Solution of Block Diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref}\frac{G_cG_{vd}/V_M}{1+HG_cG_{vd}/V_M} + \hat{v}_g\frac{G_{vg}}{1+HG_cG_{vd}/V_M} - \hat{i}_{load}\frac{Z_{out}}{1+HG_cG_{vd}/V_M}$$

which is of the form
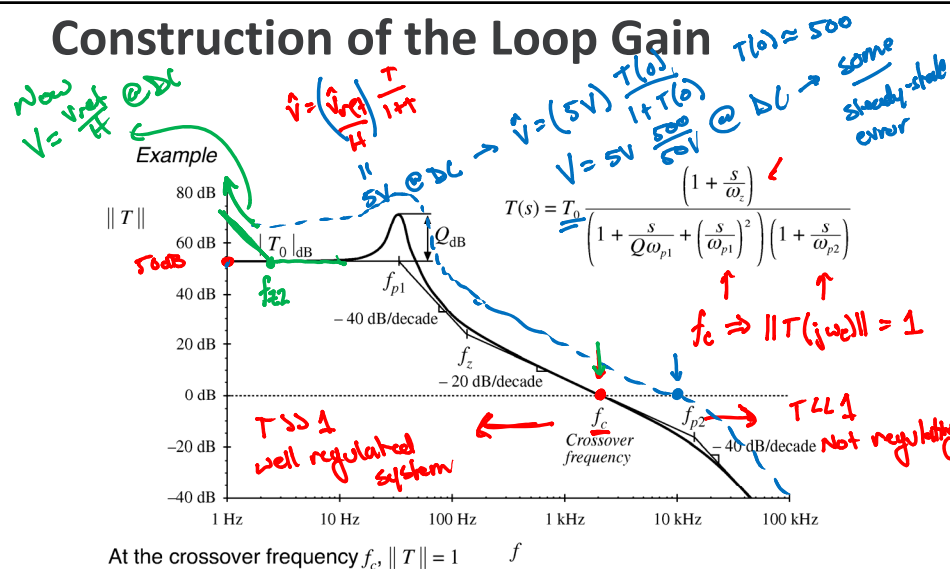
$$\hat{v} = \hat{v}_{ref}\frac{1}{H}\frac{T}{1+T} + \hat{v}_g\frac{G_{vg}}{1+T} - \hat{i}_{load}\frac{Z_{out}}{1+T}$$

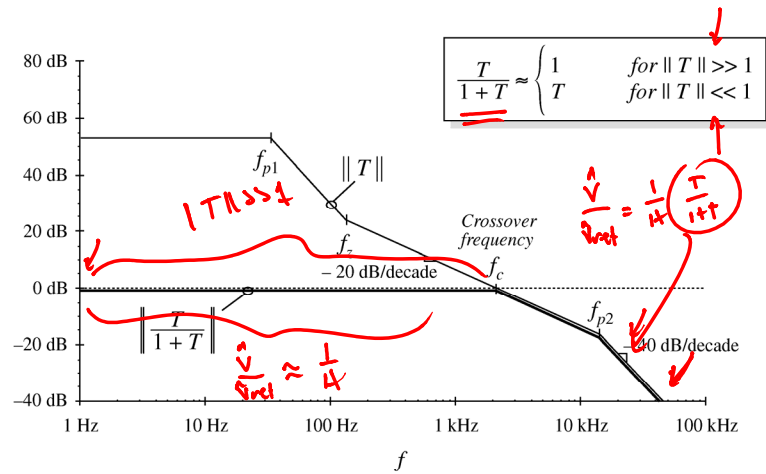*if* $T \to \infty$
$\hat{v} \approx \hat{v}_{ref}\frac{1}{H}$

with $T(s) = H(s)\,G_c(s)\,G_{vd}(s)/V_M$ = "*loop gain*"

Loop gain $T(s)$ = products of the gains around the negative feedback loop.
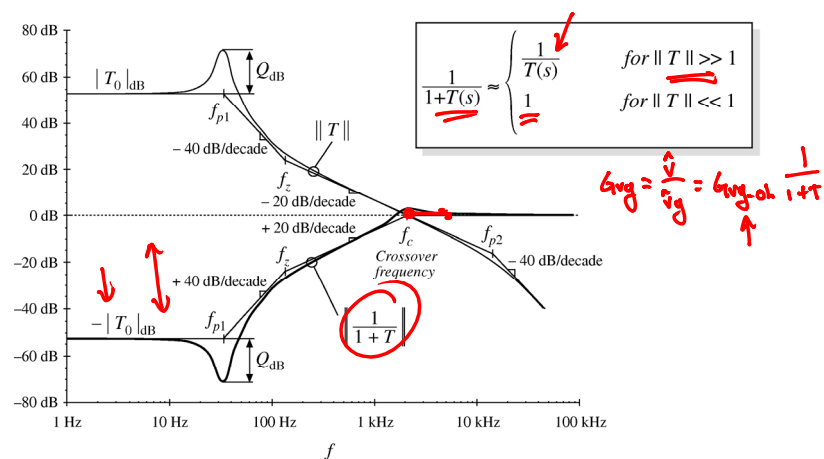
Construction of the Loop Gain

Now $V = \hat{v}_{ref}\frac{1}{H}$ @ DC

$\hat{v} = \left(\frac{\hat{v}_{ref}}{H}\right)\frac{T}{1+T}$

5V @ DC → $\hat{v} = (5V)\frac{T(o)}{1+T(o)}$

$T(o) \approx 500$

$V = 5V\frac{500}{501V}$ @ DC → some steady-state error

$T(o) \approx 500$

$$T(s) = T_0\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{Q\omega_{p1}}+\left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$f_c \Rightarrow \|T(j\omega_c)\| = 1$

Example

$\|T\|$

$500B$

f_{z1}

$T_0|_{dB}$

Q_{dB}

f_{p1}

– 40 dB/decade

f_z

– 20 dB/decade

$T \gg 1$ well regulated system

f_c *Crossover frequency*

f_{p2}

$T \ll 1$ Not regulated

– 40 dB/decade

80 dB, 60 dB, 40 dB, 20 dB, 0 dB, –20 dB, –40 dB

1 Hz   10 Hz   100 Hz   1 kHz   10 kHz   100 kHz

At the crossover frequency f_c, $\|T\| = 1$ f

Effect of Loop Gain

$$\frac{T}{1+T} \approx \begin{cases} 1 & for \; \| T \| \gg 1 \\ T & for \; \| T \| \ll 1 \end{cases}$$

80 dB
60 dB
f_{p1}
40 dB
$\| T \|$
20 dB
f_z
Crossover frequency
− 20 dB/decade
f_c
0 dB
$\left\| \dfrac{T}{1+T} \right\|$
−20 dB
f_{p2}
−40 dB/decade
−40 dB

1 Hz   10 Hz   100 Hz   1 kHz   10 kHz   100 kHz

f

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Rejection of Disturbances

$$\frac{1}{1+T(s)} \approx \begin{cases} \dfrac{1}{T(s)} & for \; \| T \| \gg 1 \\ 1 & for \; \| T \| \ll 1 \end{cases}$$

80 dB
$| T_0 |_{dB}$ Q_{dB}
60 dB
f_{p1}
40 dB
− 40 dB/decade
$\| T \|$
20 dB
f_z
− 20 dB/decade
0 dB
+ 20 dB/decade
f_c f_{p2}
f_z
Crossover frequency
−20 dB
+ 40 dB/decade
− 40 dB/decade
−40 dB
$-| T_0 |_{dB}$ f_{p1}
$\dfrac{1}{1+T}$
−60 dB
Q_{dB}
−80 dB

1 Hz   10 Hz   100 Hz   1 kHz   10 kHz   100 kHz

f

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Open-Loop and Closed-Loop

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$$G_{vd}(s) \qquad G_{vg}(s) \qquad Z_{out}(s)$$

Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \qquad \frac{G_{vg}(s)}{1 + T(s)} \qquad \frac{Z_{out}(s)}{1 + T(s)}$$

The loop gain:

$$T(s)$$

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Effect of Feedback

Feedback moves the poles of the system transfer functions

- Good news: we can use feedback to alter the poles and improve the frequency response
- Bad news: if you're not careful, feedback can move the poles into the right half of the complex s-plane (poles have positive real parts), leading to an unstable system

Open loop

Closed loop

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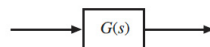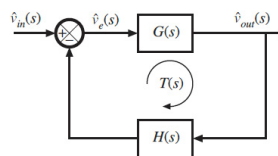
Determining Stability From T(s)

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

Phase margin test is valid if:

(1) $f_c \Rightarrow \| T(j2\pi f_c) \| = 1$ is uniquely defined.

(2) $T(s)$ has no RHP poles

9.4.1 – The Phase Margin Test

"Phase Margin" $= \varphi_m = 180° + \angle T(j\omega_c)$

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

The phase margin φ_m is determined from the phase of $T(s)$ at f_c, as follows:
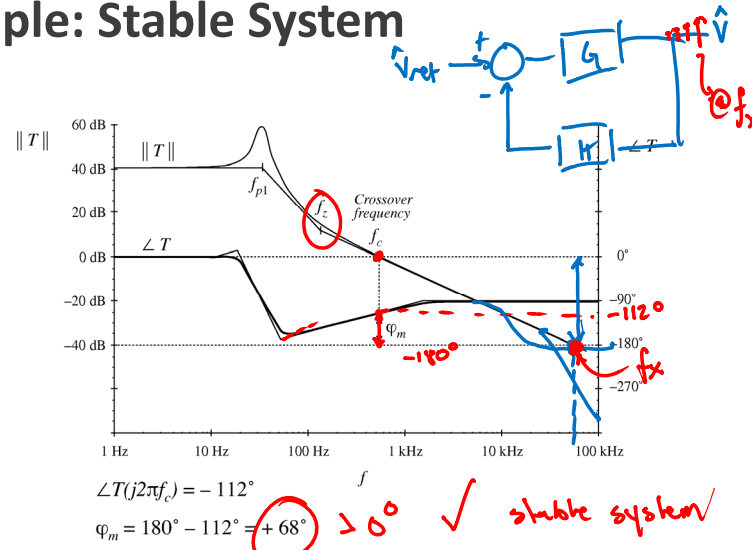
$$\varphi_m = 180° + \angle T(j2\pi f_c)$$

System stable when $\varphi_m > 0°$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.
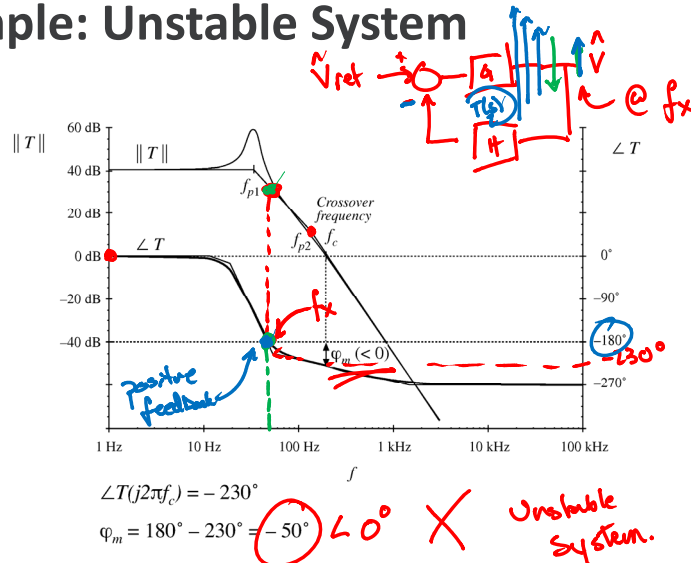
Example: Stable System

$\|T\|$

$\|T\|$

60 dB

40 dB

20 dB

0 dB

−20 dB

−40 dB

f_{p1}

f_z

Crossover frequency

f_c

$\angle T$

φ_m

0°

−90°

−112°

180°

−270°

f_x

−180°

1 Hz   10 Hz   100 Hz   1 kHz   10 kHz   100 kHz

f

$\angle T(j2\pi f_c) = -112°$

$\varphi_m = 180° - 112° = +68°$ $> 0°$ ✓ stable system ✓

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Example: Unstable System

$\|T\|$

$\|T\|$

60 dB

40 dB

20 dB

0 dB

−20 dB

−40 dB

f_{p1}

f_{p2}

Crossover frequency

f_c

$\angle T$

$\angle T$

f_x

$\varphi_m (< 0)$

0°

−90°

−180°

−230°

−270°

positive feedback

1 Hz   10 Hz   100 Hz   1 kHz   10 kHz   100 kHz

f

$\angle T(j2\pi f_c) = -230°$

$\varphi_m = 180° - 230° = -50°$ $< 0°$ ✗ Unstable System.

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Selecting Phase Margin

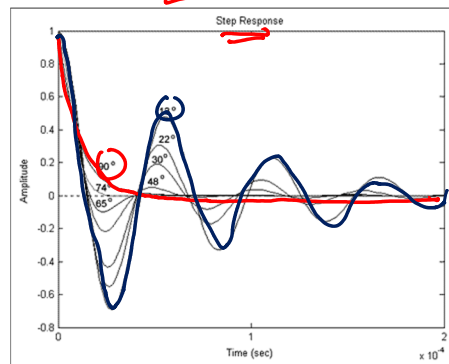*Stable for $\psi_m > 0°$*

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.
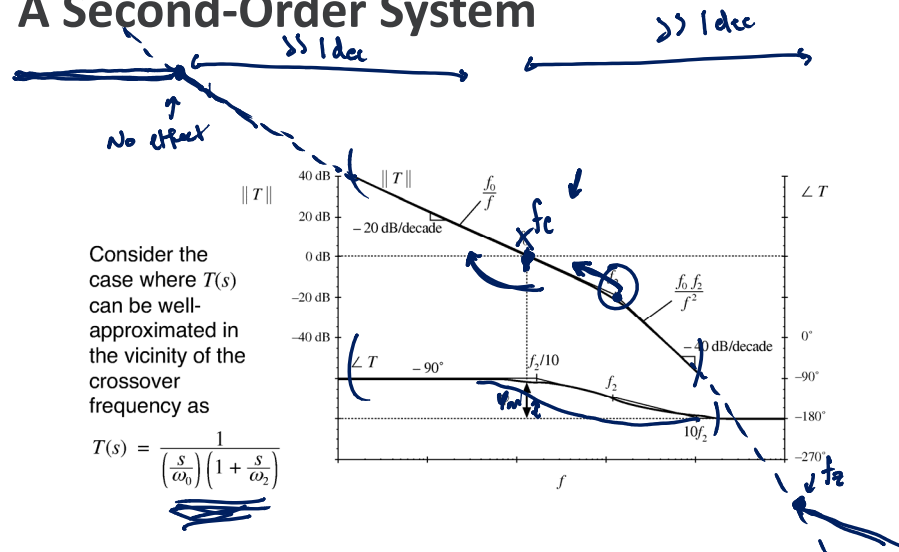
The relation between phase margin and closed-loop Q is quantified in this section.

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Step Response of Second-Order System

$$T(s) = \frac{1000}{\left(\frac{s}{\omega_1}\right)\left(\frac{s}{\omega_2}+1\right)}.$$

Maxim, App. Note 3443, "Line and Load Transient Testing for Power Supplies"

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A Second-Order System

>> 1 dec *>> 1 dec*

No effect

$\|T\|$

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

40 dB — $\|T\|$, $\frac{f_0}{f}$

20 dB — -20 dB/decade

0 dB

−20 dB

−40 dB — -40 dB/decade, $\frac{f_0 f_2}{f^2}$

$\angle T$ — $-90°$, $f_2/10$, f_2, $10f_2$

$\angle T$

$0°$

$-90°$

$-180°$

$-270°$

fc

f

Closed-Loop Response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0\omega_2}}$$
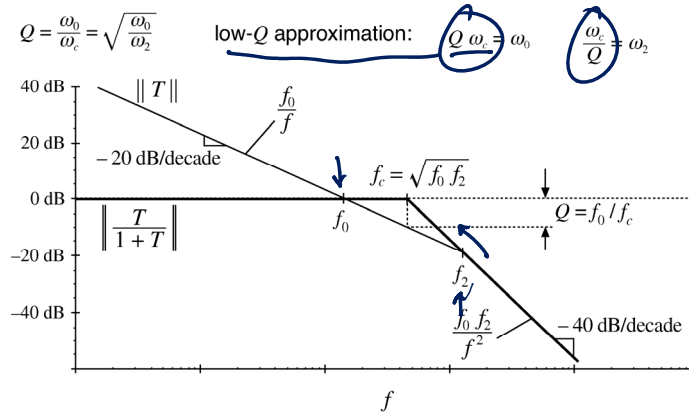
or,

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0\omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$
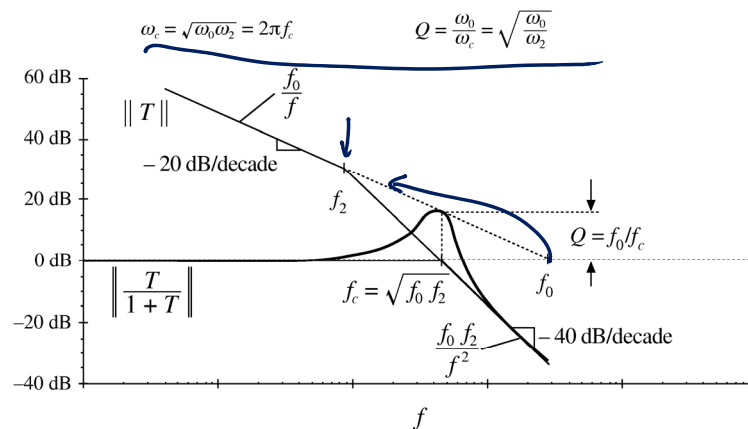
7

Low-Q Case

$$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

low-Q approximation: $Q\,\omega_c = \omega_0$ $\qquad \frac{\omega_c}{Q} = \omega_2$

40 dB

$\|T\|$ $\qquad \frac{f_0}{f}$

20 dB

-20 dB/decade

0 dB $\qquad f_c = \sqrt{f_0 f_2}$

f_0 $\qquad Q = f_0 / f_c$

$\left\| \dfrac{T}{1+T} \right\|$

-20 dB

f_2

-40 dB $\qquad \dfrac{f_0 f_2}{f^2}$ $\qquad -40$ dB/decade

f

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High-Q Case

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad\qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

60 dB

$\|T\|$ $\qquad \frac{f_0}{f}$

40 dB

-20 dB/decade

20 dB $\qquad f_2$

$\qquad Q = f_0/f_c$

0 dB

$\left\| \dfrac{T}{1+T} \right\|$ $\qquad f_c = \sqrt{f_0 f_2}$ $\qquad f_0$

-20 dB $\qquad \dfrac{f_0 f_2}{f^2}$ $\qquad -40$ dB/decade

-40 dB

f

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Q vs. *φ*ₘ

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}$$
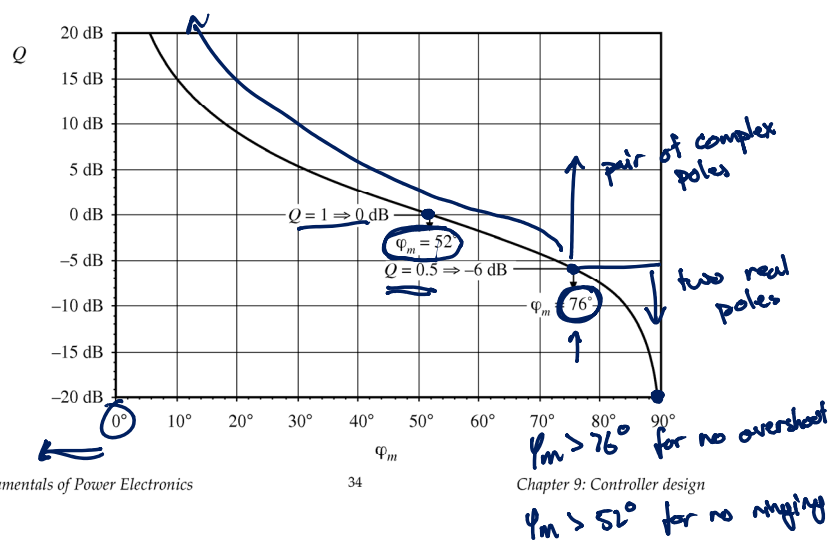
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Q vs. *φ*ₘ

$Q_{CL} \rightarrow Q$ of Closed-loop Tf

$Q = 1 \Rightarrow 0$ dB

$\varphi_m = 52°$

$Q = 0.5 \Rightarrow -6$ dB

$\varphi_m = 76°$

pair of complex poles

two real poles

$\psi_m > 76°$ for no overshoot

$\psi_m > 52°$ for no ringing

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Closed-Loop Step Response vs. *Q*
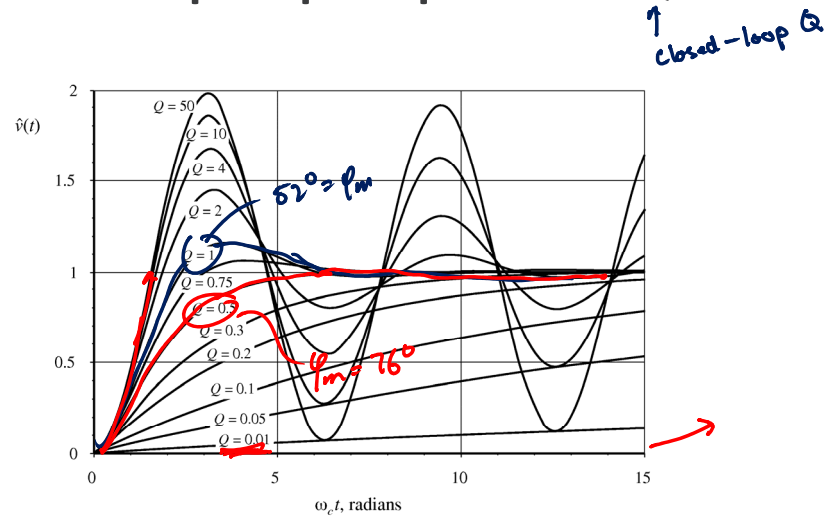
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9.5 – Compensator Design

Typical specifications:

- Effect of load current variations on output voltage regulation

    This is a limit on the maximum allowable output impedance

- Effect of input voltage variations on the output voltage regulation

    This limits the maximum allowable line-to-output transfer function

- Transient response time

    This requires a sufficiently high crossover frequency

- Overshoot and ringing

    An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

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Design Approach

- Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency gain amplitude large enough to result in low steady-state error?
  - Is ϕ_m sufficient for stability and requirements on ringing/overshoot?
  - Is f_c high enough for a sufficiently fast response?
- Construct compensator to address shortcomings of $T_u(s)$
  - Use "toolbox" of compensators on following slides

$T(s)$
w/ our designed
→ $G_c(s)$

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