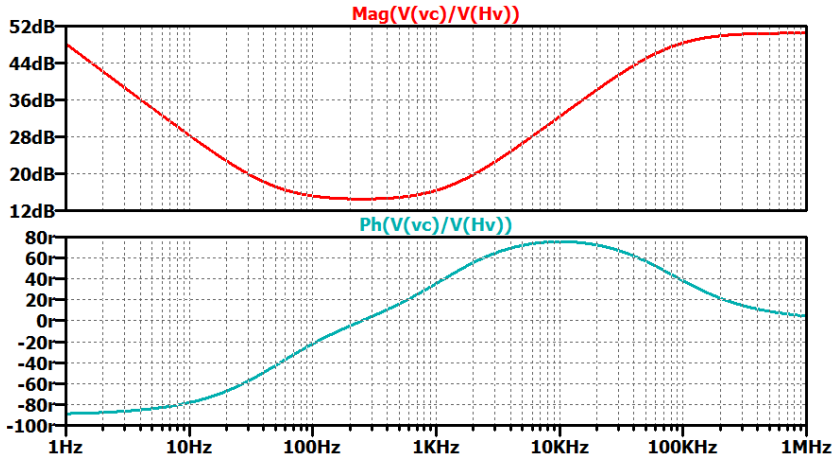
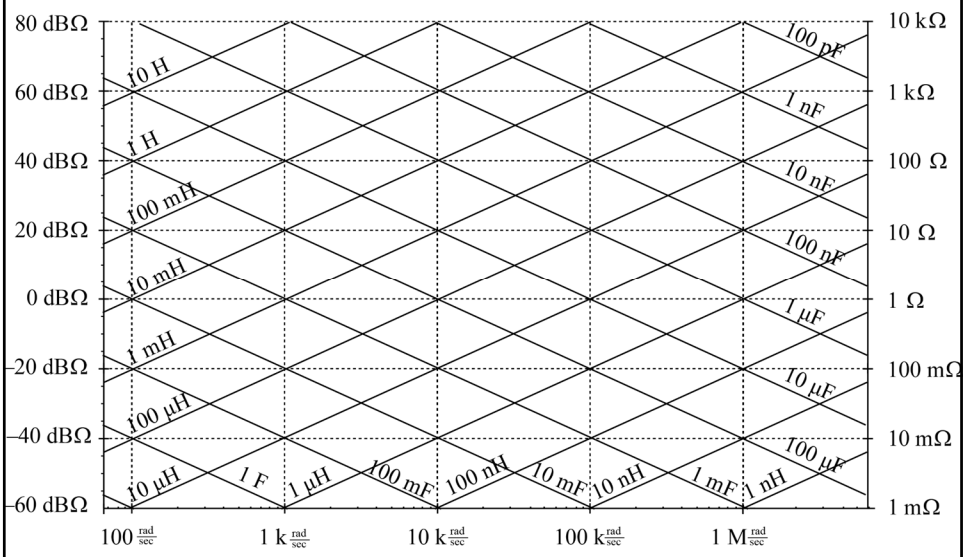
# Complete Compensator



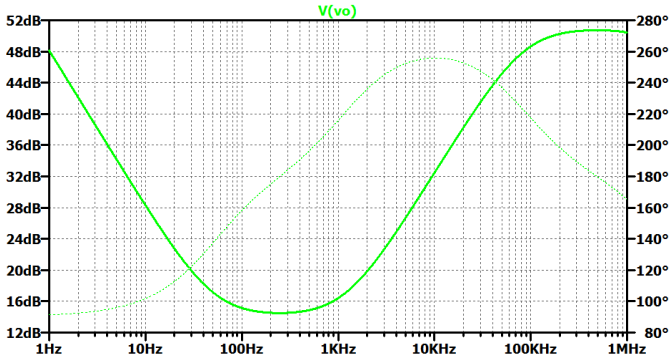

$$G_c = G_{cm}\left(1 + \frac{2\pi(50\text{Hz})}{s}\right)\frac{\left(1 + \frac{s}{2\pi(1.2\text{kHz})}\right)}{\left(1 + \frac{s}{2\pi(81k\text{Hz})}\right)}$$

# Compensator Realization

# Reactance Paper

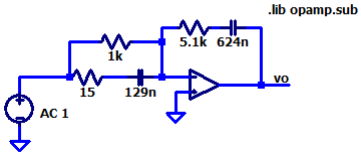



# Compensator TF

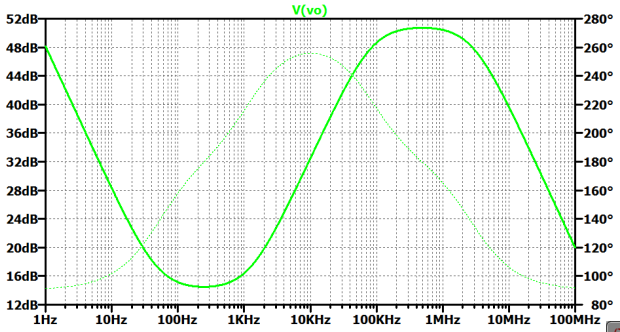


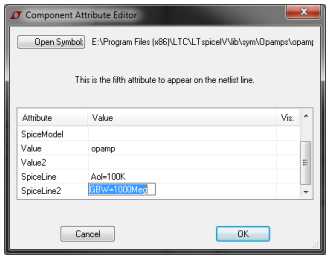

.ac dec 1000 1 1Meg

.lib opamp.sub

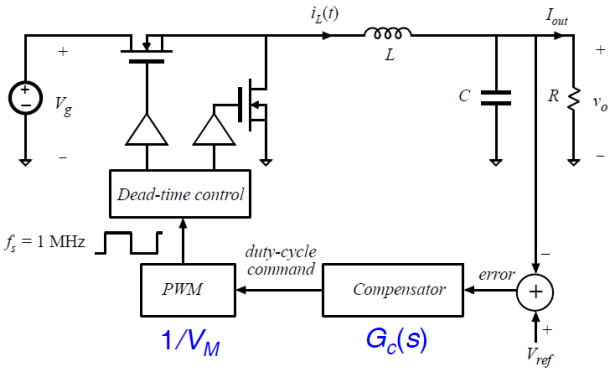


# Op-amp GBW




# Another Example




Point-of-Load Synchronous Buck Regulator

Power stage parameters

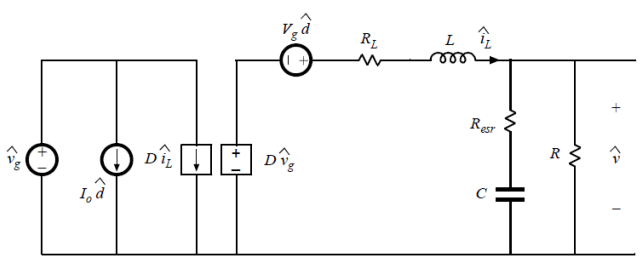
- Switching frequency: $f_s$ = 1MHz
- $V_{ref}$ = 1.8 V
- $I_{out}$ = 0 to 5 A
- $V_g$ = 5 V
- $L$ = 1 μH
- $R_L$ = 30 mΩ
- $C$ = 200 μF
- $R_{esr}$ = 0.8 mΩ
- $V_M$ = 1 V
- $H$ = 1

# Design Goals

- Set $f_c$ = 100 kHz
- Set $\phi_m > 52°$ ($Q$=1)
- Obtain $\|T_0\| \rightarrow \infty$



# AC Modeling




$$G_{vd}(s) = \frac{\hat{v}_o}{\hat{d}}$$

$$G_{vd}(s) = V_g \frac{1 + \frac{s}{\omega_{esr}}}{1 + \frac{1}{Q}\frac{s}{\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

Pair of poles:

$$f_o = \frac{1}{2\pi\sqrt{CL}} = 11\,\text{kHz}$$

$$Q_{loss} = \frac{\sqrt{L/C}}{R_{esr} + R_L} = 2.3 \rightarrow 7.2\,\text{dB} \qquad Q_{load} = \frac{R}{\sqrt{L/C}} > 5$$

$$Q = Q_{loss} \,\|\, Q_{load} = \frac{Q_{loss} Q_{load}}{Q_{loss} + Q_{load}} < 2.3 \rightarrow 7.2\,\text{dB}$$

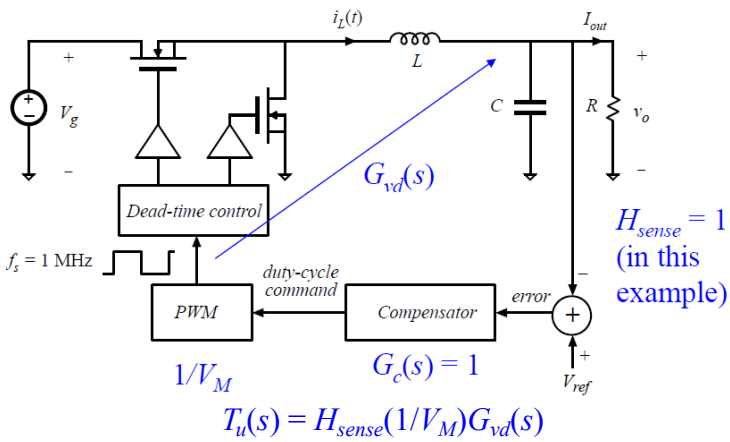
Low-frequency gain (including PWM gain):

$$G_{vdo} \frac{1}{V_M} = 5 \rightarrow 14\text{dB}$$

ESR zero:

$$f_{esr} = \frac{1}{2\pi C R_{esr}} = 1\,\text{MHz}$$

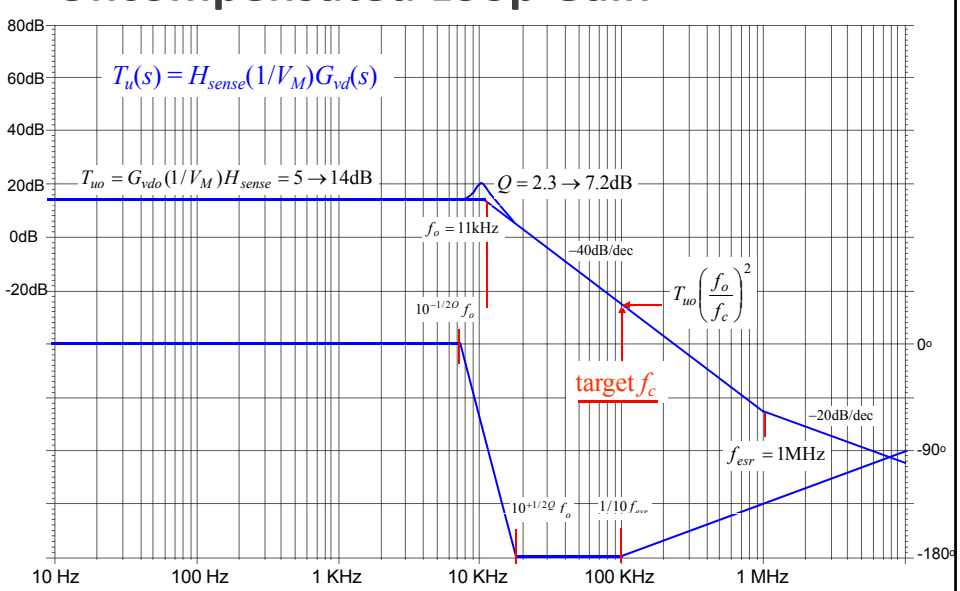
# Loop Gain



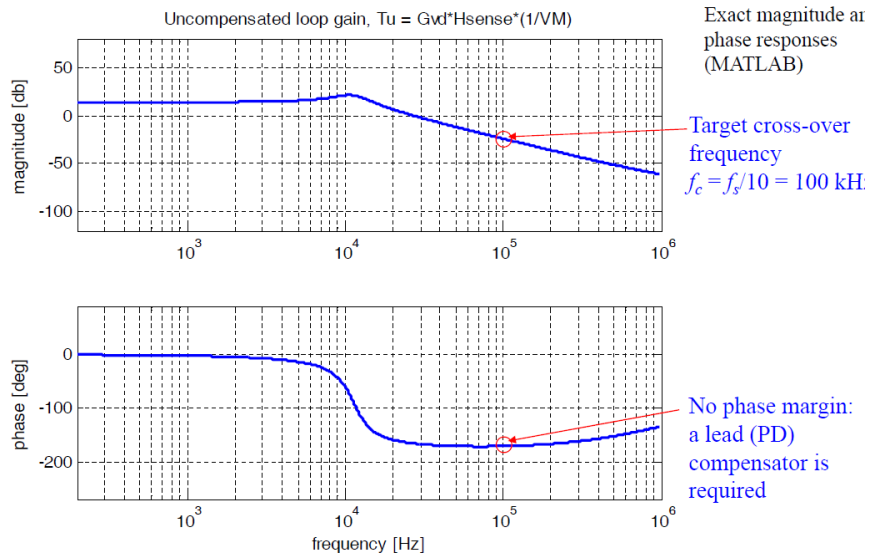

$$T_u(s) = H_{sense}(1/V_M)G_{vd}(s)$$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

# Uncompensated Loop Gain



# Exact Bode Plot of $T_u$




Exact magnitude ar
phase responses
(MATLAB)

Target cross-over frequency
$f_c = f_s/10 = 100$ kH

No phase margin: a lead (PD) compensator is required

# Lead Compensator Design

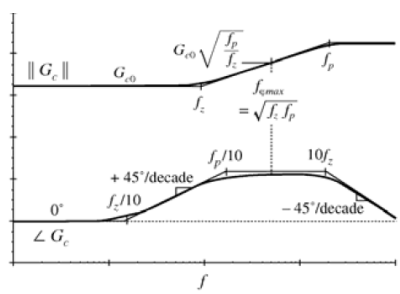
1. Choose: $f_c = 100$ kHz

$\theta = \varphi_m = 53^o$

2. Compute:

$$f_z = f_c \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 33\text{ kHz}$$

$$f_p = f_c \sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 300\text{ kHz}$$




3. Find $G_{co}$ to position the crossover frequency:

$$\underbrace{T_{uo}\left(\frac{f_o}{f_c}\right)^2}_{\text{Magnitude of } T_u \text{ at } f_c} \underbrace{G_{co}\sqrt{\frac{f_p}{f_z}}}_{\text{Magnitude of } G_c \text{ at } f_c} = 1 \quad \longrightarrow \quad G_{co} = \frac{1}{T_{uo}}\left(\frac{f_c}{f_o}\right)^2 \sqrt{\frac{f_z}{f_p}} = 5.45 \rightarrow 15\text{ dB}$$

# Lead Compensator Summary

$$G_c(s) = G_{co} \underbrace{\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)}}_{\text{Lead compensator}} \underbrace{\frac{1}{\left(1+\frac{s}{\omega_{p2}}\right)}}_{\text{HF pole}}$$

$G_{co} = 5.45 \rightarrow 15\,\text{dB}$

$f_z = 33\,\text{kHz}$

$f_{p1} = 300\,\text{kHz}$

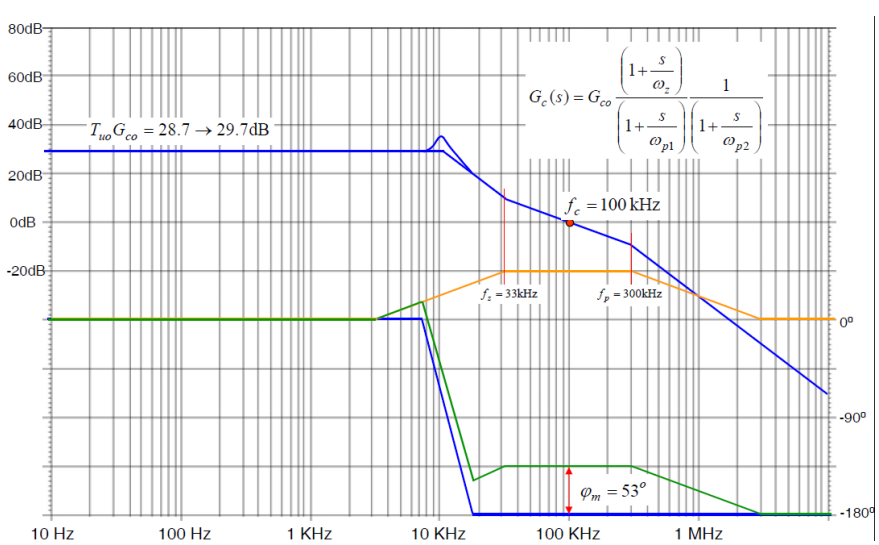
$f_c = 100\,\text{kHz}$ (=1/10 of $f_s$)

High-frequency gain of the lead compensator: $G_{co} f_{p1}/f_z = 49$ (34 dB)

Added high-frequency pole: $f_{p2} = 1\,\text{MHz}$ ($= f_{esr} = f_s$ in this example)

Practical implementation would require an op-amp with a gain bandwidth product of at least $49 * f_{p2} = 49$ MHz

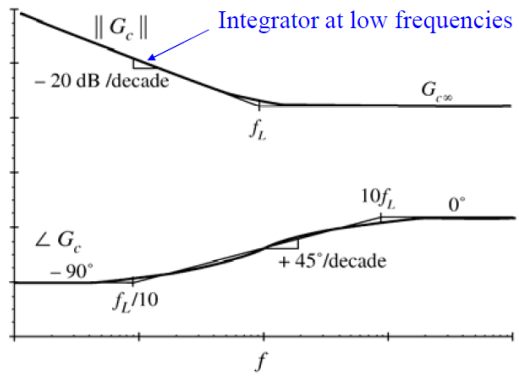
# Loop Gain With Lead Compensator



# Lag Compensator Design

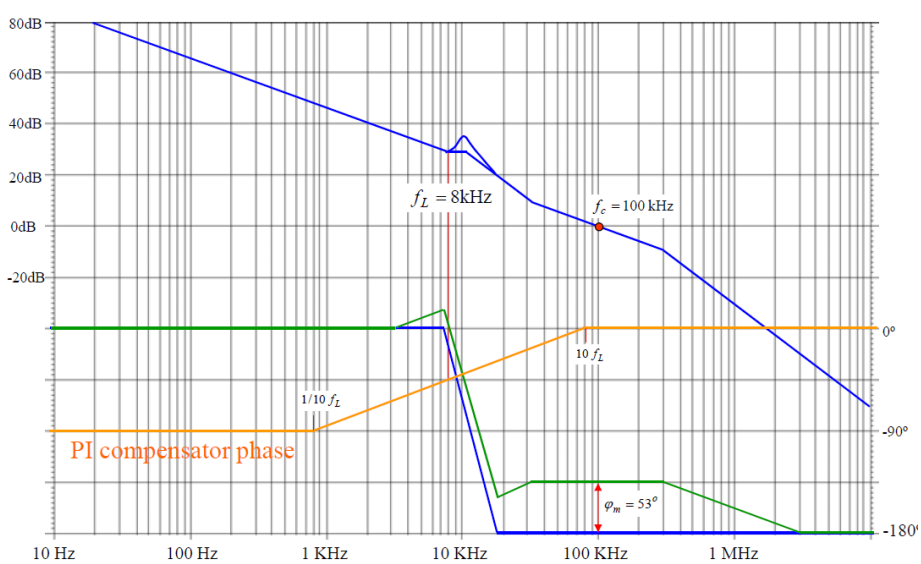
$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation

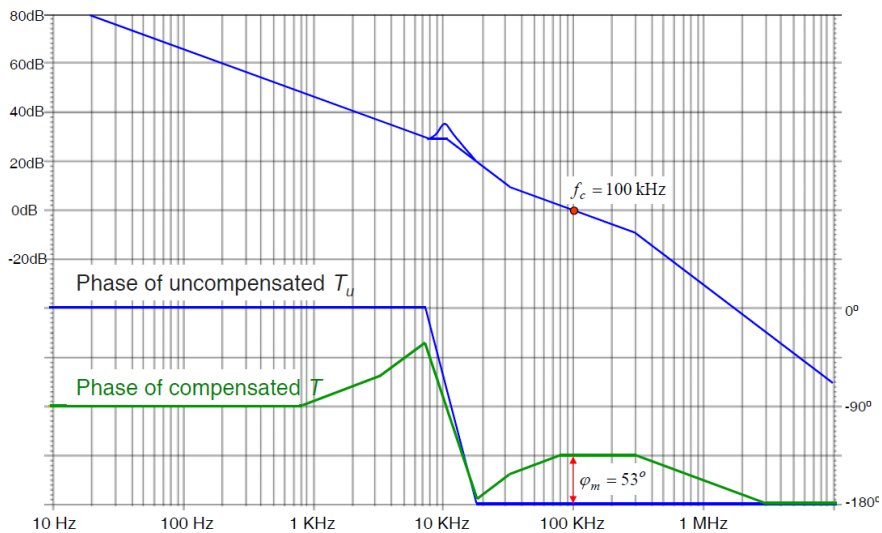



Choose $10f_L < f_c$ so that phase margin stays approximately the same: $f_L = 8$ kHz

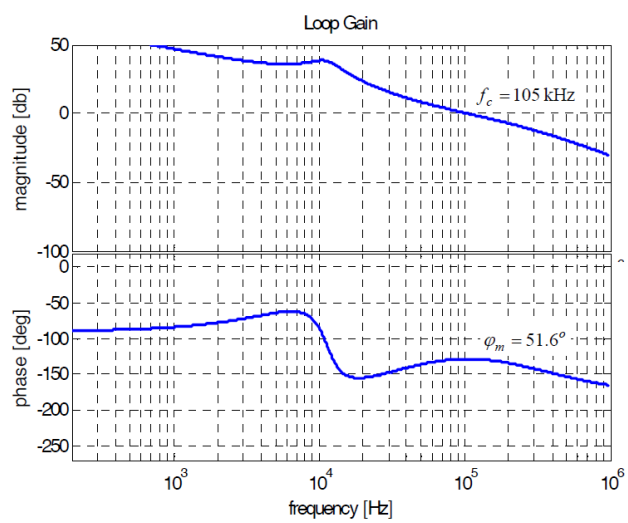
Keep the same cross-over frequency: $G_{c\infty} = G_{co} = G_{cm} = 5.45 \rightarrow 15$ dB



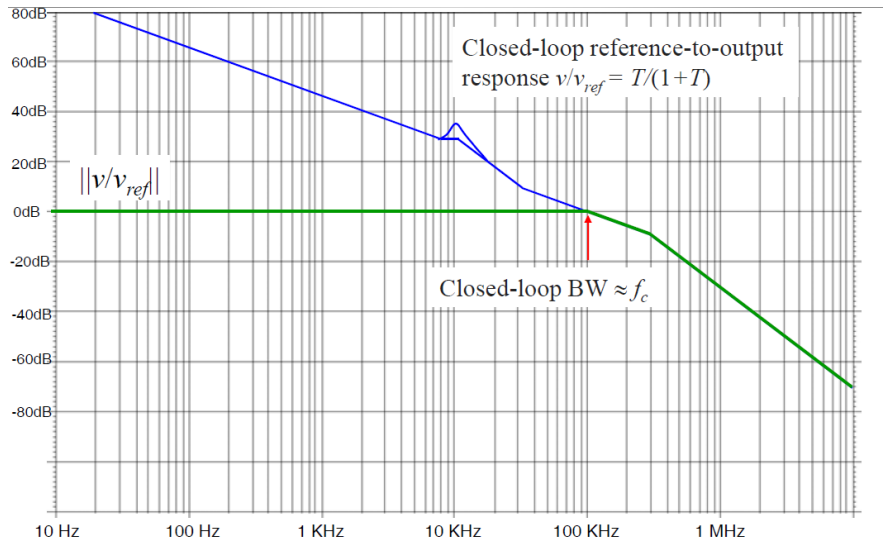
# Loop Gain with PID Compensator



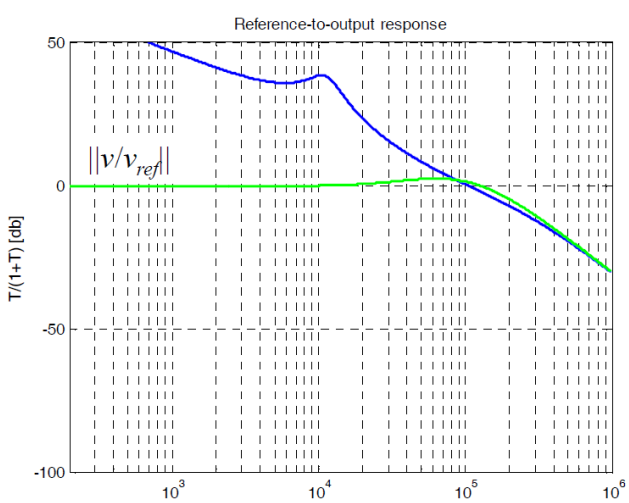

# Exact Compensated Loop Gain



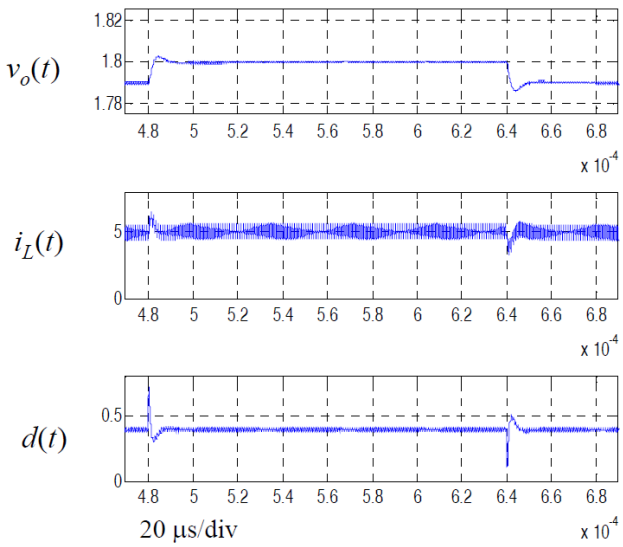
# Construction of T/(1+T)




# Closed-Loop Reference-to-Output

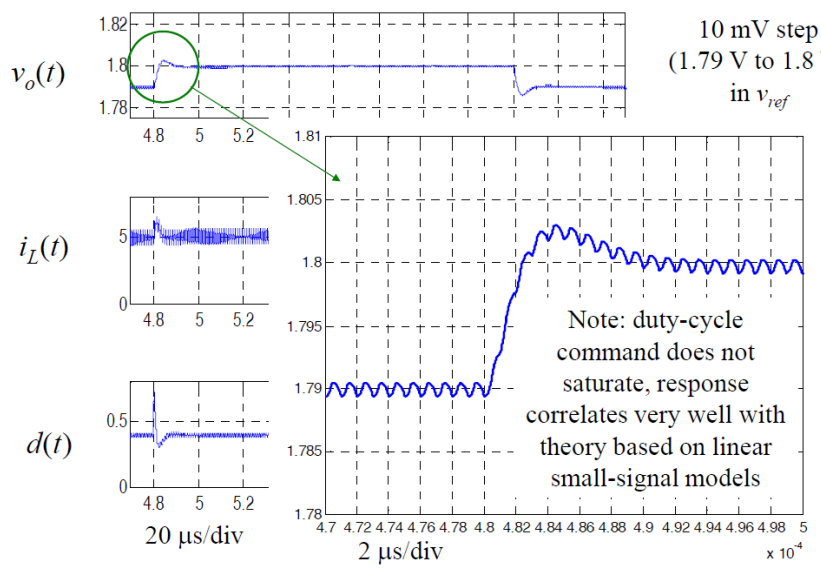


# Reference Step Response




10 mV step
(1.79 V to 1.8 V)
in $v_{ref}$

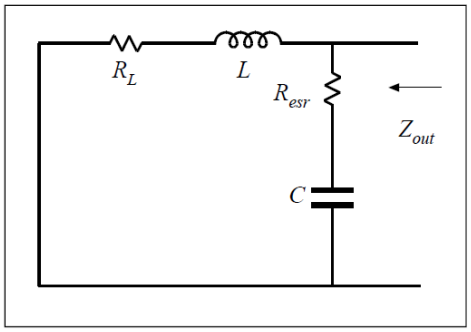
# Reference Step Response




10 mV step
(1.79 V to 1.8 V)
in $v_{ref}$

# Output Impedance

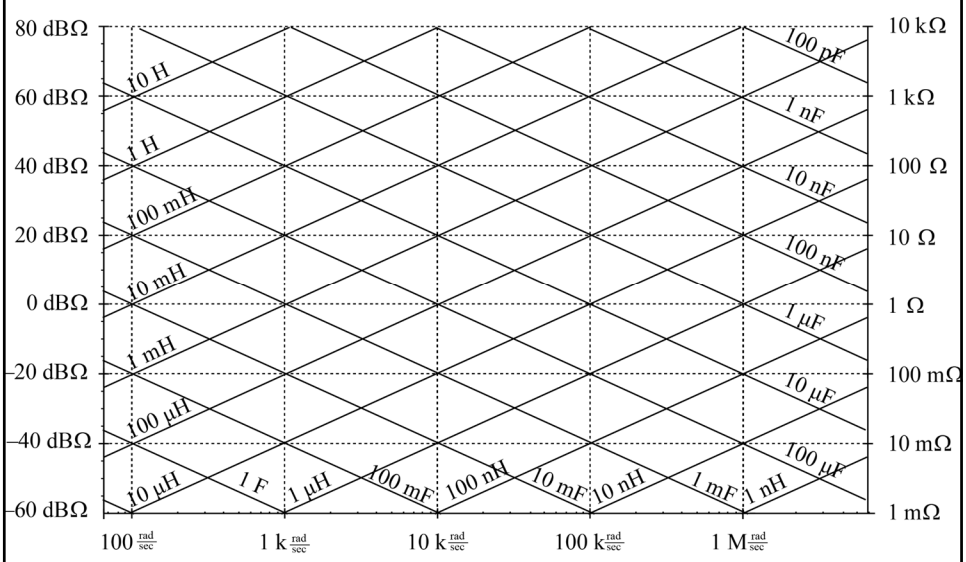
Synchronous buck open-loop output impedance



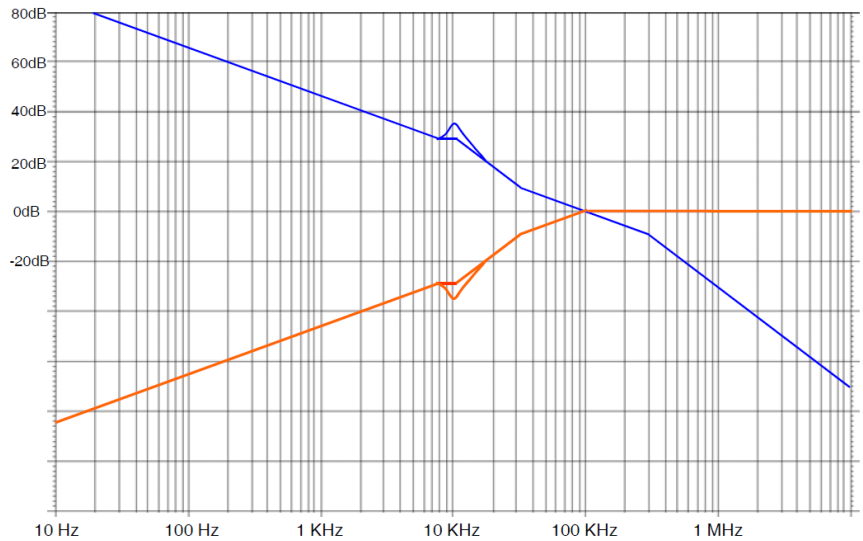

$$Z_{out}(s) = \left(R_{esr} + \frac{1}{sC}\right) \| \left(R_L + sL\right)$$

- $L$ = 1 μH
- $R_L$ = 30 mΩ
- $C$ = 200 μF
- $R_{esr}$ = 0.8 mΩ

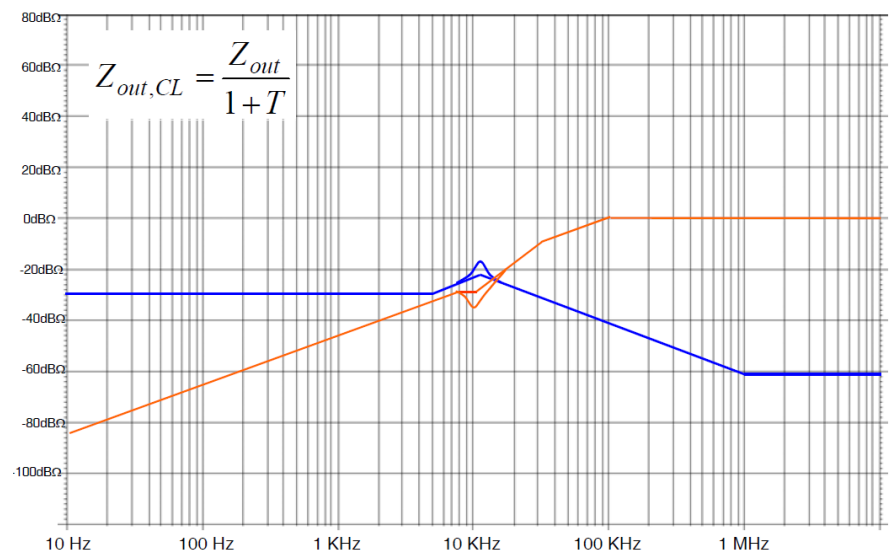
# Reactance Paper



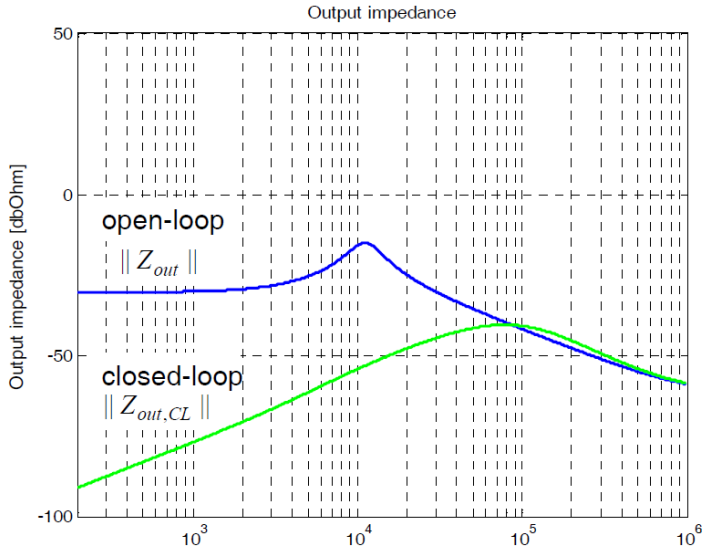
# Construction of 1/(1+T)



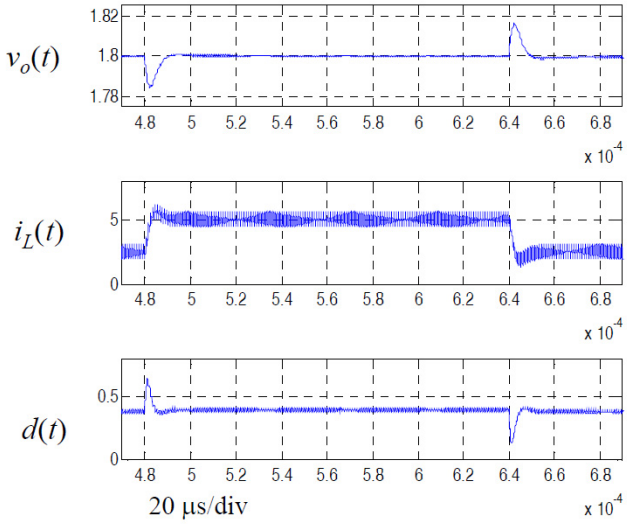

# Closed-Loop $Z_{out}$



# Exact $Z_{out}$

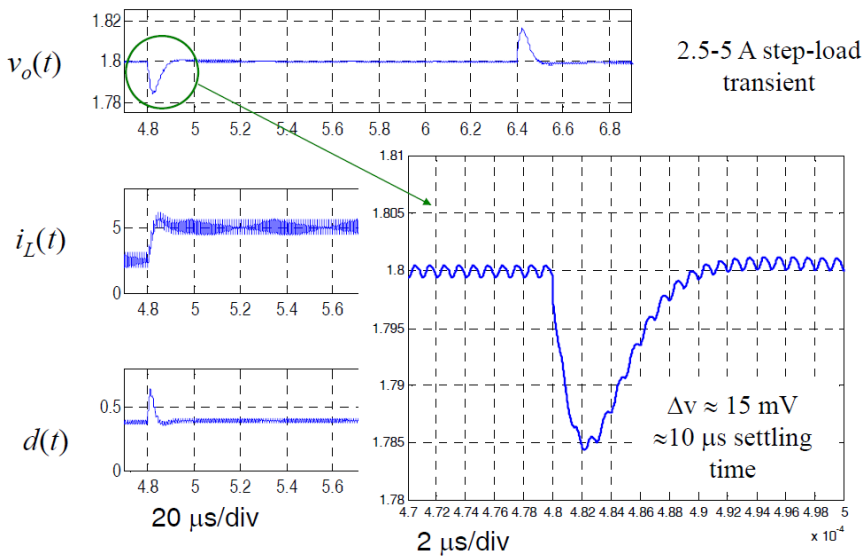



# Load Step Response




2.5-5 A step-load transient

# Load Step Response




# Chapter 9: Summary

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $1/H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.
2. The loop gain $T(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency $f_c$ is the frequency at which the loop gain $T$ has unity magnitude, and is a measure of the bandwidth of the control system.

# Chapter 9: Summary

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where $T$ is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where $T$ is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.
4. Stability can be assessed using the phase margin test. The phase of $T$ is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.



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# Chapter 9: Summary

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or *PD* controllers, are added to improve the phase margin and extend the control system bandwidth. *PI* controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.
6. Loop gains can be experimentally measured by use of voltage or current injection. This approach avoids the problem of establishing the correct quiescent operating conditions in the system, a common difficulty in systems having a large dc loop gain. An injection point must be found where interstage loading is not significant. Unstable loop gains can also be measured.



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