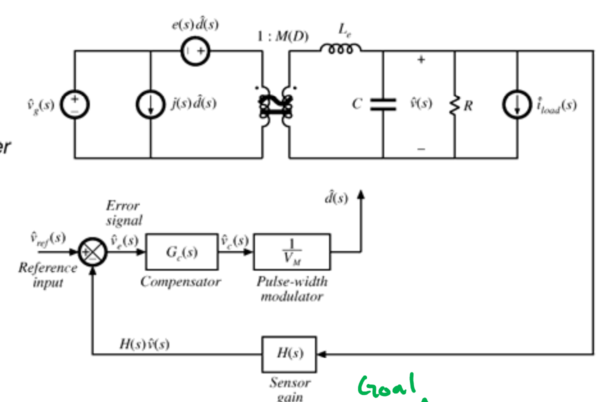
Small-Signal Closed-Loop Model

- *Use small-signal converter model*
- *Perturb and linearize remainder of feedback loop:*

$v_{ref}(t) = V_{ref} + \hat{v}_{ref}(t)$

$v_e(t) = V_e + \hat{v}_e(t)$

etc.

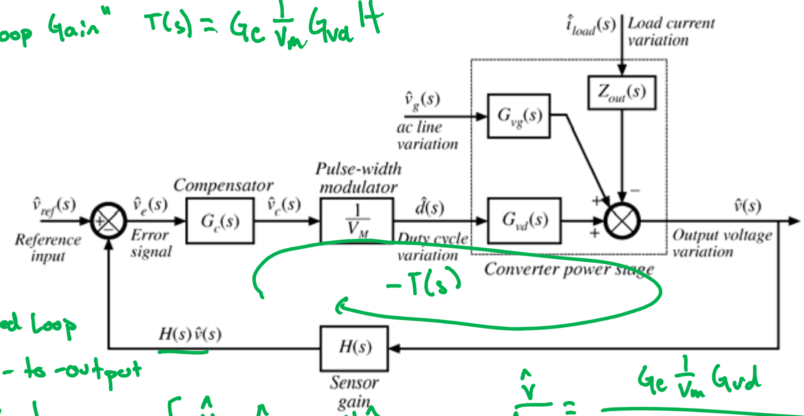


Goal
$\hat{v} H = \hat{v}_{ref}$

$\rightarrow \hat{v} = \frac{1}{H}\hat{v}_{ref}$

Block Diagram

"Loop Gain" $T(s) = G_c \frac{1}{V_m} G_{vd} H$



Closed Loop

Ref-to-output

$$\frac{\hat{v}}{\hat{v}_{ref}}\Bigg|_{\substack{\hat{v}_g \to 0 \\ \hat{i}_{load} \to 0}}$$

$$\left\{\begin{aligned} &\hat{v}_e = \hat{v}_{ref} - H\hat{v} \\ &\hat{v} = \hat{v}_e G_c \frac{1}{V_m} G_{vd} \\ &\hat{v} = (\hat{v}_{ref} - H\hat{v}) G_c \frac{1}{V_m} G_{vd} \end{aligned}\right.$$

$$\frac{\hat{v}}{\hat{v}_{ref}} = \frac{G_c \frac{1}{V_m} G_{vd}}{1 + H G_c \frac{1}{V_m} G_{vd}}$$

$$\frac{\hat{v}}{\hat{v}_{ref}} = \underbrace{\frac{1}{H}}_{\text{Ideal}} \frac{T}{1+T} \quad \Big\} \text{ want } T \to \infty$$

Closed-Loop Transfer Functions

Disturbance-to-output

$$\frac{\hat{v}}{\hat{i}_{out}} \longrightarrow \hat{v} = -Z_{out}\hat{i}_{load} + \hat{v} H G_c \frac{1}{V} G_{vd}$$

$$\frac{\hat{v}}{\hat{i}_{out}} = -\underset{\substack{\uparrow \\ \text{open-loop}}}{Z_{out}} \frac{1}{1+T} \longrightarrow$$

if $T \to \infty$ $\hat{i}_{out}$ has no effect on $\hat{v}$

$$\frac{\hat{v}}{\hat{v}_g} = G_{vg} \frac{1}{1+T} \longrightarrow \underline{\text{same}}$$

Why not $T(s) \to \infty$? <u>Stability</u>