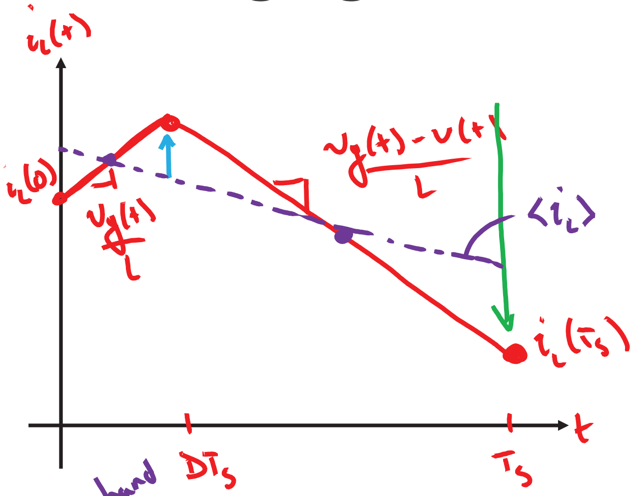
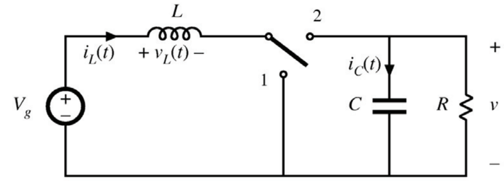
Averaging in Transient Operation





$$i_L(T_s) = i_L(0) + \frac{v_g(t)}{L} DT_s + \frac{v_g(t) - v(t)}{L} D'T_s$$

Small Ripple $\rightarrow$ Averaging over T_s

short-hand $\langle x(t) \rangle_{T_s} \updownarrow \langle x \rangle$

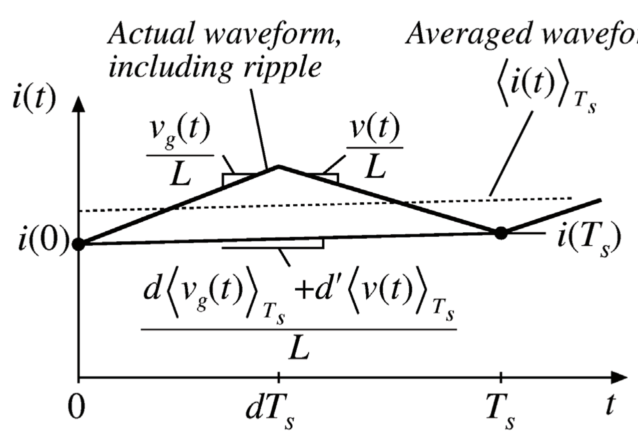
$$\frac{L}{T_s}\left[i_L(T_s) - i_L(0)\right] = D\langle v_g(t)\rangle_{T_s} + D'\left(\langle v_g(t)\rangle_{T_s} - \langle v(t)\rangle_{T_s}\right)$$

$$\langle v_L \rangle = \frac{L}{T_s}\left[i_L(T_s) - i_L(0)\right] = \langle v_g \rangle - D'\langle v \rangle$$

$$\langle v_L \rangle = L\frac{d\langle i_L \rangle}{dt} = \langle v_g \rangle - d'(t)\langle v \rangle$$

Averaging: Correct Prediction

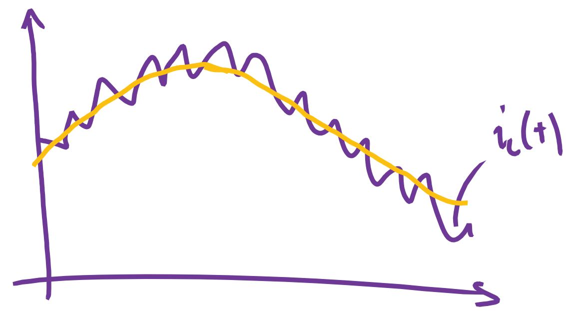
Buck-Boost



The net change in inductor current over one switching period is exactly equal to the period T_s multiplied by the average slope $\langle v_L \rangle_{T_s}/L$.

Averaging: Discussion

Averaging will yield a correct prediction of <u>averaged</u> behavior



<u>Limitations</u>:

(1) Ripple << average value

(2) $\omega_m << \omega_s$

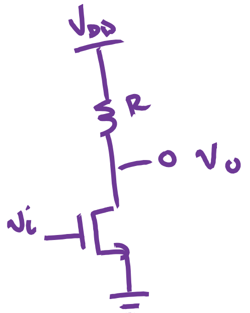
(3) circuit may still be nonlinear

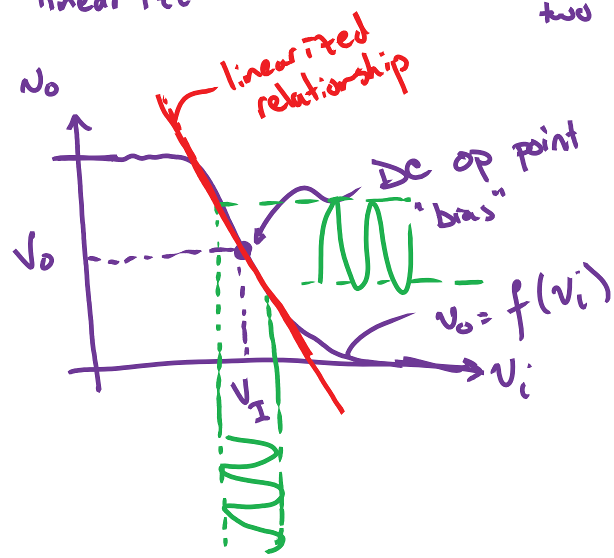
Small Signal Modeling: Linearization

$$\langle v_L \rangle = L \frac{d\langle i_L \rangle}{dt} = \langle v_g \rangle - \underbrace{d'(t)\langle v \rangle}$$

↳ Nonlinear, product of two time-varying signals

- want to linearize





$\hat{v}_i << V_I$

Small signal linearization

if $v_o = f(v_i)$

$$v_o \approx f(V_I) + \hat{v}_i \left.\frac{\partial f}{\partial v_i}\right|_{dc}$$

$\updownarrow$

V_o

"hats" denote small-signal variables

(1) Perturb and Linearize

$$L\frac{d\langle i_L\rangle}{dt} = \langle v_g\rangle - d'(t)\langle v\rangle$$

$\hat{x} \ll X$

① Replace all $\langle x\rangle \rightarrow X + \hat{x}$

X: DC op point; $\hat{x}$: AC small signal

② Eliminate all 2nd order or higher terms (nonlinear)

$$L\frac{d}{dt}(I_L + \hat{i}_L) = (V_g + \hat{v}_g) - (1 - D - \hat{d})(V + \hat{v})$$

$$L\frac{dI_L}{dt} + L\frac{d\hat{i}_L}{dt} = V_g + \hat{v}_g - V - \hat{v} + DV + D\hat{v} + \hat{d}V + \cancel{\hat{d}\hat{v}}$$

Neglect 2nd order terms

DC: $L\frac{dI_L}{dt} = V_g - D'V$

AC: $\boxed{L\frac{d\hat{i}_L}{dt} = \hat{v}_g - D'\hat{v} + \hat{d}V}$

(2) 1st order Taylor Series Expansion

Recall Taylor Series:

$$f(x_1, x_2, \dots) \approx f(X_1, X_2, \dots) + \hat{x}_1 \frac{\partial f}{\partial x_1}\bigg|_{DC} + \hat{x}_2 \frac{\partial f}{\partial x_2}\bigg|_{DC} + \dots$$

+ 2nd order & higher terms

$$L\frac{d\langle i_L\rangle}{dt} = \overbrace{\langle v_g\rangle - d'(t)\langle v\rangle}^{f(v_g, d, v)} =$$

AC: $\boxed{L\frac{d\hat{i}_L}{dt} = \underbrace{\hat{v}_g(1)}_{\frac{\partial f}{\partial \langle v_g\rangle}} + \underbrace{\hat{v}(-D')}_{\frac{\partial f}{\partial \langle v\rangle}} + \underbrace{\hat{d}(V)}_{\frac{\partial f}{\partial \langle d\rangle}}}$