Selecting Phase Margin

Stable for $\varphi_m > 0°$

How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop Q is quantified in this section.

When stable ($\varphi_m > 0°$)
↓ φ_m ⟹ ↑ Q_{CL} (freq domain)
⟹ ↑ Ringing (time domain)

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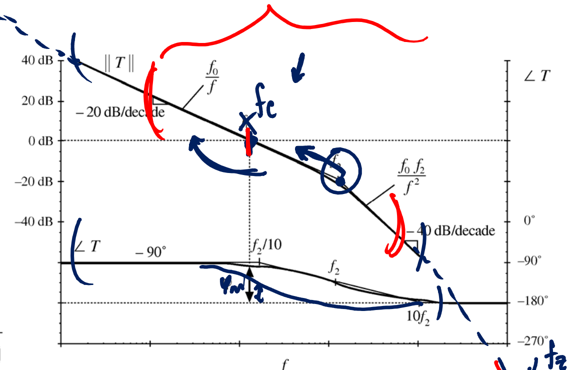
A Second-Order System

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

$\| T \|$



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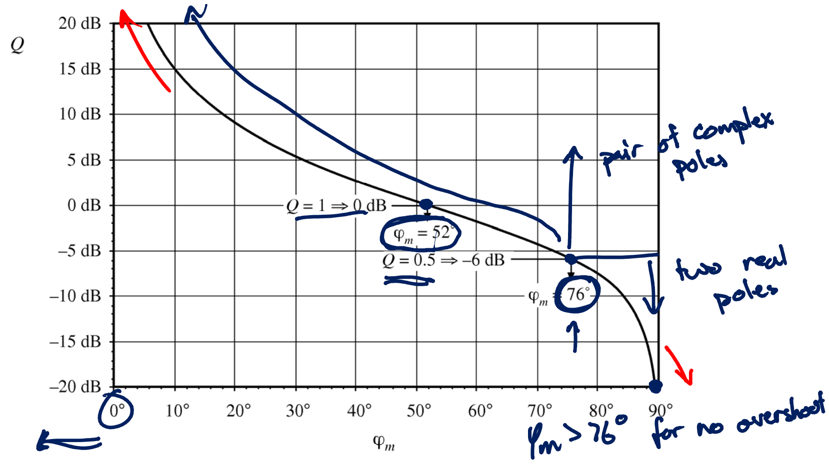
Q vs. φ_m

Q_{CL} → Q of Closed-loop Tf



$\varphi_m > 76°$ for no overshoot

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$\varphi_m > 52°$ for no ringing

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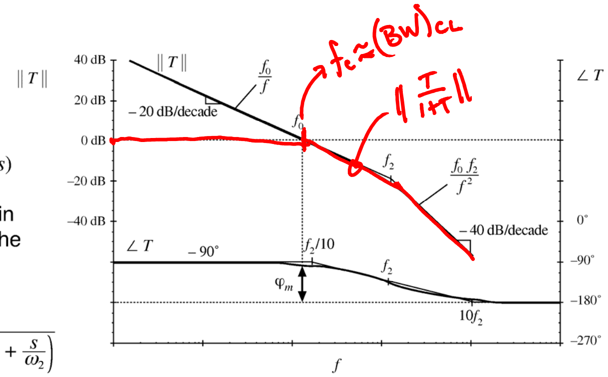
Closed-Loop Bandwidth

$H = 1$

$$\frac{\hat{v}}{\hat{v}_{ref}} = \frac{1}{H}\frac{T}{1+T}$$

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$



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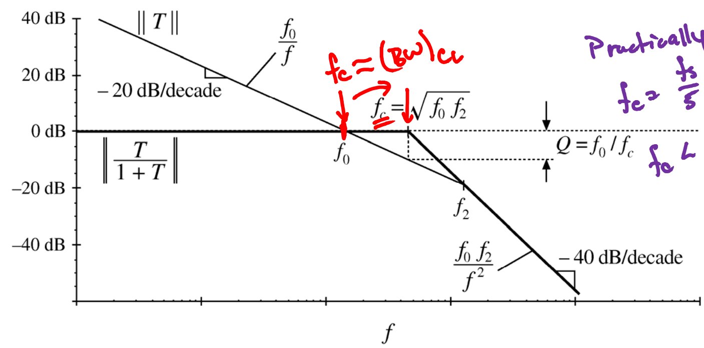
Low-Q Case

What are the limits to f_c? → must be well below f_s

$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$ low-Q approximation: $Q\,\omega_c = \omega_0$ $\frac{\omega_c}{Q} = \omega_2$

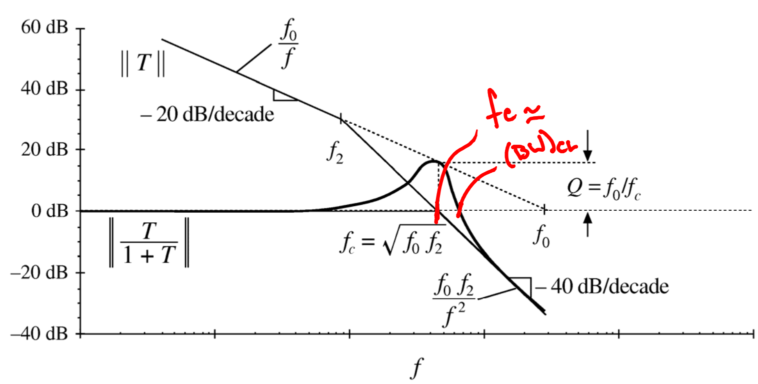
Practically $f_c \approx \frac{f_s}{5} \Rightarrow$ aggressive

$f_c < \frac{f_s}{10} \Rightarrow$ conservative



High-Q Case

$\omega_c = \sqrt{\omega_0 \omega_2} = 2\pi f_c$ $Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}}$

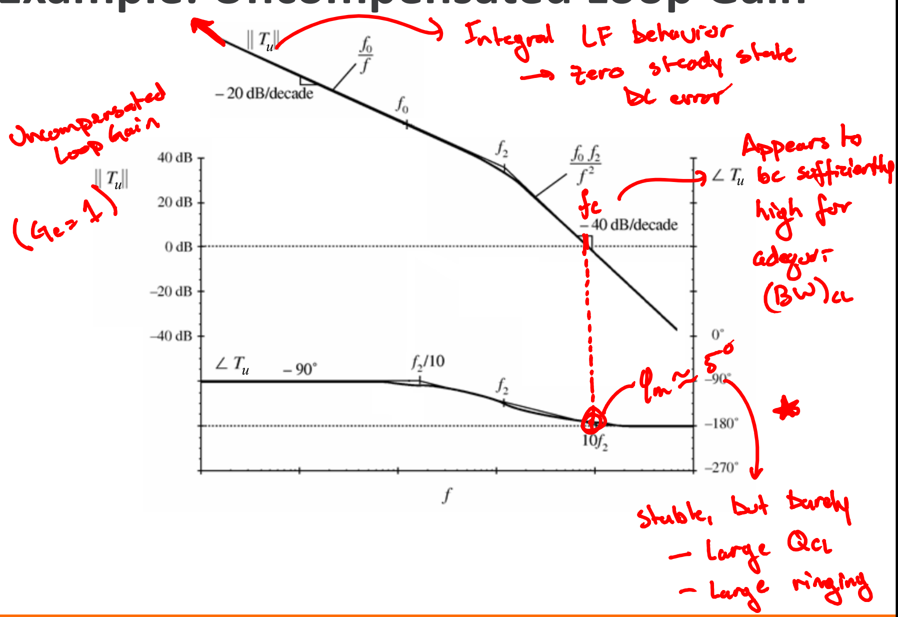


Design Approach

- Assume $G_c(s) = 1$, and plot the resulting uncompensated loop gain $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency gain amplitude large enough to result in low steady-state error?
  - Is ϕ_m sufficient for stability and requirements on ringing/overshoot?
  - Is f_c high enough for a sufficiently fast response?
- Construct compensator to address shortcomings of $T_u(s)$
  - Use “toolbox” of compensators on following slides

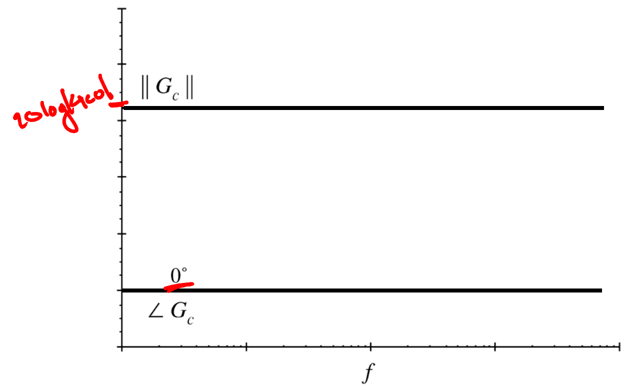
T(s) w/ our designed → Gc(s)

Example: Uncompensated Loop Gain

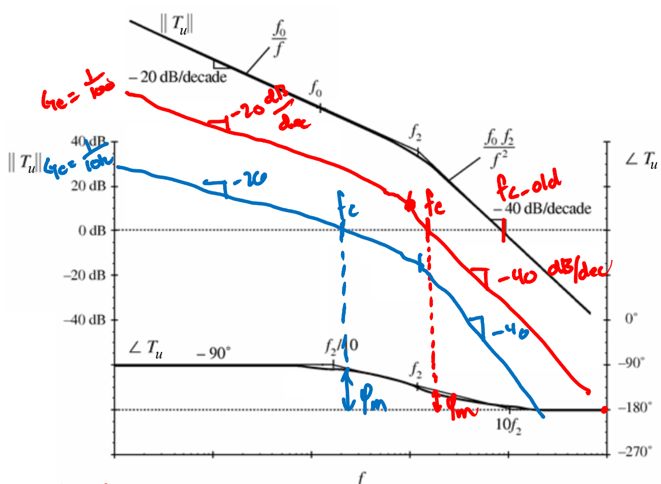


Proportional (P) Compensator

$G_c(s) = G_{c0}$

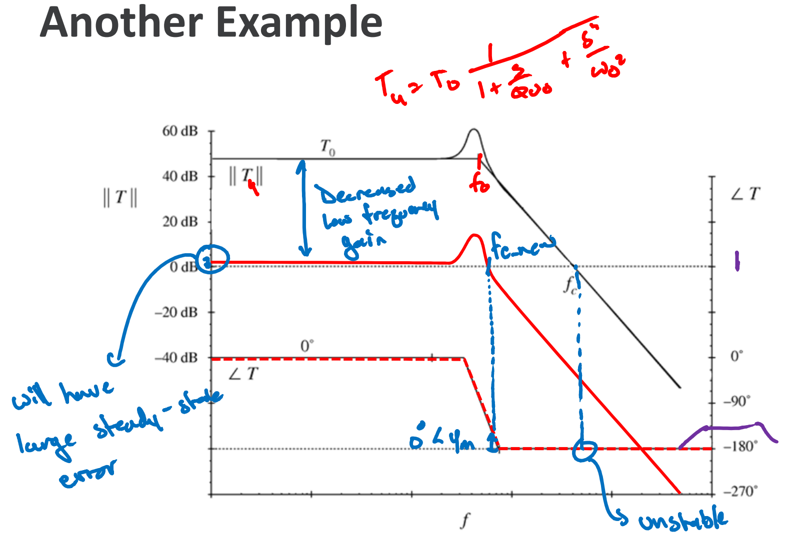


Stabilization by (P) Compensator



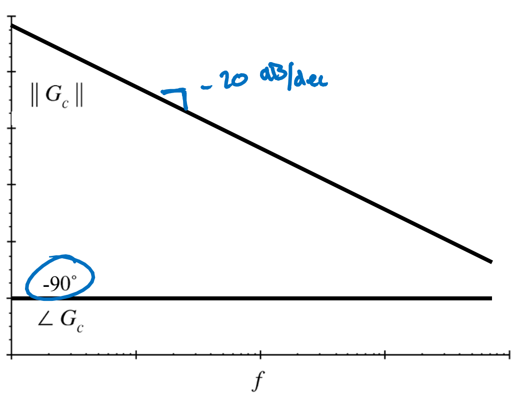
- Improved φm
- Reduced fc

Another Example

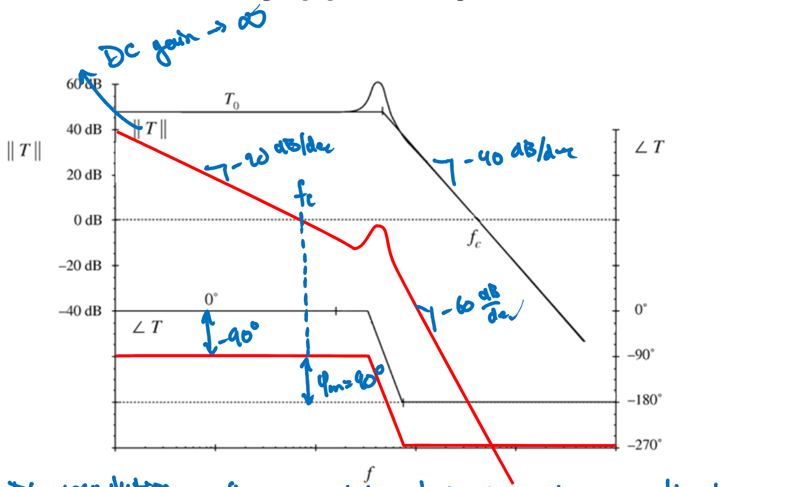


Integral (I) Compensator

$$G_c(s) = \frac{K}{s}$$



Stabilization by (I) Compensator

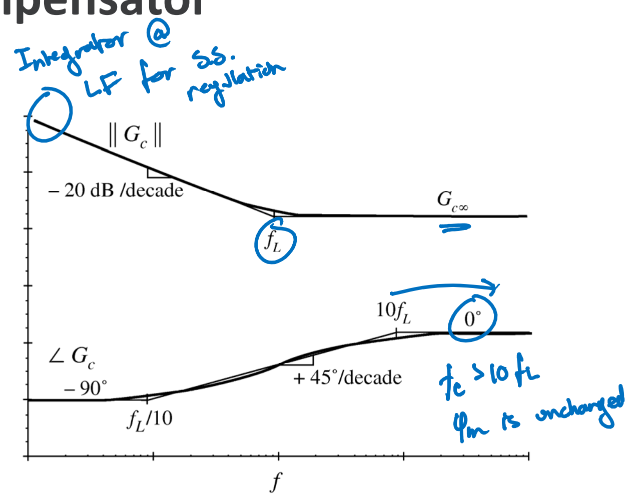


For DC regulation, a pure integral compensator with low enough K will usually stabilize a system with zero steady-state DC error (as long as they have 0 dB/dec LF slope)

Lag (PI) Compensator

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



Example Lag Compensator Design

original (uncompensated) loop gain is

$$T_u(s) = \frac{T_{u0}}{\left(1 + \frac{s}{\omega_0}\right)}$$

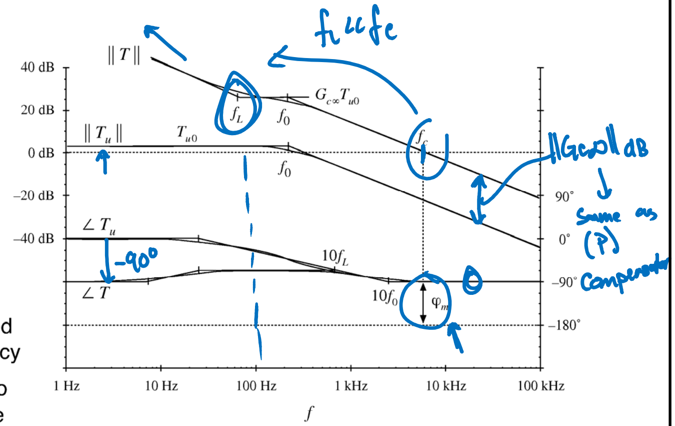
compensator:

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Design strategy: choose

$G_{c\infty}$ to obtain desired crossover frequency

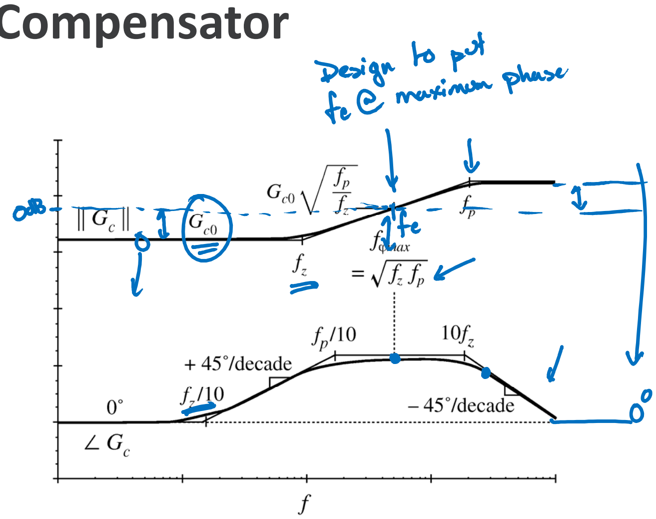
ω_L sufficiently low to maintain adequate phase margin



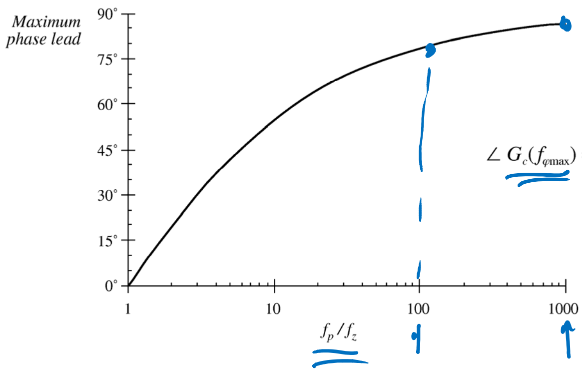
Lead (PD) Compensator

$$G_c(s) = G_{c0}\frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Maximum Phase Lead



$$f_{\varphi \max} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{\varphi \max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2} \right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

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Lead Compensator Design

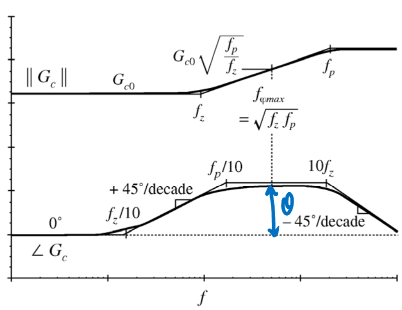
To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



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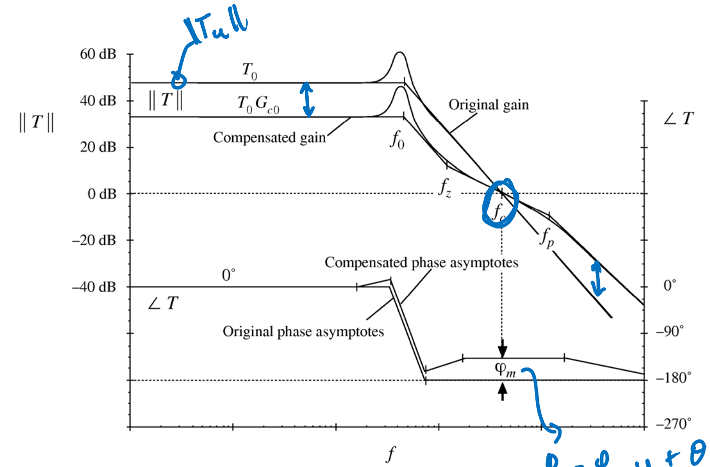
Example Lead Compensator Design



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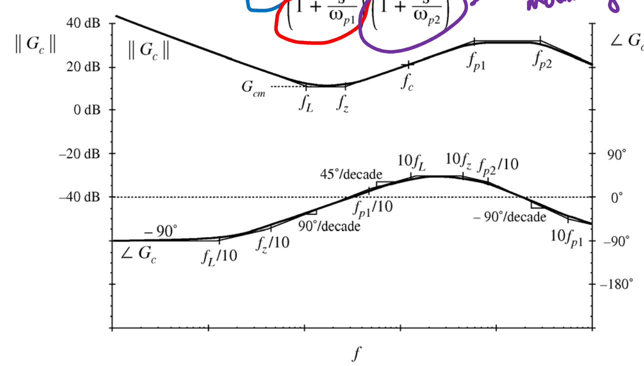
Combined (PID) Compensator

PI PD

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{\omega_L}{s}\right)\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

extra pole? modeling HF pole



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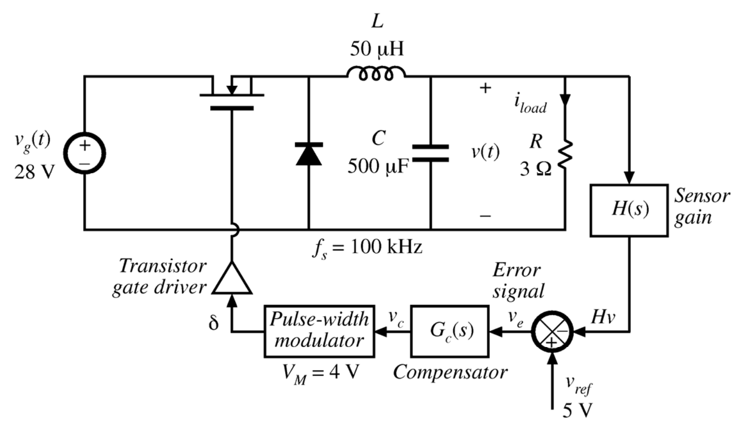
Example Design of Buck Compensator



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