Chapter 7: AC equivalent circuit modeling
Chapter 7: AC Equivalent Circuit Modeling

7.1 Introduction
7.2 The basic AC modeling approach

\{ 7.3 State-space averaging
7.4 Circuit averaging and averaged switch modeling
\}
7.5 The canonical circuit model
7.6 Modeling the pulse-width modulator
7.7 Summary of key points
Objective: maintain $v(t)$ equal to an accurate, constant value $V$.

There are disturbances:
- in $v_g(t)$
- in $R$

There are uncertainties:
- in element values
- in $V_g$
- in $R$

- loss mechanism

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Control Objectives and Inputs

\[ V_{out} \text{ control} \]
- Power supplies (DC)
- Off-grid inverter (AC)

\[ I_{out} \text{ control} \]
- Grid-tied inverter (e.g. PV)

\[ V_{in} \text{ control} : \]
- PV MPP control

\[ I_{in} \text{ control :} \]
- Grid-tied rectifier
Objectives of Part II

Develop tools for modeling, analysis, and design of converter control systems.

Need dynamic models of converters:

- How do ac variations in $v_g(t)$, $R$, or $d(t)$ affect the output voltage $v(t)$?
- What are the small-signal transfer functions of the converter?
  - Extend the steady-state converter models of Chapters 2 and 3, to include CCM converter dynamics (Chapter 7)
  - Construct converter small-signal transfer functions (Chapter 8)
  - Design converter control systems (Chapter 9)

- Design input EMI filters that do not disrupt control system operation (Chapter 10)

- Model converters operating in DCM (Chapter 11)
- Current-programmed control of converters (Chapter 12)
PWM Spectrum

\[ f_s = 150kHz \]

\[ G_{ud}(s) = \frac{v}{d} \]

\[ D = 0.5 \]

\[ d(t) = 0.5 + D_m \cos(2\pi f_mt) \]

\[ f_m = 50 \text{ kHz} \]

\[ f_s \]

\[ 3f_s \]

\[ f_m < f_s \]

What we want to control

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Neglecting The Switching Ripple

Suppose the duty cycle is modulated sinusoidally:

\[ d(t) = D + D_m \cos \omega_m t \]

where \( D \) and \( D_m \) are constants, \( |D_m| \ll D \), and the modulation frequency \( \omega_m \) is much smaller than the converter switching frequency \( \omega_s = 2\pi f_s \).

The resulting variations in transistor gate drive signal and converter output voltage:

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Output Voltage Spectrum

Contains frequency components at:
- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.
Objectives of AC Modeling

- Predict how low-frequency variations in duty cycle induce low-frequency variations in the converter voltages and currents
- Ignore the switching ripple
- Ignore complicated switching harmonics and sidebands

Approach:
- Remove switching harmonics by averaging all waveforms over one switching period
Low-frequency Averaging

Average over one switching period to remove switching ripple:

\[
L \frac{d\langle i_L(t) \rangle_{T_s}}{dt} = \langle v_L(t) \rangle_{T_s}
\]

\[
C \frac{d\langle v_C(t) \rangle_{T_s}}{dt} = \langle i_C(t) \rangle_{T_s}
\]

where

\[
\langle x(t) \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} x(\tau) d\tau
\]

Note that, in steady-state,

\[
\langle v_L(t) \rangle_{T_s} = 0
\]

\[
\langle i_C(t) \rangle_{T_s} = 0
\]

by inductor volt-second balance and capacitor charge balance.

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Averaging in Steady-State

In steady-state:

\[ i_c(0) = i_c(T_s) \]

\[ i_c(T_s) = i_c(0) + \frac{v_g(t)}{L} DT_s + \frac{v_d(t) - v(t)}{L} DT_s \]

\[ \frac{L}{T_b} [i_c(T_s) - i_c(0)] = (D + D') v_g(t) - D' v(t) \] (small ripple)

\[ \frac{L}{T_b} \frac{\Delta i_c}{\Delta t} = \frac{L}{T_b} [i_c(T_s) - i_c(0)] = v_g - D' v \]