

## Derivation of Volt-second Balance

Inductor defining relation:

$$\int_{t_0}^{T_s} v_L(t) dt = L \int_{i_L(0)}^{i_L(T_s)} i_L(t) dt = L \int_{i_L(0)}^{i_L(T_s)} i_L(t) dt = L(i_L(T_s) - i_L(0))$$

Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L} \int_0^{T_s} v_L(t) dt$$

In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t) dt$$

Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

An equivalent form:

$$0 = \frac{1}{T_s} \int_0^{T_s} v_L(t) dt = \langle v_L \rangle T_s$$

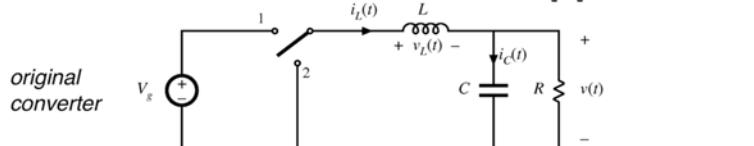
The average inductor voltage is zero in steady state.

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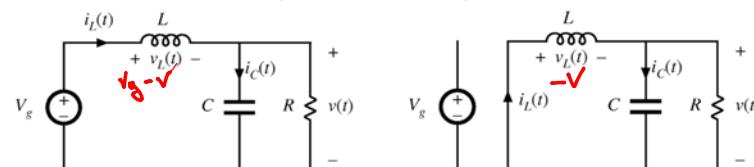
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## Volt-Second Balance: Direct Application



switch in position 1

switch in position 2



$$\begin{aligned} \langle v_L \rangle &= 0 \\ &= \frac{1}{T_s} (Dv_g + (1-D)v_g) \\ &= Dv_g - (Dv_g - v_g) \end{aligned}$$

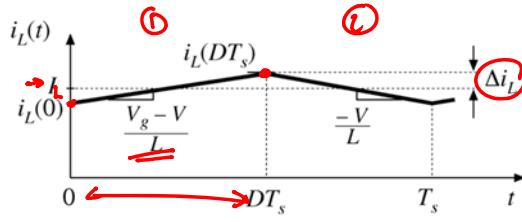
$$\frac{V_g}{\sqrt{2}} = M = D$$

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## Current Ripple Magnitude

$$2\Delta i_L = \frac{V_g - V}{L} DT_s$$

$$\Delta i_L = \frac{V_g - V}{2L} DT_s$$



If I want to reduce  $\Delta i_L$  in a given application

to reduce  $\Delta i_L$ :

- ↑ L
- ↓ T\_s ( $= \uparrow f_s$ )

$(\text{change in } i_L) = (\text{slope})(\text{length of subinterval})$

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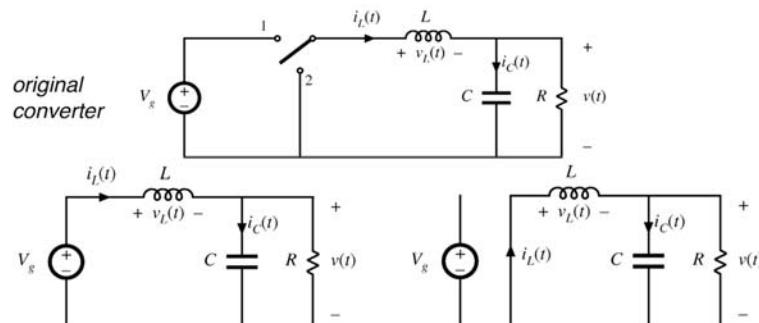
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## Capacitor Charge Balance

→ Dual of volt-second balance

if  $v_L(t_s) = v_C(\infty)$ ,  $\langle i_c \rangle|_{t_s} = 0$



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## Derivation of Capacitor Charge Balance

Capacitor defining relation:

$$i_c(t) = C \frac{dv_c(t)}{dt}$$

Integrate over one complete switching period:

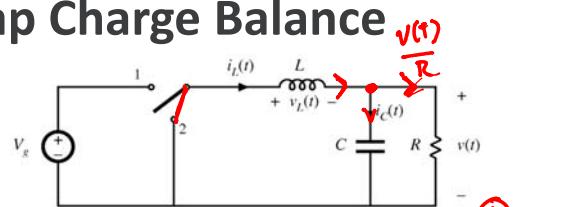
$$v_c(T_s) - v_c(0) = \frac{1}{C} \int_0^{T_s} i_c(t) dt$$

In periodic steady state, the net change in capacitor voltage is zero:

$$0 = \frac{1}{T_s} \int_0^{T_s} i_c(t) dt = \langle i_c \rangle$$

Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

## Buck Cap Charge Balance



$$\textcircled{1} \quad i_c(t) = i_L(t) - \frac{v(t)}{R}$$

$$\textcircled{2} \quad i_c(t) = i_L(t) - \frac{v(t)}{R}$$

Apply small Ripple Approx:  $i_L(t) = I_L$

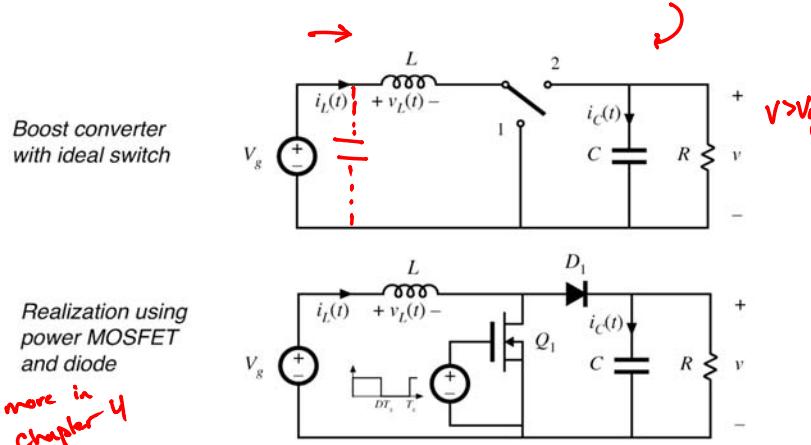
$$i_c(t) = I_L - \frac{v(t)}{R}$$

Apply Cap Charge Balance

$$\langle i_c \rangle|_{T_s} = \emptyset = D(I_L - \frac{V}{R}) + D'(I_L - \frac{V}{R})$$

$$\emptyset = I_L - \frac{V}{R} \rightarrow I_L = \frac{V}{R}$$

## The Boost Converter

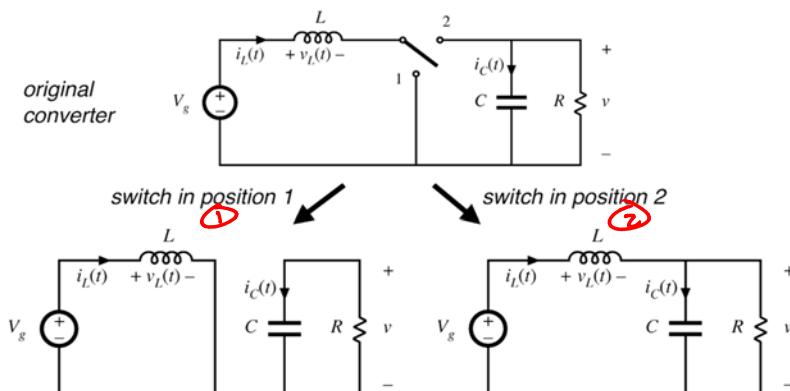


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## Boost Subintervals



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## Boost: Subinterval 1

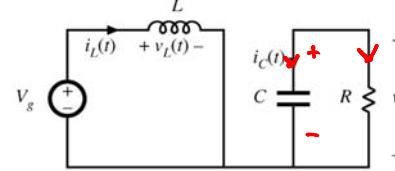
$$v_L(t) = V_g$$

$$i_c(t) = -\frac{V(t)}{R}$$

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Δ<sub>App</sub> SRA

$$v_L(t) = V_g$$

$$i_c(t) = -\frac{V}{R}$$



## Boost: Subinterval 2

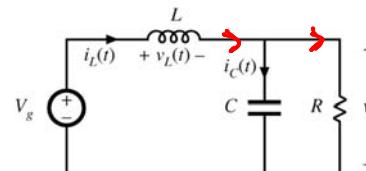
$$v_L(t) = V_g - v(t)$$

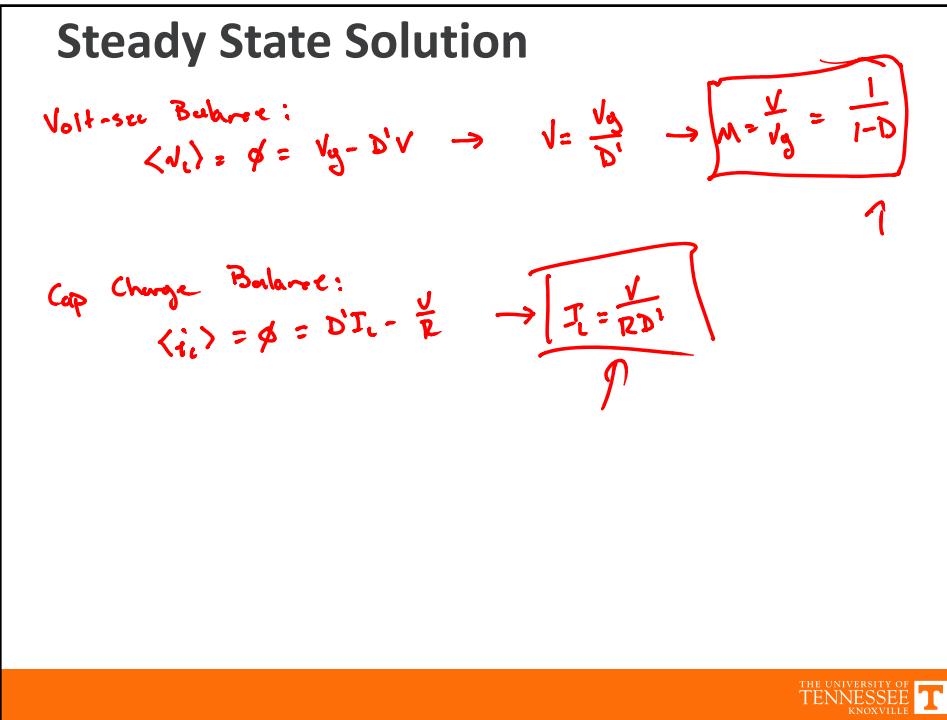
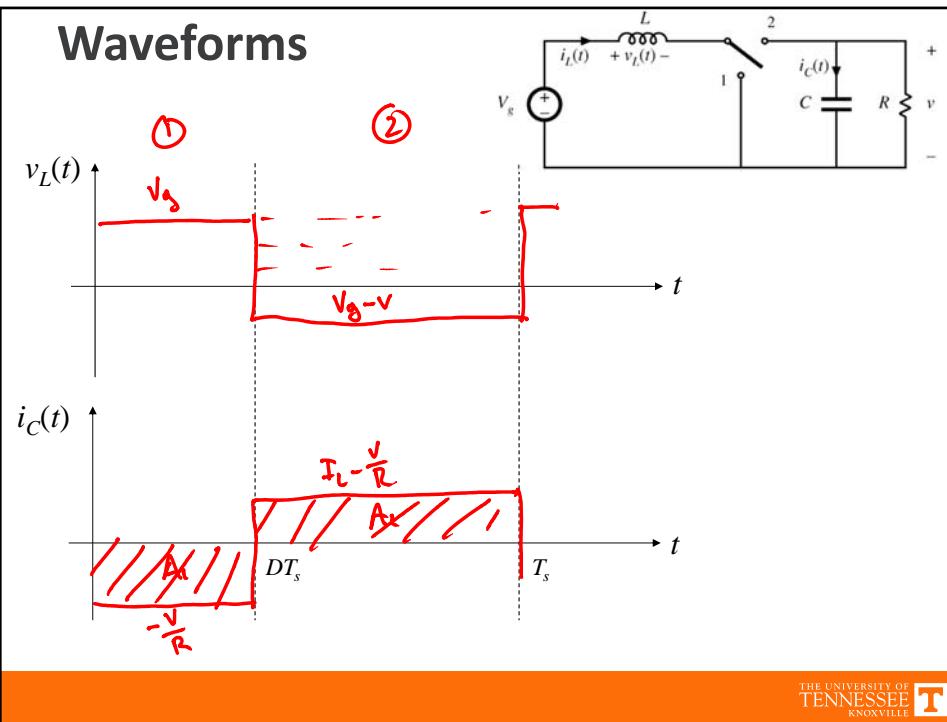
$$i_c(t) = i_L(t) - \frac{v(t)}{R}$$

|  
SRA

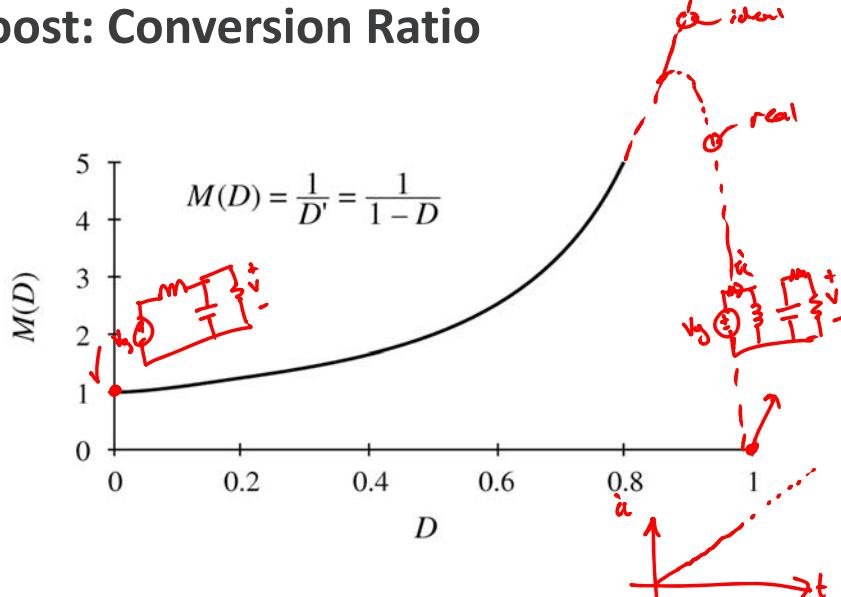
$$v_L(t) = V_g - V$$

$$i_c(t) = I_L - \frac{V}{R}$$





## Boost: Conversion Ratio



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