Finding the Conversion Ratio M(D,K)

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s}\int_0^{T_s} i_C(t)\, dt = 0$$

still apply
- true in steady-state
no approx.

Small ripple approximation sometimes applies:

$v(t) \approx V$ because $\Delta v << V$

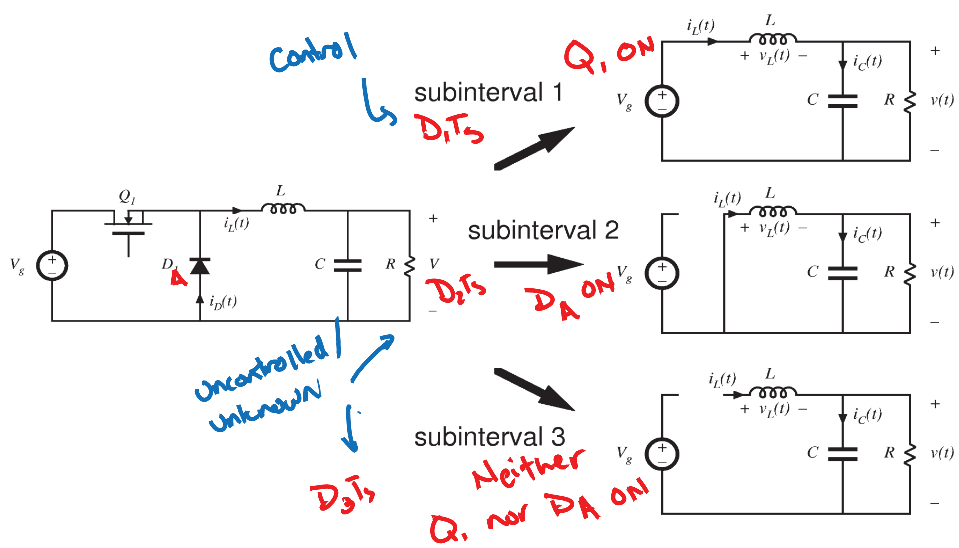
$i(t) \approx I$ is a poor approximation when $\Delta i > I$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Buck Converter in DCM

$\Delta i_L > I_L$

$\Delta v_c << V$

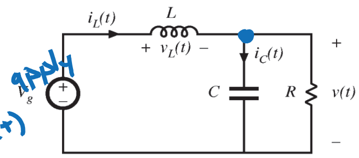


Subinterval Analysis

① $v_L(t) = V_g - v(t) \approx V_g - V$ ← SRA does apply to v_L

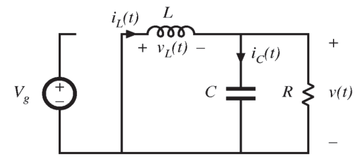
$i_C(t) = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R}$

↑ SRA doesn't apply to $i_L(t)$



② $v_L(t) = -V$

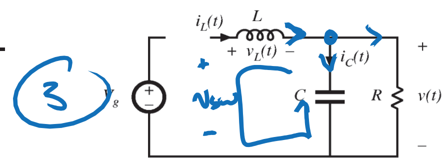
$i_C(t) = i_L(t) - \frac{V}{R}$



③ $v_L(t) = \phi$

$i_C(t) = i_L(t) - \frac{V}{R}$

$i_L(t) = \phi$



$v_L = v_{sw} - V$

Waveforms in DCM

$\langle v_L \rangle = \phi = D_1(V_g - V) - D_2 V$

$\phi = D_1 V_g - (D_1 + D_2)V$

$$\boxed{\frac{V}{V_g} = \frac{D_1}{D_1 + D_2}}$$

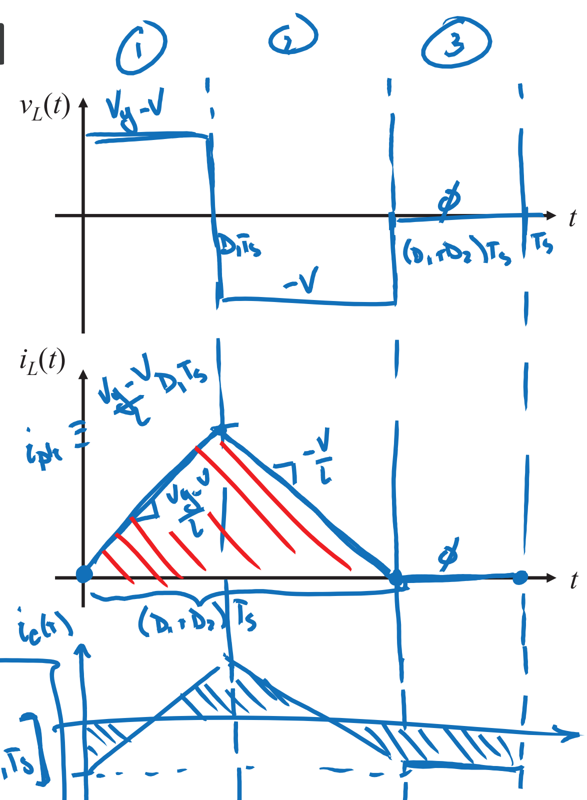
$\langle i_C \rangle = \phi = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt$

$\phi = \frac{1}{T_s}\int_0^{T_s}\left(i_L(t) - \frac{V}{R}\right)dt$

$\phi = \frac{1}{T_s}\int_0^{T_s} i_L(t)\,dt - \frac{V}{R}$

$$\boxed{\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2}(D_1 + D_2)T_s\, i_{pk}\right]}$$

$\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2}(D_1 + D_2)T_s \frac{V_g - V}{L} D_1 T_s\right]$



Solving $M(D,K)$

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2}$$ (from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L}(D_1 + D_2)(V_g - V)$$ (from capacitor charge balance)

Eliminate D_2 , solve for V :

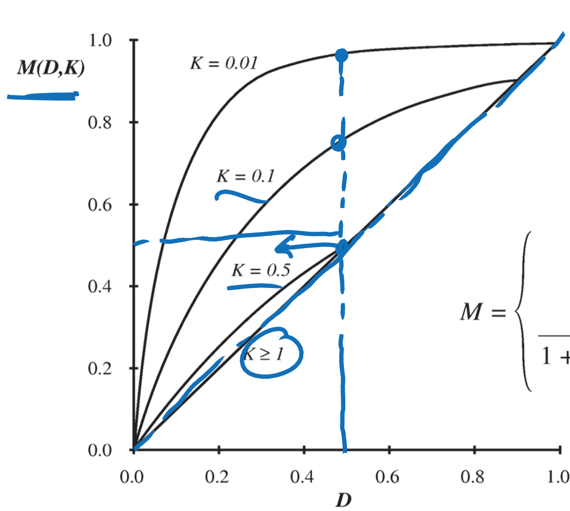
$$\boxed{\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}$

Buck Converter $M(D,K)$

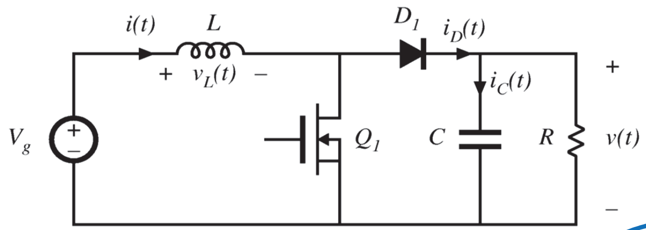
Buck in CCM $\frac{V}{V_g} = D$



$K = \frac{2L}{RT_s}$ → Design & load

$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

Boost Converter in DCM



CCM for
$I > \Delta i_L$
$\frac{\cancel{V_g}}{D'^2 R} > \frac{\cancel{V_g}}{2L} DT_s$
$\boxed{\frac{2L}{RT_s} > DD'^2}$
$\downarrow$ K $\quad \downarrow$ $K_{crit}(D)$ for Boost

Mode boundary:

$I > \Delta i_L$ *for CCM*
$I < \Delta i_L$ *for DCM*

Previous CCM soln:

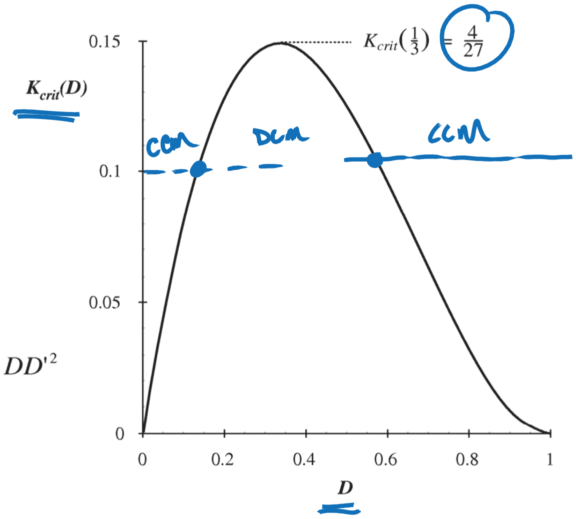
$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} DT_s$$

Boost DCM Boundary

CCM always $K > \frac{2L}{RT_s} > \frac{4}{27}$

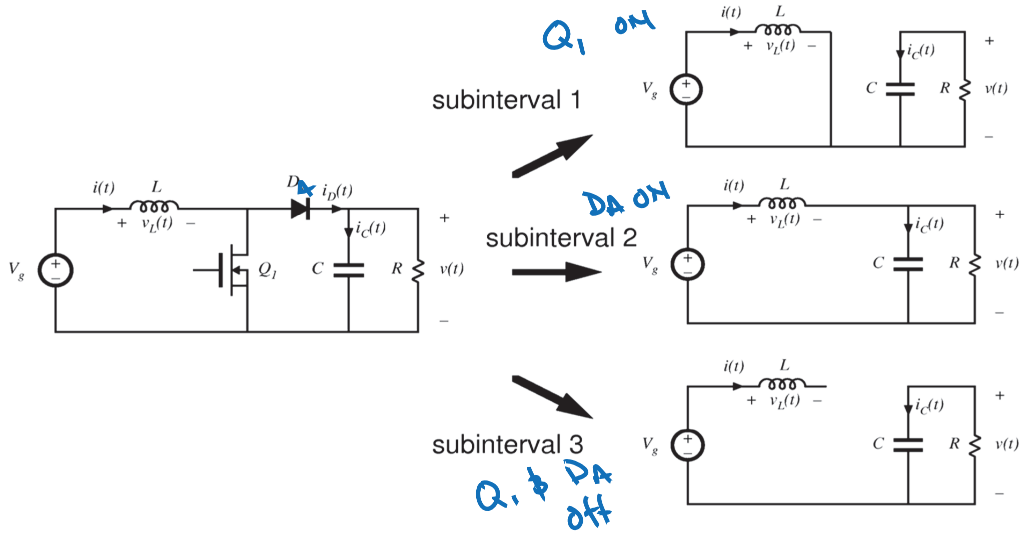
$$\frac{V_g}{D'^2 R} > \frac{DT_s V_g}{2L} \qquad \textit{for CCM}$$

$$\frac{2L}{RT_s} > DD'^2 \qquad \textit{for CCM}$$



$$K > K_{crit}(D) \qquad \textit{for CCM}$$
$$K < K_{crit}(D) \qquad \textit{for DCM}$$
$$\textit{where} \quad K = \frac{2L}{RT_s} \quad \textit{and} \quad K_{crit}(D) = DD'^2$$

Boost Converter Subintervals



Boost Conversion Ratio in DCM

① $v_L(t) = V_g$

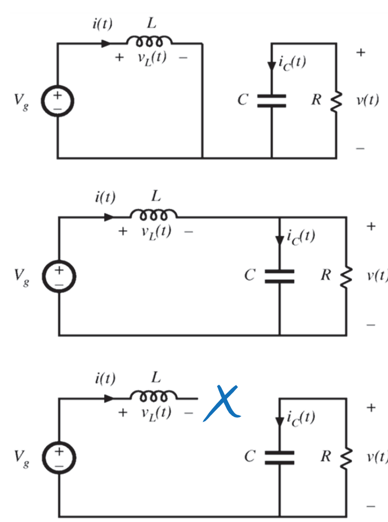
$i_C(t) = -\frac{V}{R}$

② $v_L(t) = V_g - V$

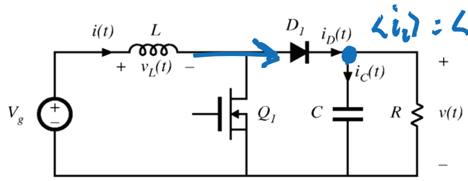
$i_C(t) = i_L(t) - \frac{V}{R}$

③ $v_L(t) = 0$

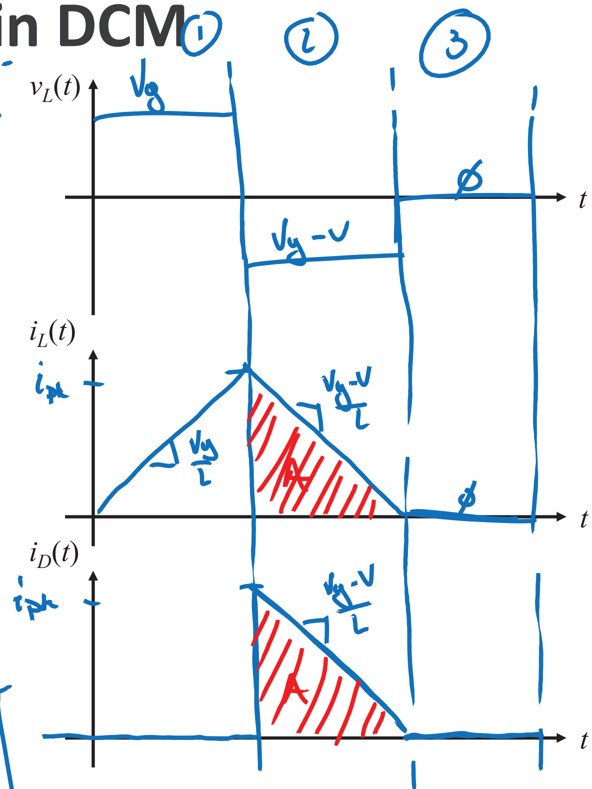
$i_C(t) = \frac{-V}{R}$



Boost Waveforms in DCM



$\langle i_c \rangle = \langle i_D \rangle - \dfrac{\langle V \rangle}{R}$



$$\langle v_L \rangle = 0 = (D_1 + D_2) V_g - D_2 V$$

$$\boxed{\frac{V}{V_g} = \frac{D_1 + D_2}{D_2}}$$

$$\langle i_c \rangle = 0 = -\frac{V}{R} + \frac{1}{T_s}\int_{D_1 T_s}^{(D_1+D_2)T_s} i_L(t)\, dt$$

$$\langle i_c \rangle = 0 = -\frac{V}{R} + \langle i_D \rangle$$

$$\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2} D_2 T_s\, i_{pk}\right]$$

$$\boxed{\frac{V}{R} = \frac{1}{T_s}\left[\frac{1}{2} D_2 T_s \frac{V_g}{L} D_1 T_s\right]}$$

Boost DCM Conversion Ratio

$$V^2 - V V_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\boxed{\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}}$$

Note that one root leads to positive V, while other leads to negative V. Select positive root:

$$\frac{V}{V_g} = \underline{M(D_1, K)} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

where $\qquad K = 2L / RT_s$

valid for $\qquad K < K_{crit}(D)$

Transistor duty cycle D = interval *1* duty cycle D_1

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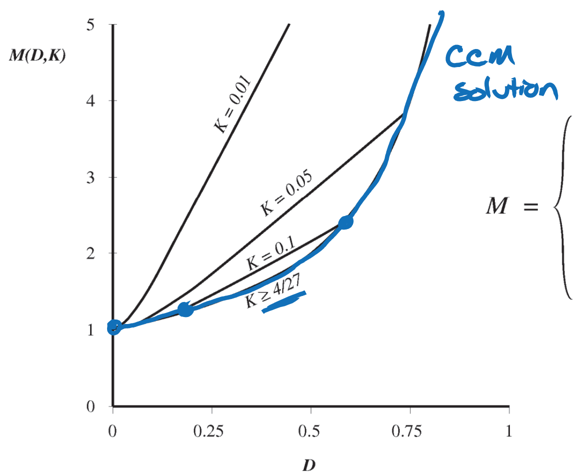
Boost Conversation Ratio



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\[2ex] \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Summary of DCM Characteristics

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	DCM $M(D,K)$	DCM $D_2(D,K)$	CCM $M(D)$
Buck	$(1-D)$	$\dfrac{2}{1+\sqrt{1+4K/D^2}}$	$\dfrac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\dfrac{1+\sqrt{1+4D^2/K}}{2}$	$\dfrac{K}{D}M(D,K)$	$\dfrac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\dfrac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\dfrac{D}{1-D}$

with $K = 2L/RT_s$. DCM *occurs for* $K < K_{crit}$.

Chapter 5 Summary

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.
2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.
3. The dc conversion ratio M of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.
4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.
5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.