Solution of Block Diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + H G_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + H G_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + H G_c G_{vd} / V_M}$$

which is of the form

$$\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1+T} + \hat{v}_g \frac{G_{vg}}{1+T} - \hat{i}_{load} \frac{Z_{out}}{1+T}$$

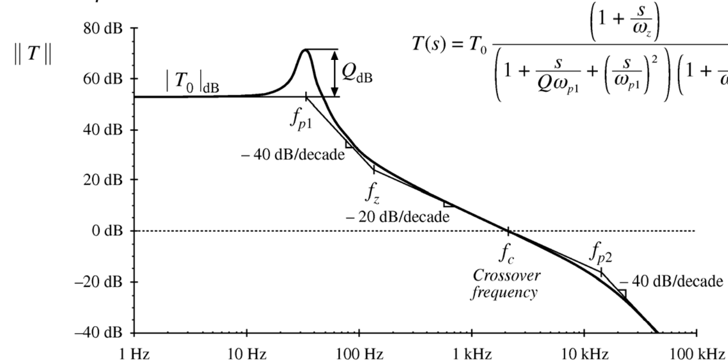
$$with \quad T(s) = H(s)\, G_c(s)\, G_{vd}(s) / V_M \quad = \text{"}loop\ gain\text{"}$$

Loop gain $T(s)$ = products of the gains around the negative feedback loop.

Construction of the Loop Gain

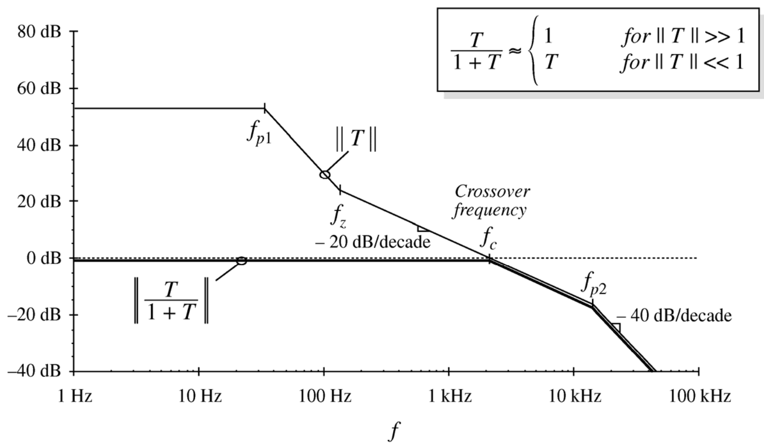
Example

$$T(s) = T_0 \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_{p1}} + \left(\frac{s}{\omega_{p1}}\right)^2\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$



At the crossover frequency f_c, $\| T \| = 1$ $\quad f$

Effect of Loop Gain



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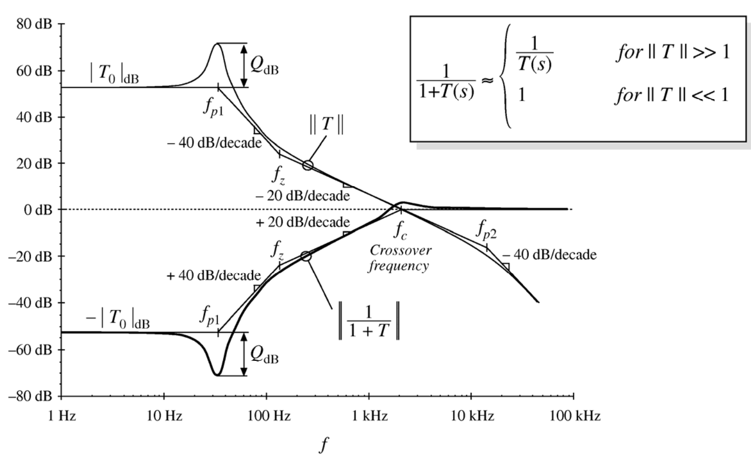
Rejection of Disturbances



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Open-Loop and Closed-Loop

Original transfer functions, before introduction of feedback ("open-loop transfer functions"):

$$G_{vd}(s) \qquad G_{vg}(s) \qquad Z_{out}(s)$$

Upon introduction of feedback, these transfer functions become ("closed-loop transfer functions"):

$$\frac{1}{H(s)}\frac{T(s)}{1+T(s)} \qquad \frac{G_{vg}(s)}{1+T(s)} \qquad \frac{Z_{out}(s)}{1+T(s)}$$

The loop gain:

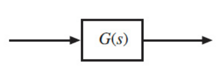
$$T(s)$$

Effect of Feedback

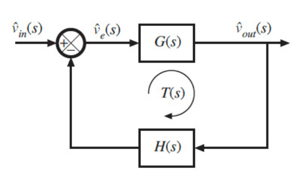
Feedback moves the poles of the system transfer functions

- Good news: we can use feedback to alter the poles and improve the frequency response
- Bad news: if you're not careful, feedback can move the poles into the right half of the complex *s*-plane (poles have positive real parts), leading to an unstable system

Open loop



Closed loop



Determining Stability From T(s)

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

  Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

  A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

9.4.1 – The Phase Margin Test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

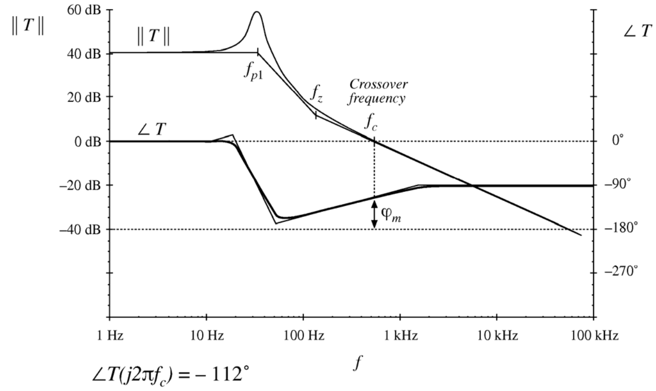
The phase margin φ_m is determined from the phase of $T(s)$ at f_c , as follows:

$$\varphi_m = 180^\circ + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

Example: Stable System



$\angle T(j2\pi f_c) = -\ 112^\circ$

$\varphi_m = 180^\circ - 112^\circ = +\ 68^\circ$

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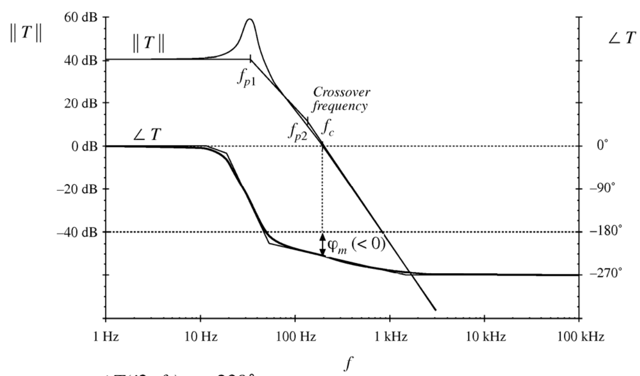
Example: Unstable System



$\angle T(j2\pi f_c) = -\ 230^\circ$

$\varphi_m = 180^\circ - 230^\circ = -\ 50^\circ$

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Selecting Phase Margin

How much phase margin is required?

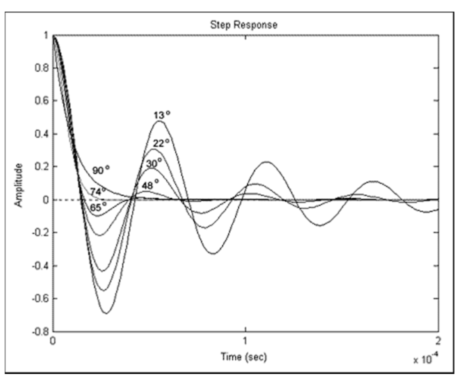
A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop Q is quantified in this section.

Step Response of Second-Order System

$$T(s)=\frac{1000}{\left(\frac{s}{\omega_1}\right)\left(\frac{s}{\omega_2}+1\right)}$$

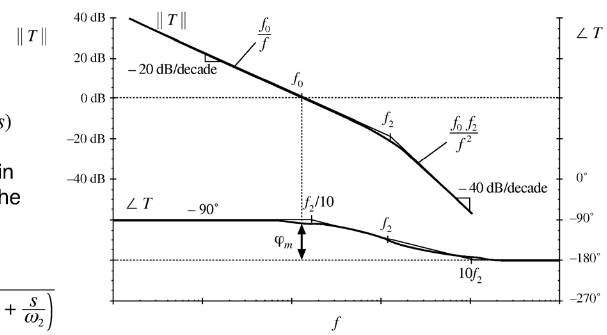


Maxim, App. Note 3443, "Line and Load Transient Testing for Power Supplies"

A Second-Order System

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$



Closed-Loop Response

If

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Then

$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{1}{T(s)}} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0 \omega_2}}$$

or,

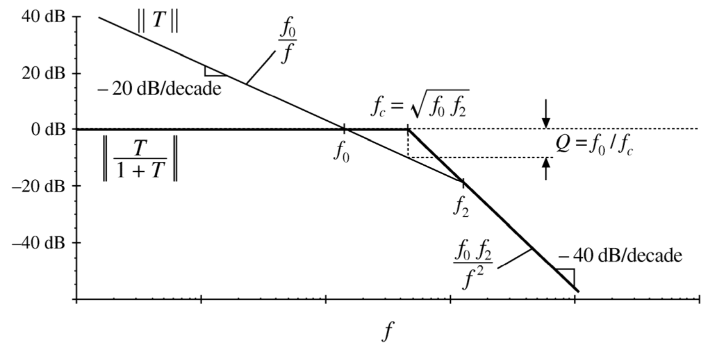
$$\frac{T(s)}{1 + T(s)} = \frac{1}{1 + \frac{s}{Q\omega_c} + \left(\frac{s}{\omega_c}\right)^2}$$

where

$$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c \qquad Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$$

Low-Q Case

$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$ low-Q approximation: $Q\,\omega_c = \omega_0$ $\frac{\omega_c}{Q} = \omega_2$



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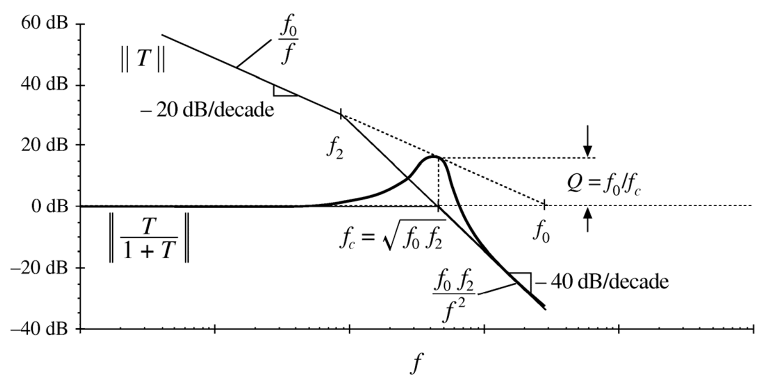
High-Q Case

$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c$ $Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$



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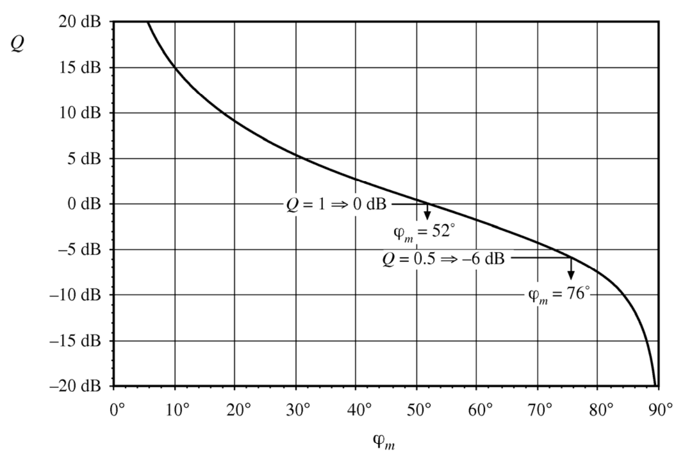
Q vs. φ_m

Solve for exact crossover frequency, evaluate phase margin, express as function of φ_m. Result is:

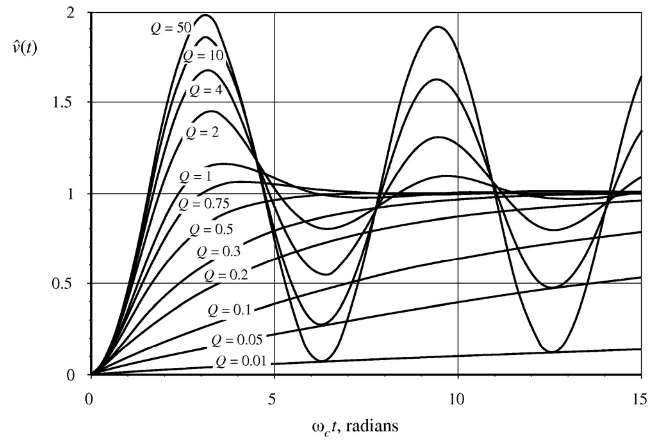
$$Q = \frac{\sqrt{\cos \varphi_m}}{\sin \varphi_m}$$

$$\varphi_m = \tan^{-1}\sqrt{\frac{1 + \sqrt{1 + 4Q^4}}{2Q^4}}$$

Q vs. φ_m



Closed-Loop Step Response vs. *Q*



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9.5 – Compensator Design

Typical specifications:

- Effect of load current variations on output voltage regulation
  This is a limit on the maximum allowable output impedance
- Effect of input voltage variations on the output voltage regulation
  This limits the maximum allowable line-to-output transfer function
- Transient response time
  This requires a sufficiently high crossover frequency
- Overshoot and ringing
  An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

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Design Approach

- Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency gain amplitude large enough to result in low steady-state error?
  - Is ϕ_m sufficient for stability and requirements on ringing/overshoot?
  - Is f_c high enough for a sufficiently fast response?
- Construct compensator to address shortcomings of $T_u(s)$
  - Use "toolbox" of compensators on following slides

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