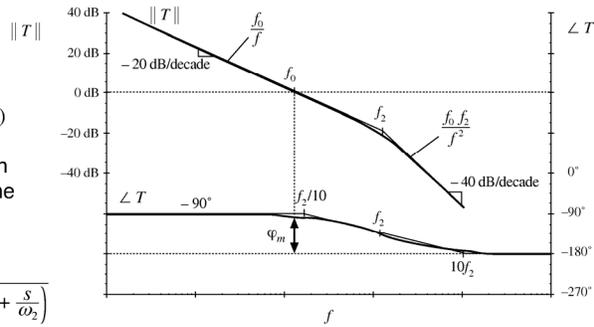


## Closed-Loop Bandwidth

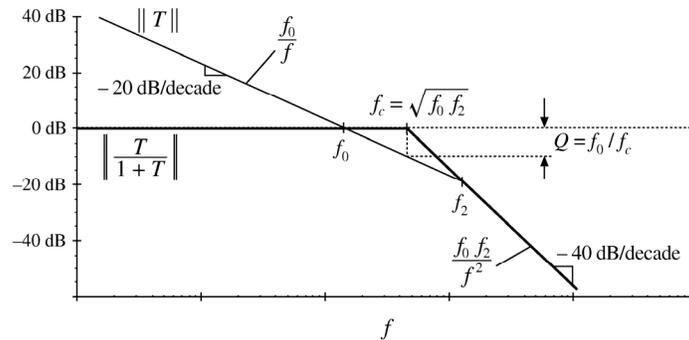
Consider the case where  $T(s)$  can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

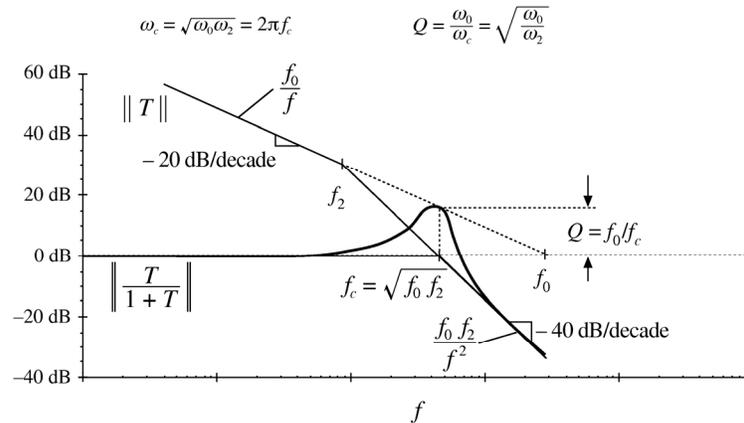


## Low-Q Case

$$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_2}} \quad \text{low-}Q \text{ approximation: } Q \omega_c = \omega_0, \quad \frac{\omega_c}{Q} = \omega_2$$



## High-Q Case



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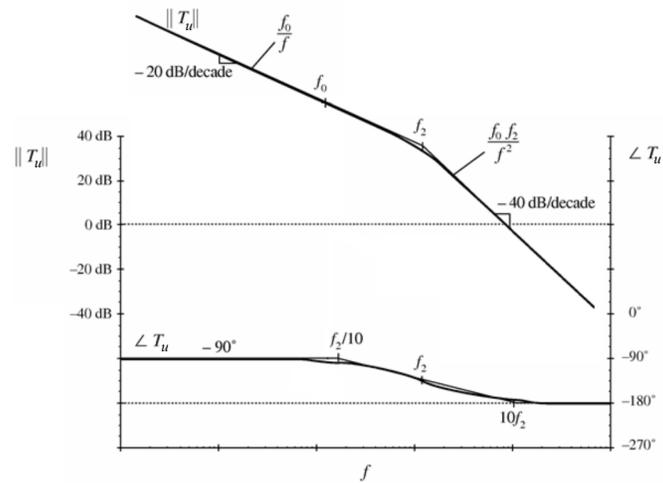
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## Design Approach

- Assume  $G_c(s) = 1$ , and plot the resulting uncompensated loop gain  $T_u(s)$
- Examine uncompensated loop gain to determine the needs of the compensator
  - Is low-frequency gain amplitude large enough to result in low steady-state error?
  - Is  $\phi_m$  sufficient for stability and requirements on ringing/overshoot?
  - Is  $f_c$  high enough for a sufficiently fast response?
- Construct compensator to address shortcomings of  $T_u(s)$ 
  - Use "toolbox" of compensators on following slides

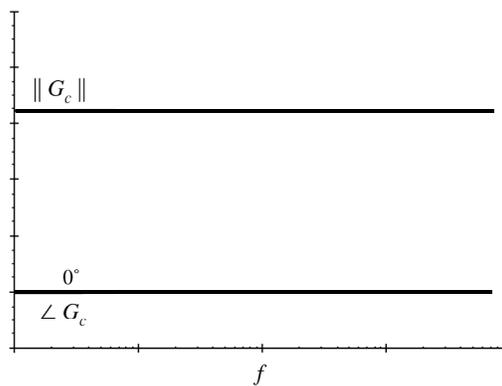
$T(s)$   
 w/ our designed  
 $\rightarrow G_c(s)$

## Example: Uncompensated Loop Gain

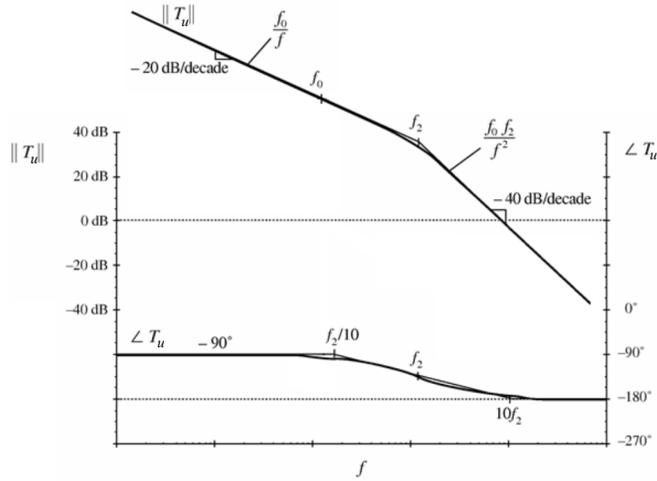


## Proportional (P) Compensator

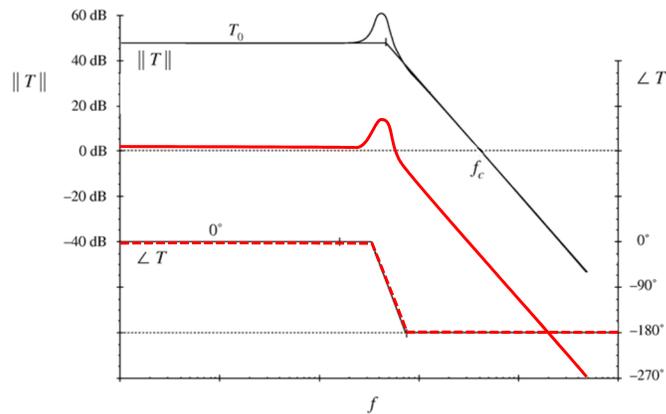
$$G_c(s) = G_{c0}$$



## Stabilization by (P) Compensator

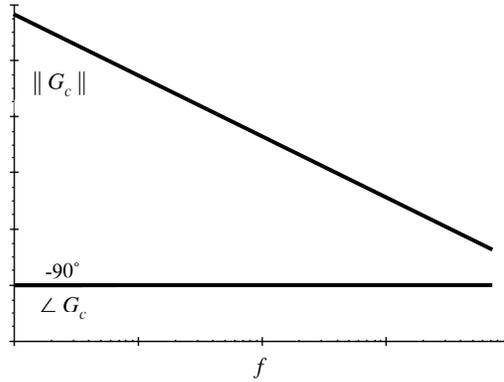


## Another Example

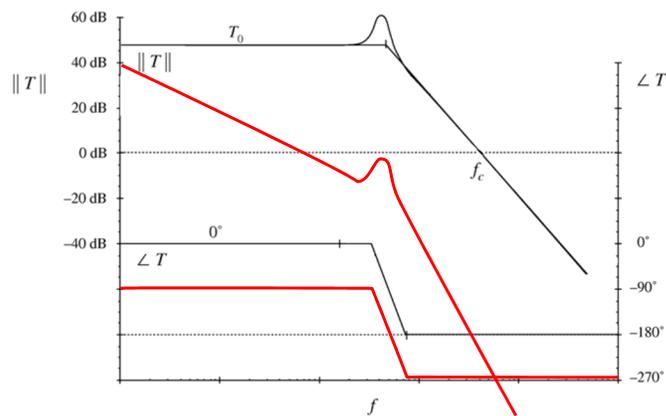


## Integral (I) Compensator

$$G_c(s) = \frac{K}{s}$$



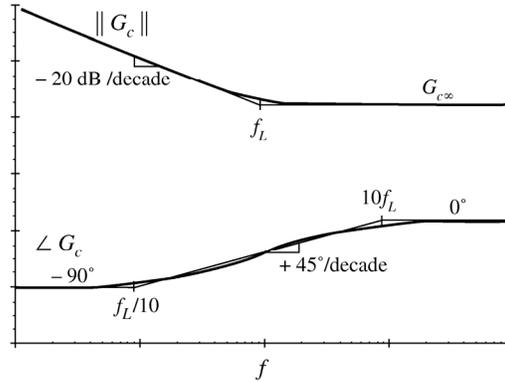
## Stabilization by (I) Compensator



## Lag (PI) Compensator

$$G_c(s) = G_{c\infty} \left( 1 + \frac{\omega_L}{s} \right)$$

Improves low-frequency loop gain and regulation



## Example Lag Compensator Design

original (uncompensated) loop gain is

$$T_u(s) = \frac{T_{u0}}{\left( 1 + \frac{s}{\omega_0} \right)}$$

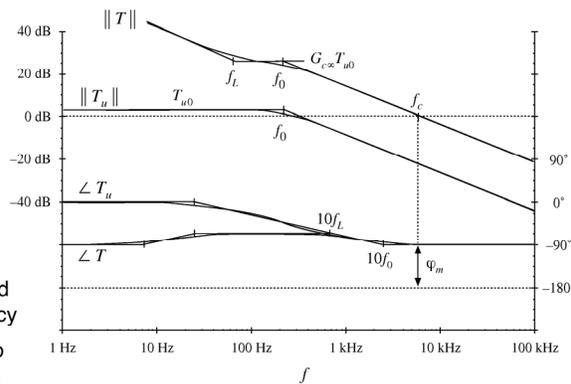
compensator:

$$G_c(s) = G_{c\infty} \left( 1 + \frac{\omega_L}{s} \right)$$

Design strategy: choose

$G_{c\infty}$  to obtain desired crossover frequency

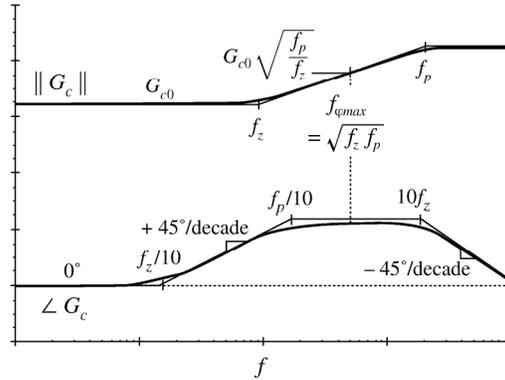
$\omega_L$  sufficiently low to maintain adequate phase margin



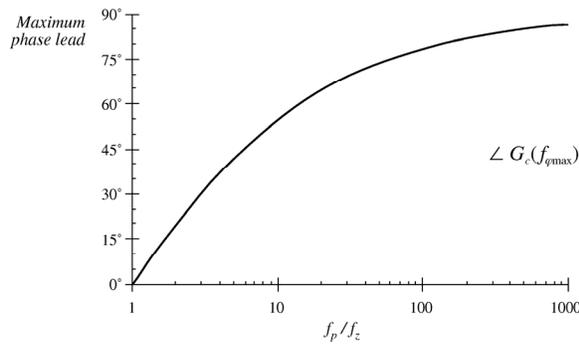
## Lead (PD) Compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



## Maximum Phase Lead



$$f_{qmax} = \sqrt{f_z f_p}$$

$$\angle G_c(f_{qmax}) = \tan^{-1} \left( \frac{\sqrt{\frac{f_p}{f_z}} - \sqrt{\frac{f_z}{f_p}}}{2} \right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 - \sin(\theta)}$$

## Lead Compensator Design

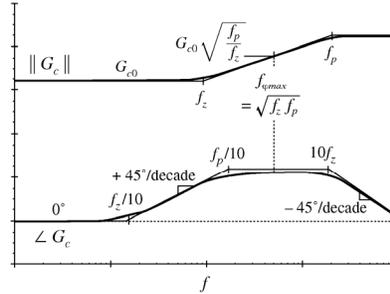
To optimally obtain a compensator phase lead of  $\theta$  at frequency  $f_c$ , the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

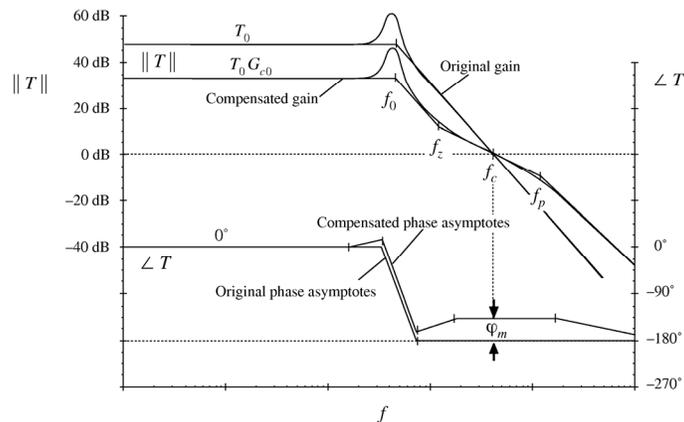
$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at  $f_c$  be unity, then  $G_{c0}$  should be chosen as

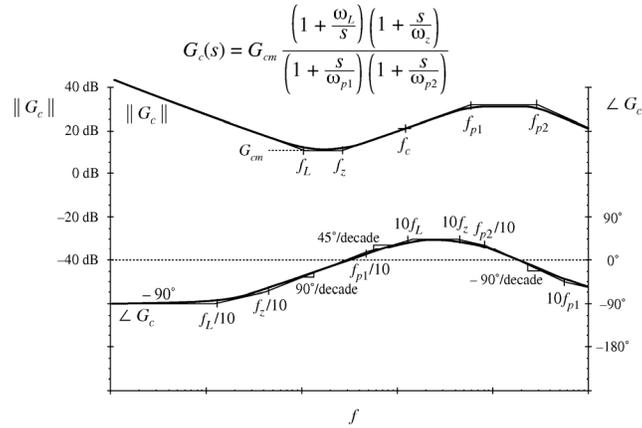
$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



## Example Lead Compensator Design



## Combined (PID) Compensator



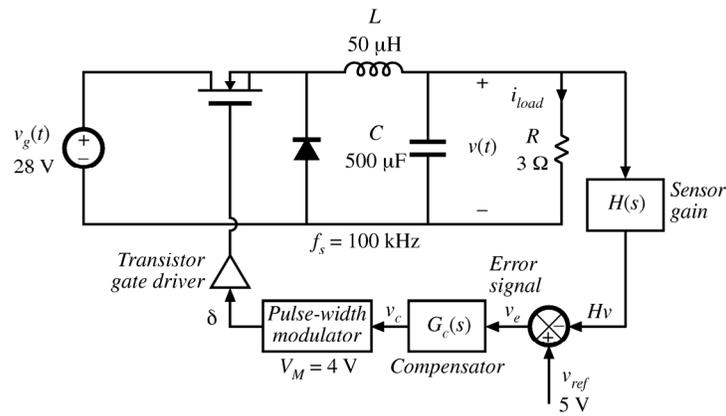
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## Example Design of Buck Compensator



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