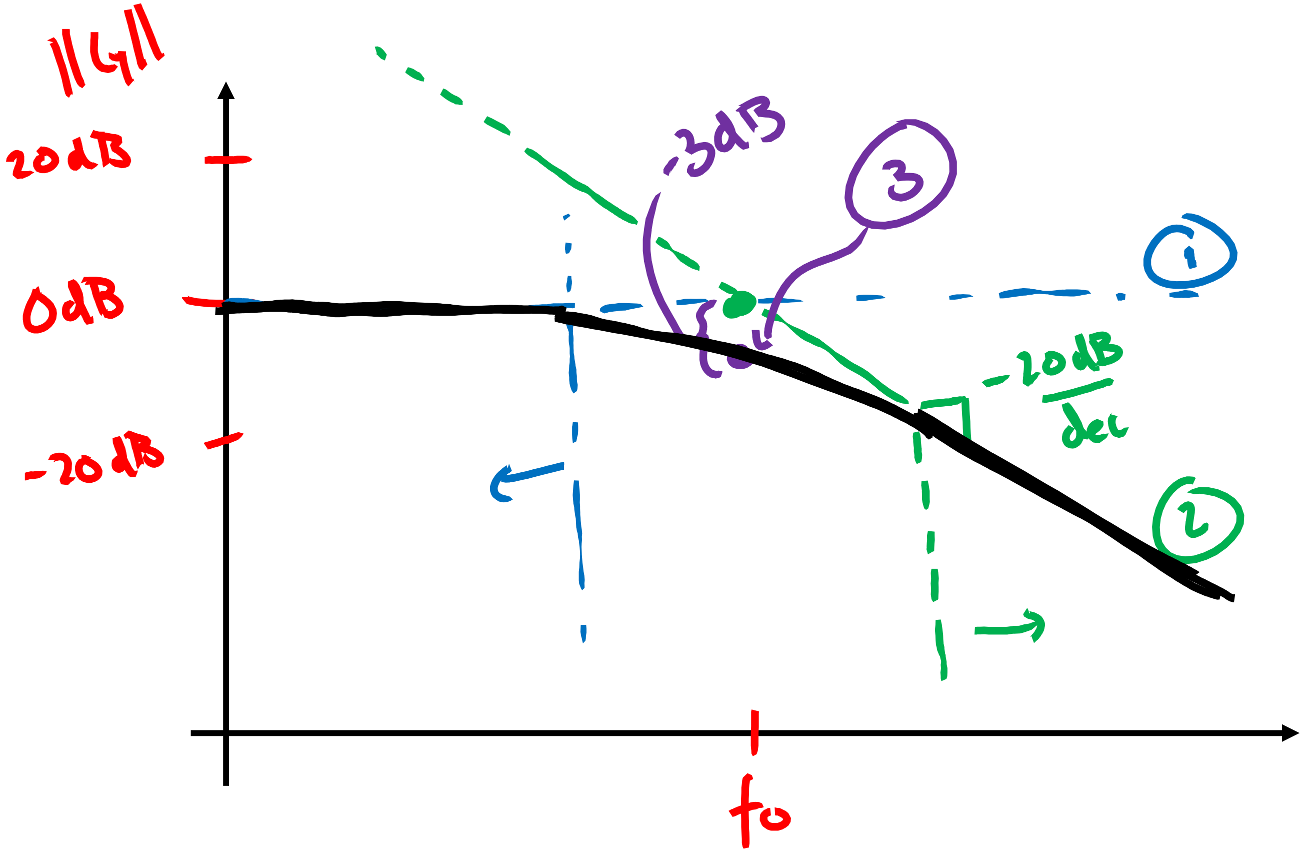
Plotting a Single Pole Response



$$\|G_{21}\| = \sqrt{\frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\|G_{21}\| \approx \begin{cases} 0\,dB, & \omega \ll \omega_0 \\ -3\,dB, & \omega = \omega_0 \\ 20\log\frac{\omega_0}{\omega}, & \omega \gg \omega_0 \end{cases}$$

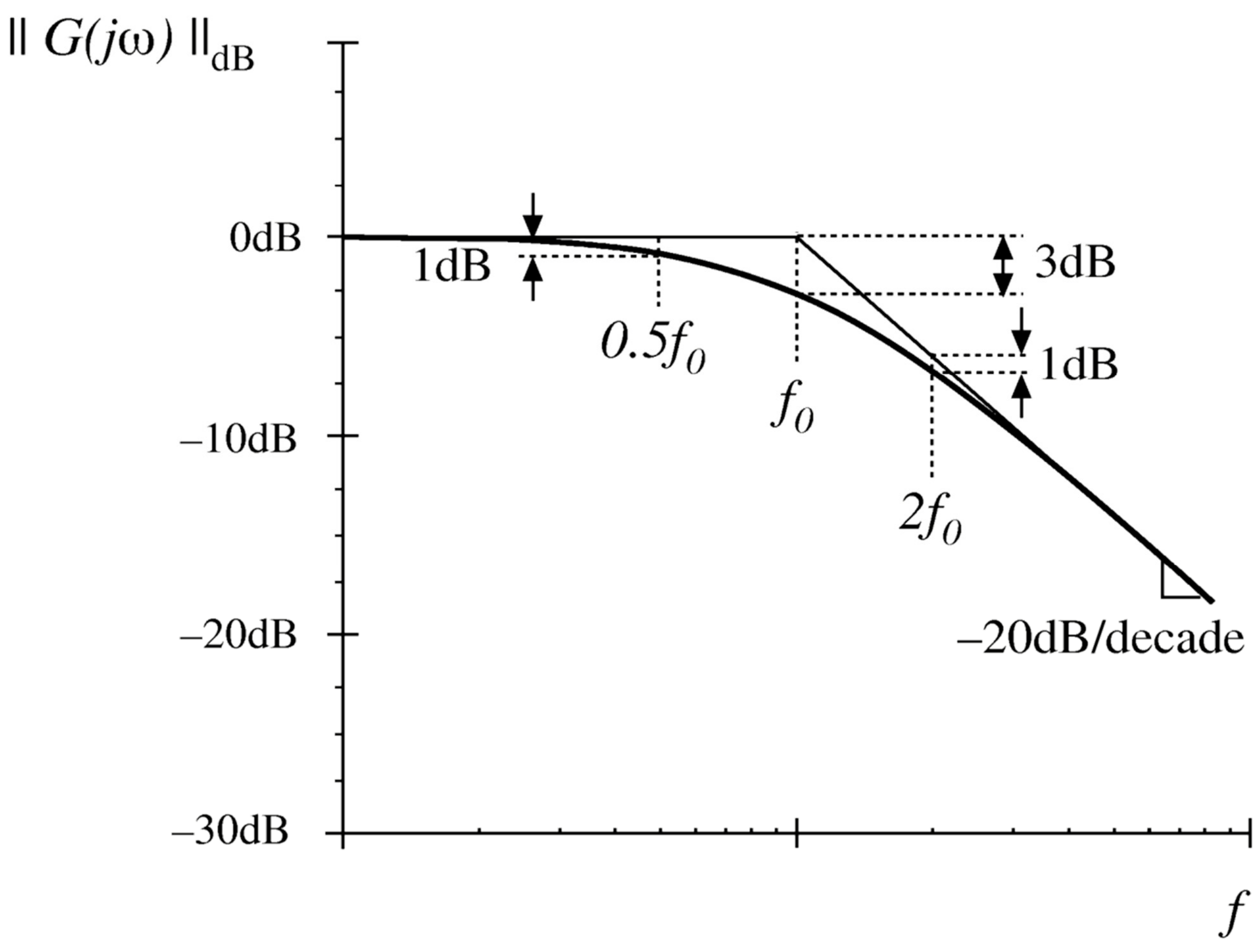
1 $\omega \ll \omega_0 \rightarrow \left(\frac{\omega}{\omega_0}\right)^2 \ll 1 \rightarrow \|G_{21}\| = \sqrt{\frac{1}{1}} = 0\,dB$

2 $\omega \gg \omega_0 \rightarrow 1+\left(\frac{\omega}{\omega_0}\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^2 \rightarrow \|G_{21}\|_{dB} = 20\log\left(\frac{\omega_0}{\omega}\right)$

$= 20\log\omega_0 - 20\log\omega$

3 $\omega = \omega_0 \rightarrow \|G_{21}\| = 20\log\sqrt{\frac{1}{2}} \approx -3\,dB$

Summary: Single Pole Magnitude



Phase of Single Pole

$$\measuredangle G(j\omega) = \tan^{-1}\left(\frac{Im(G(j\omega))}{Re(G(j\omega))}\right)$$

Phase operator identities

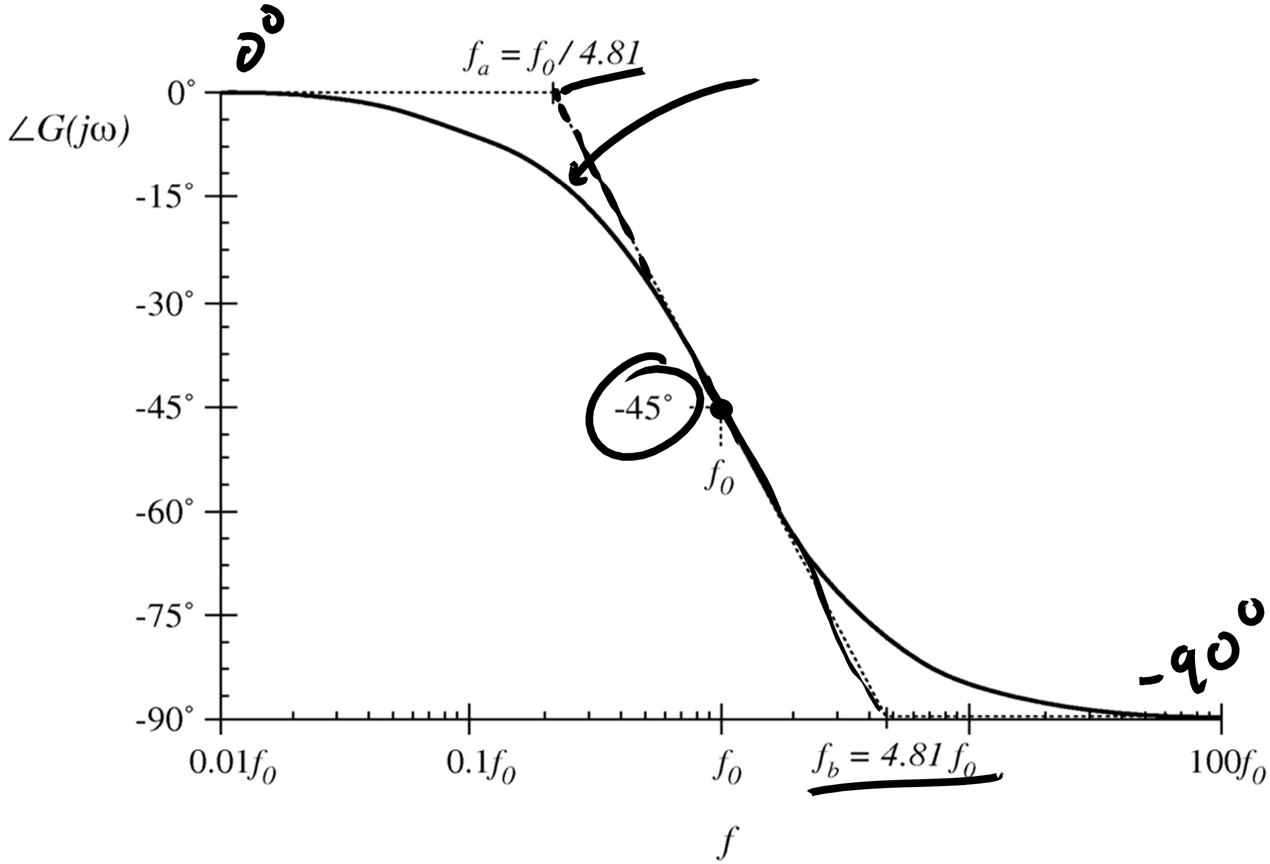
① $\measuredangle\, k\,G(j\omega) = \measuredangle\, G(j\omega)$

② $\measuredangle\, G_1(j\omega)\cdot G_2(j\omega) = \measuredangle\, G_1(j\omega) + \measuredangle\, G_2(j\omega)$

Phase Asymptotes

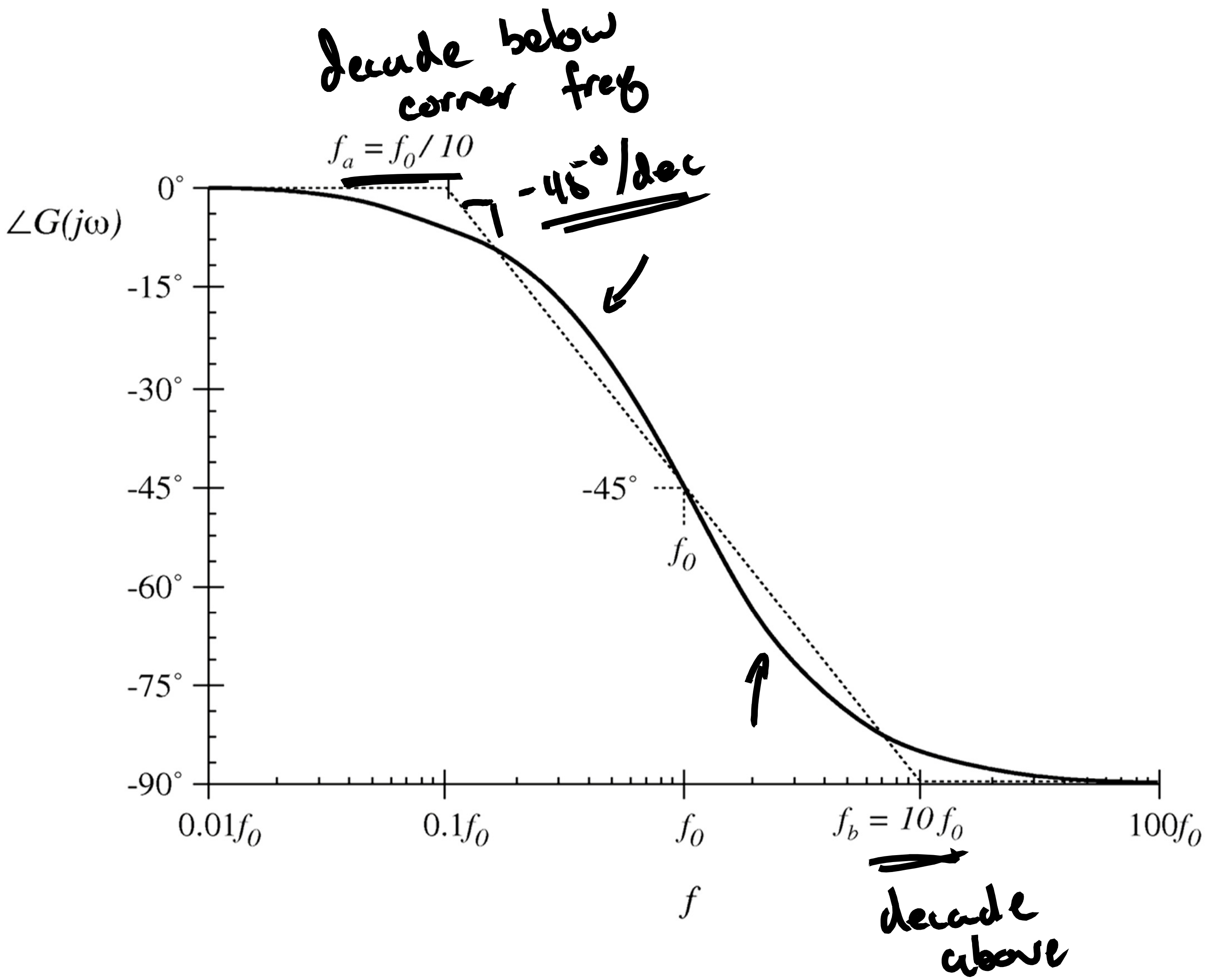
hard to remember

$$f_a = f_0\, e^{-\pi/2} \approx f_0 / 4.81$$
$$f_b = f_0\, e^{\pi/2} \approx 4.81\, f_0$$

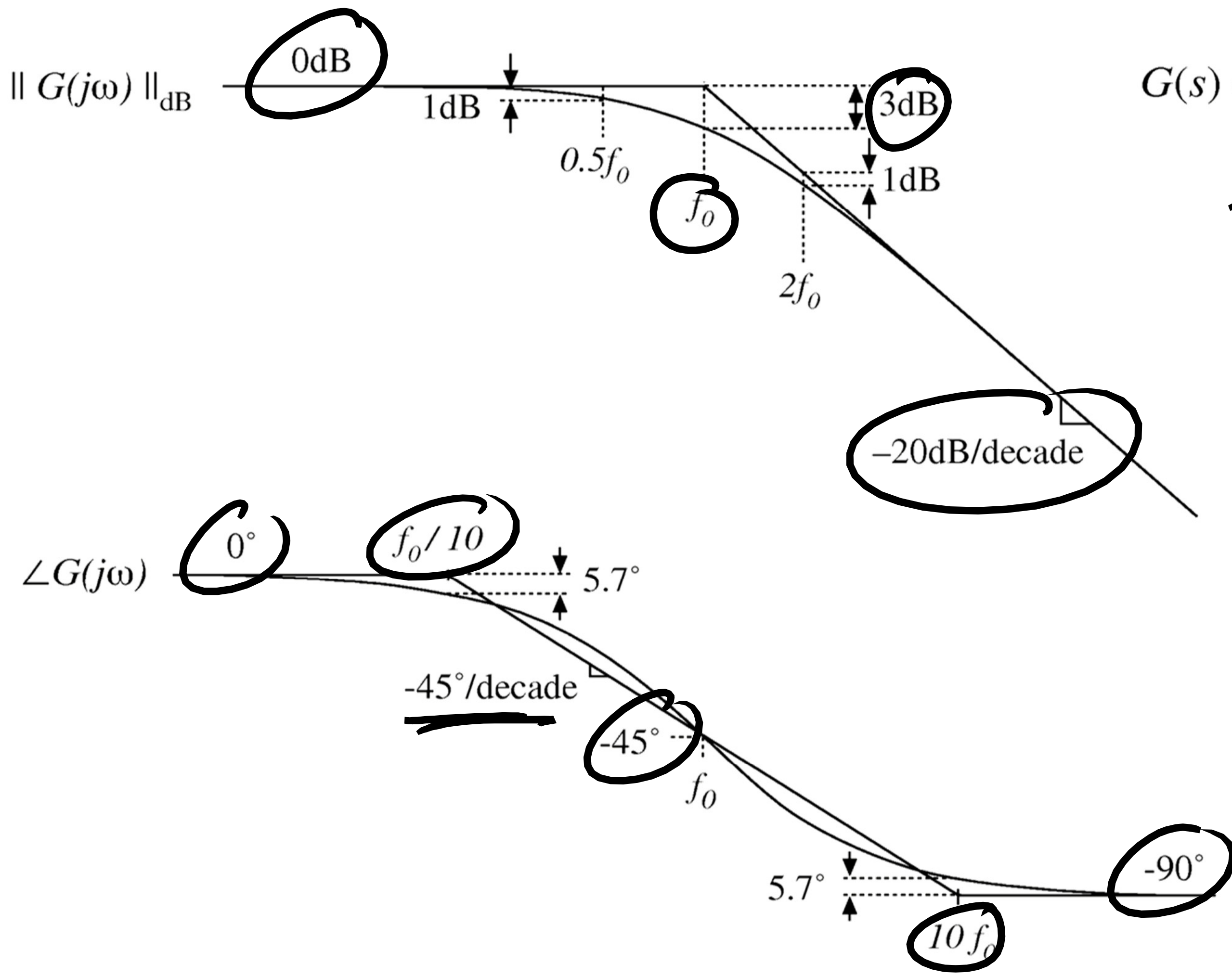


Phase Asymptotes: A Simpler Choice

$f_a = f_0 / 10$
$f_b = 10\, f_0$

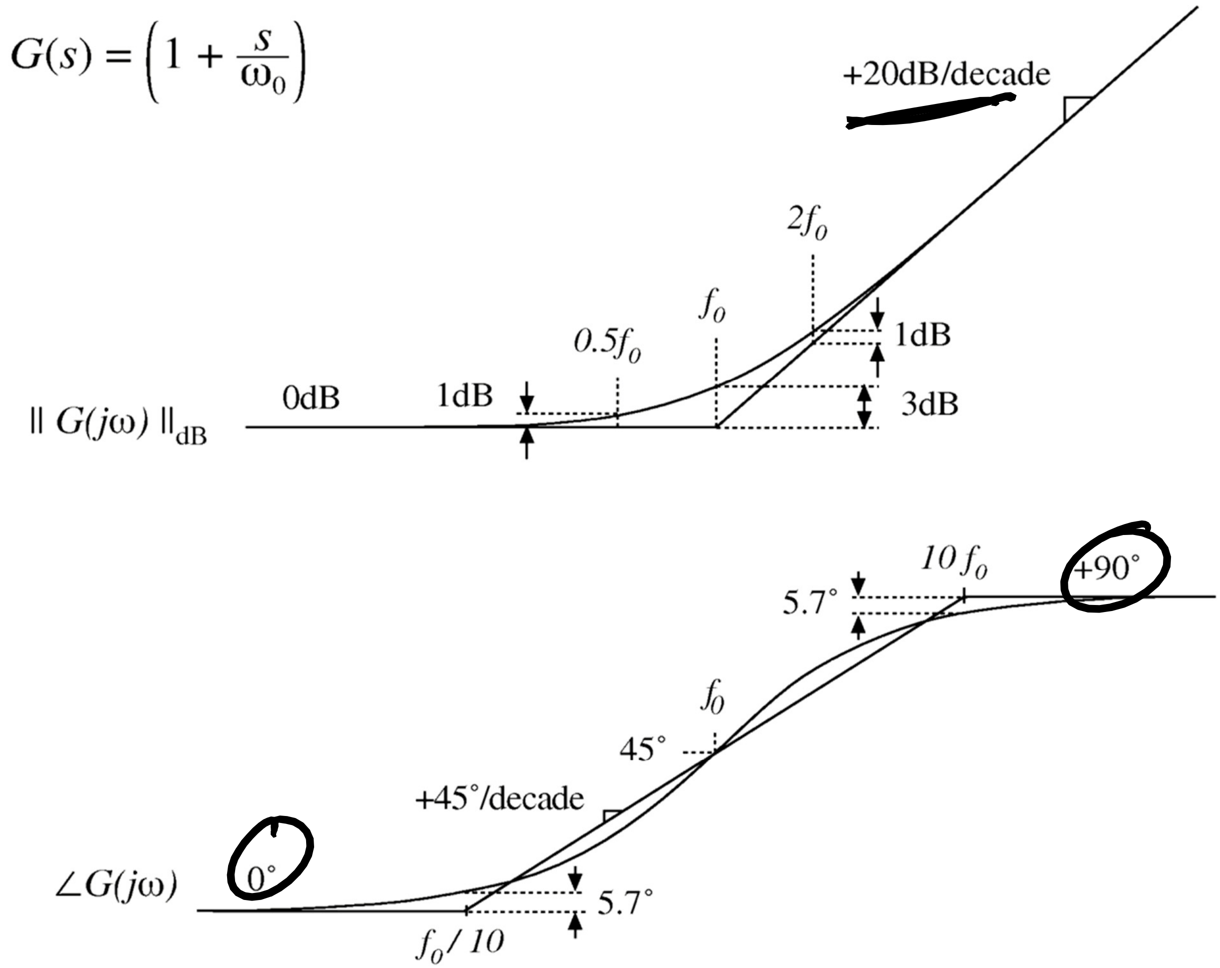


Summary: Single Real Pole



$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$

Bode Plot: Real Zero

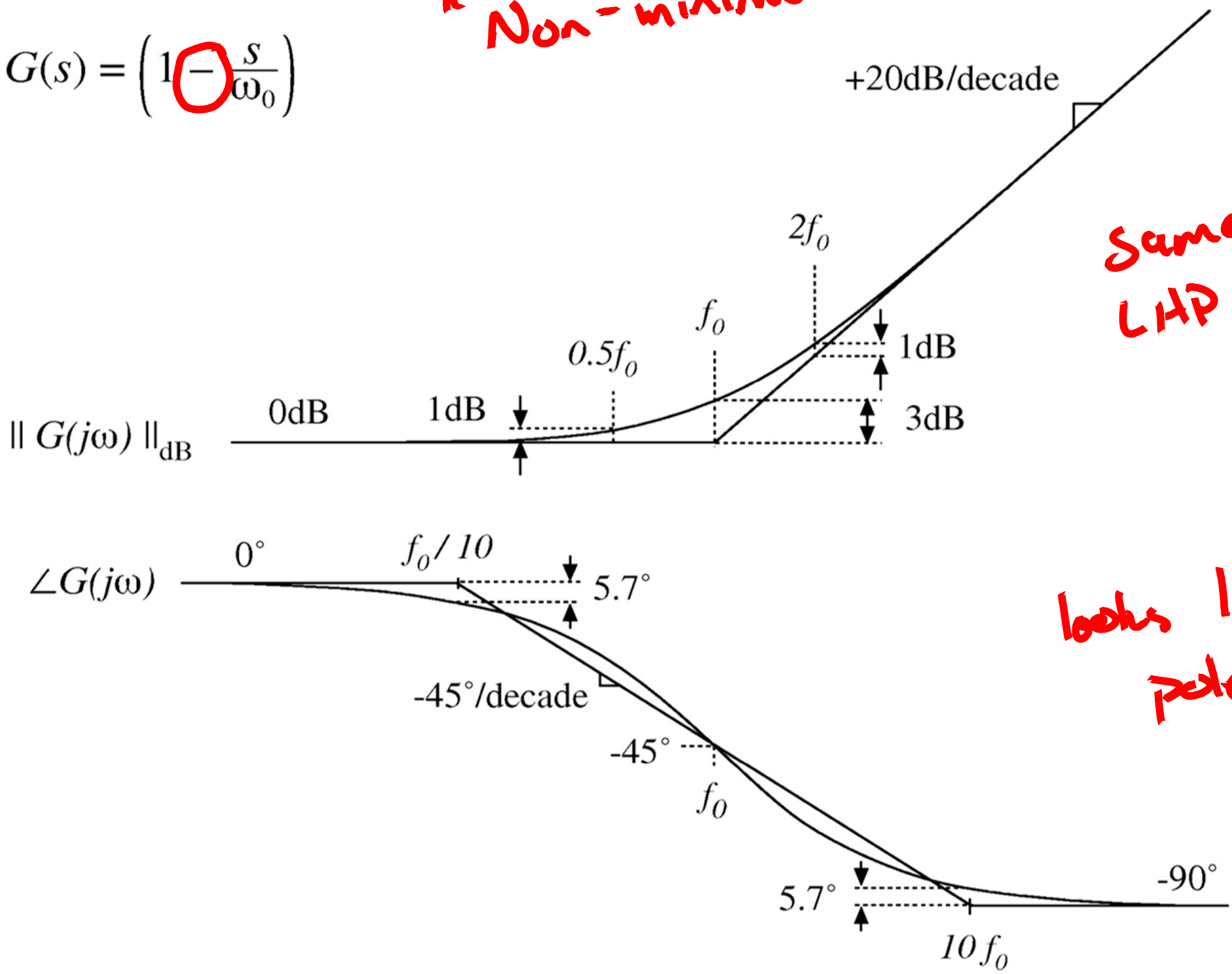


RHP Zero

"Right Half Plane"

"Non-minimum Phase"

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$



Same as LHP zero

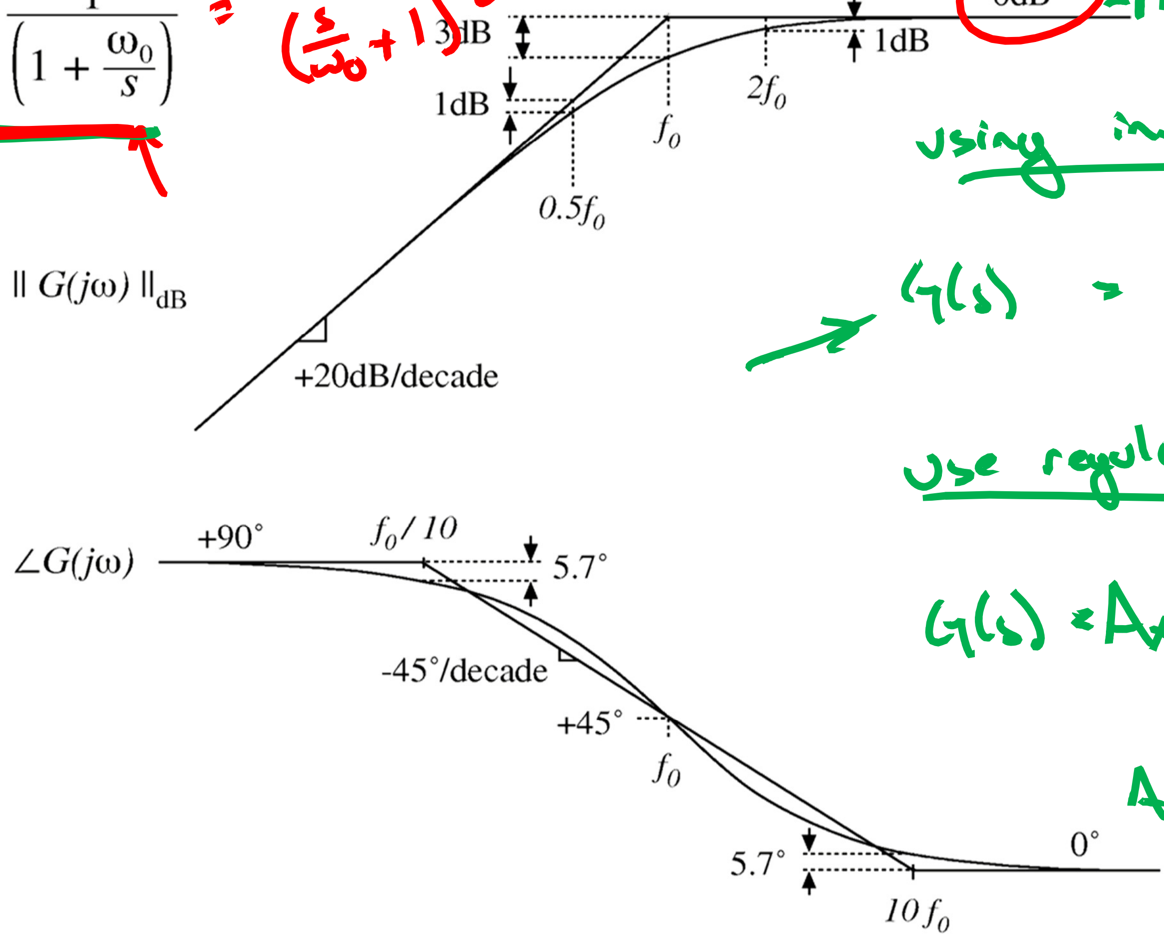
looks like a pole

Inverted Pole

zero @ DC

Pole

$$G(s) = \frac{1}{\left(1 + \frac{\omega_0}{s}\right)} = \frac{s/\omega_0}{\left(\frac{s}{\omega_0} + 1\right)}$$



$= \|A_\infty\|$

using inverted pole

$$G(s) = \frac{A_\infty}{1 + \frac{\omega_0}{s}}$$

use regular zero/pole

$$G(s) = A_x \frac{s}{\left(1 + \frac{s}{\omega_0}\right)}$$

$$A_x = \frac{A_\infty}{\omega_0}$$

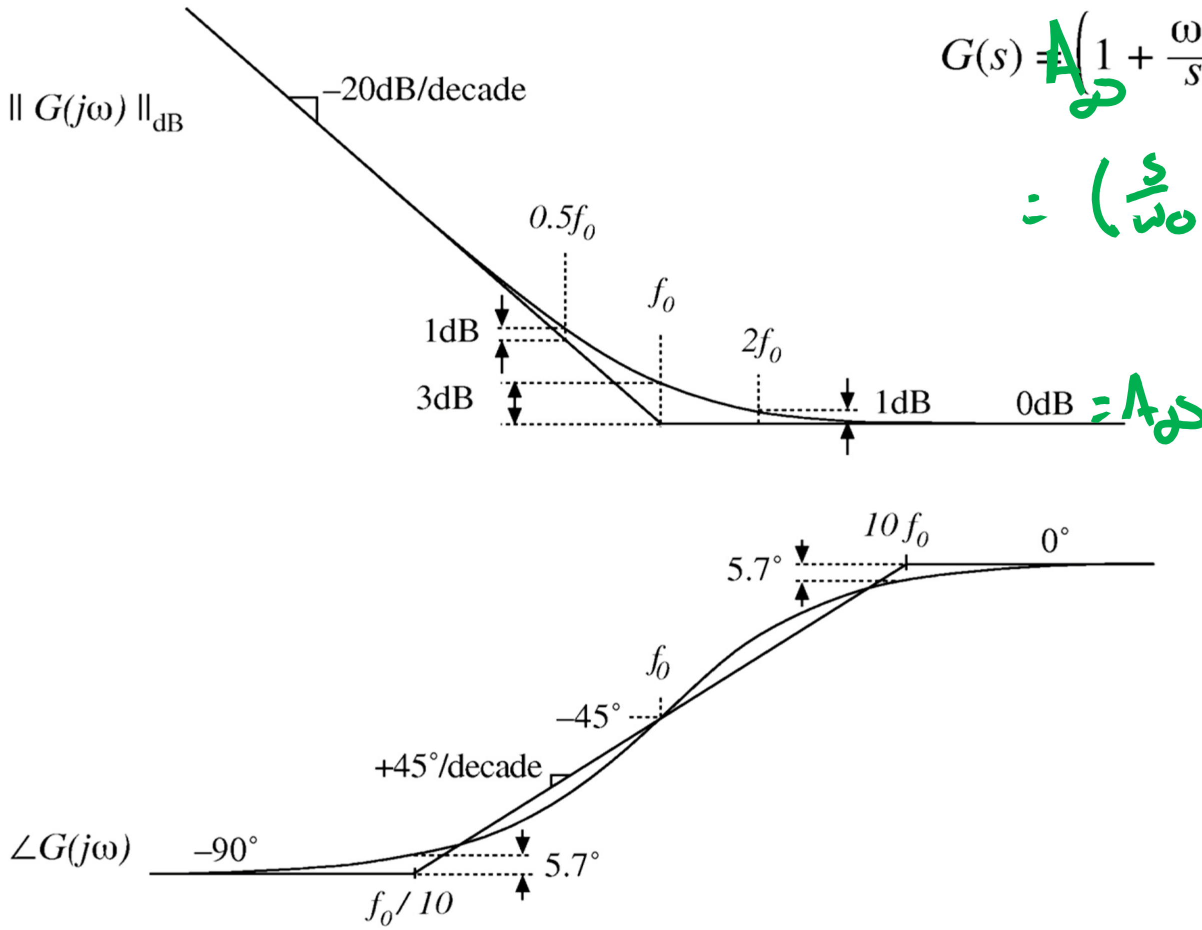
Inverted Zero

$$G(s) = A_0\left(1 + \frac{\omega_0}{s}\right)$$

$$= \left(\frac{s}{\omega_0} + 1\right)\frac{\omega_0}{s}$$

zero

DC pole



Multiplying Transfer Functions

$$G_1(s) = A_1 e^{s\theta_1}$$

$$G_2(s) = A_2 e^{s\theta_2}$$

$$G_1(s) \cdot G_2(s) = A_1 A_2 e^{s(\theta_1 + \theta_2)}$$ → phases add, magnitudes multiply

$$\|G_1 \cdot G_2\|_{dB} = 20\log\left|A_1 A_2 e^{s(\theta_1+\theta_2)}\right|$$

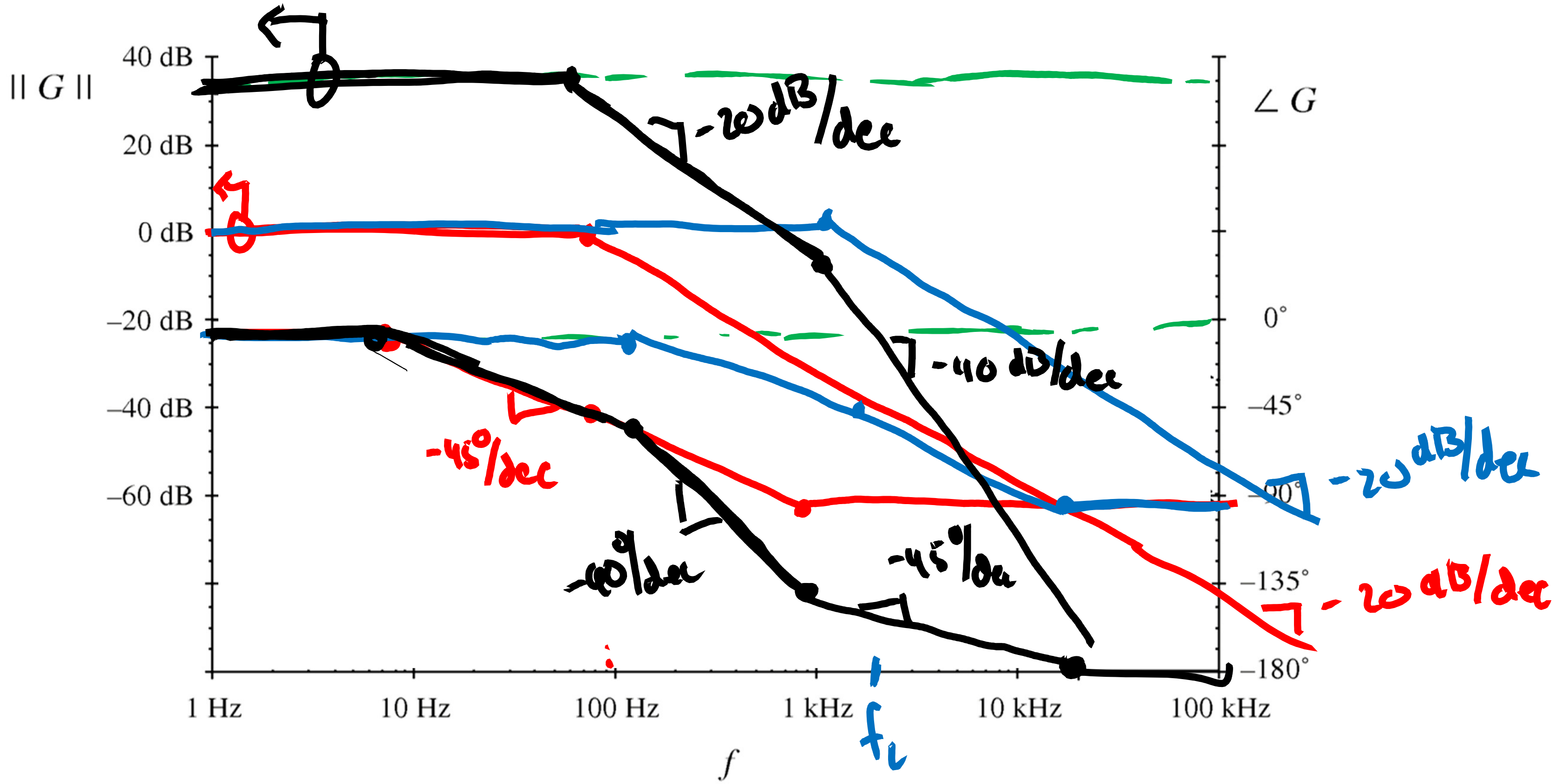
$$= \underbrace{20\log|A_1| + 20\log|A_2|}_{\text{zero phase}} + \underbrace{20\log\left|e^{s(\theta_1+\theta_2)}\right|}_{\text{magnitude} = 1 = 0\text{dB}}$$

magnitudes add in log

Example 1

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

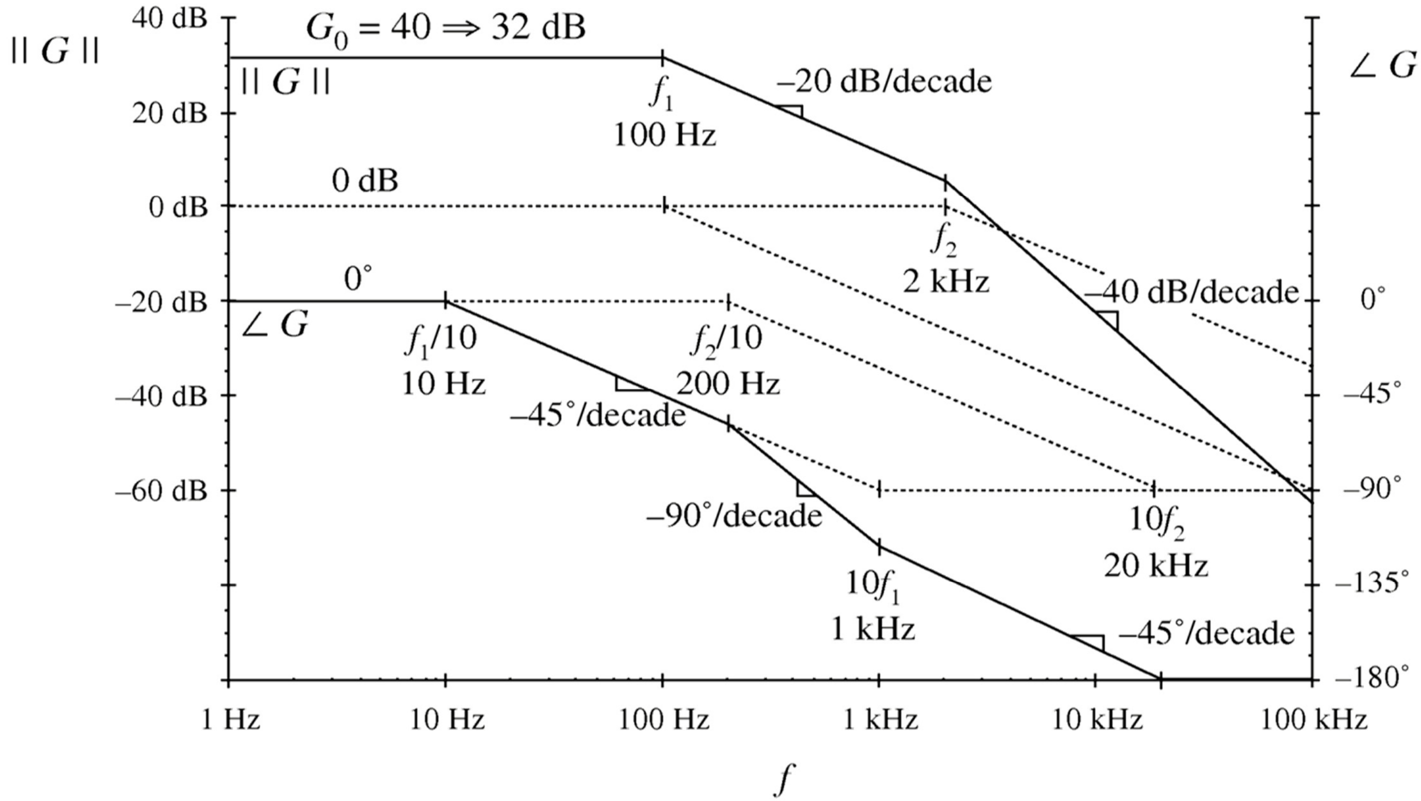
with $G_0 = 40 \Rightarrow 32$ dB, $f_1 = \omega_1/2\pi = 100$ Hz, $f_2 = \omega_2/2\pi = 2$ kHz



Example 1

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

with $G_0 = 40 \Rightarrow 32\ \text{dB}, f_1 = \omega_1/2\pi = 100\ \text{Hz}, f_2 = \omega_2/2\pi = 2\ \text{kHz}$

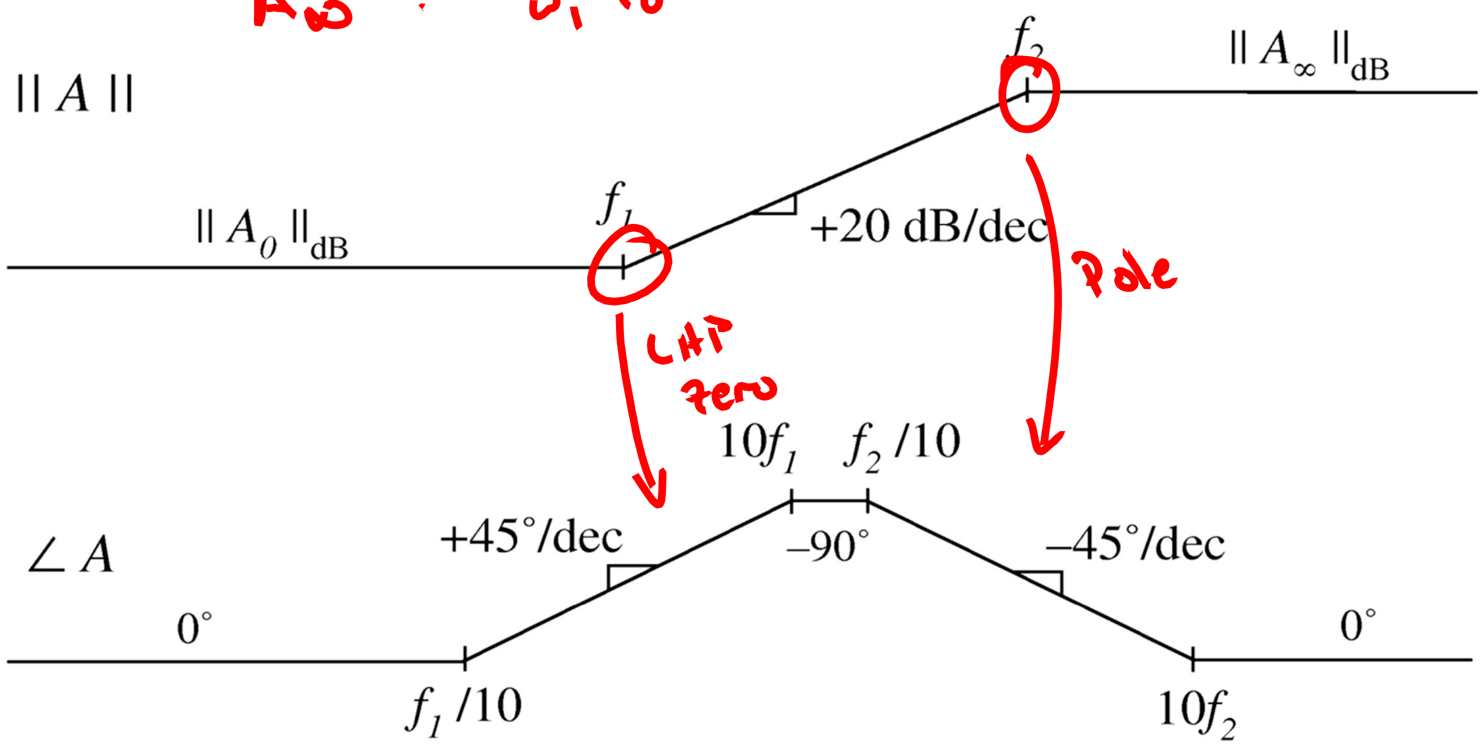


Example 2

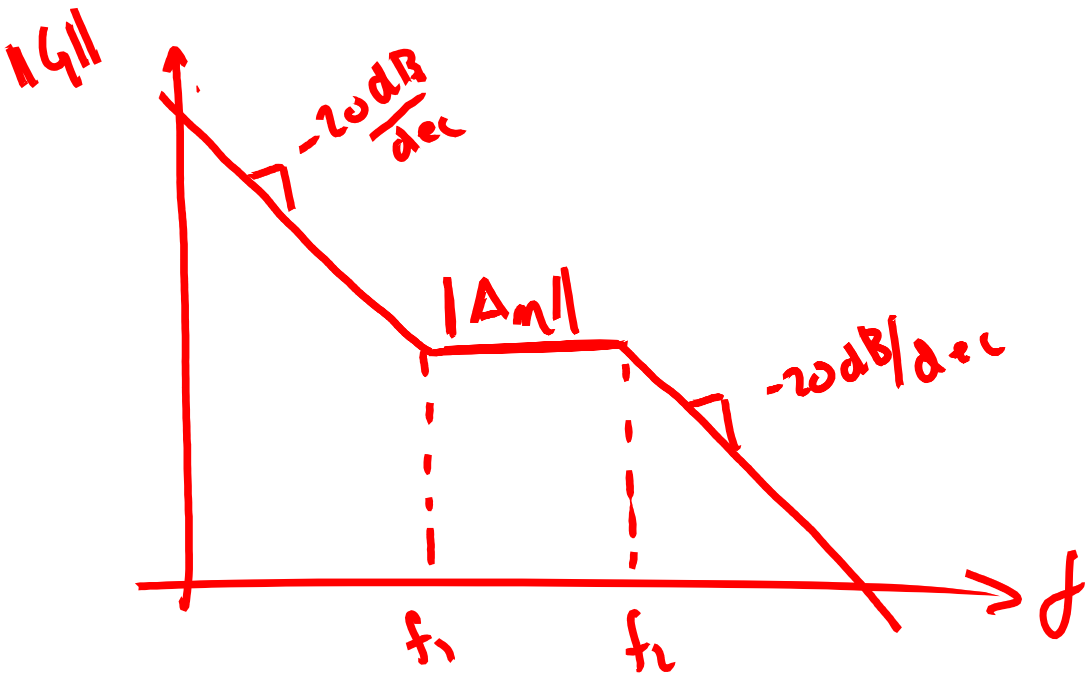
$G(s) = A_0 \frac{\left(1+\frac{s}{\omega_1}\right)}{\left(1+\frac{s}{\omega_2}\right)} \iff G(s) = A_\infty \frac{\left(1+\frac{\omega_1}{s}\right)}{\left(1+\frac{\omega_2}{s}\right)}$

Determine the transfer function $A(s)$ corresponding to the following asymptotes:

$A_\infty = ? = \frac{\omega_2}{\omega_1} A_0$



Example 3



$$G(s) = \underline{A_m} \, \frac{1 + \frac{\omega_1}{s}}{1 + \frac{s}{\omega_2}}$$

inv. zero

pole

$$G(s) = A_x \frac{1}{s} \, \frac{\left(1 + \frac{s}{\omega_1}\right)}{\left(1 + \frac{s}{\omega_2}\right)}$$

$$\underline{A_x = ?}$$