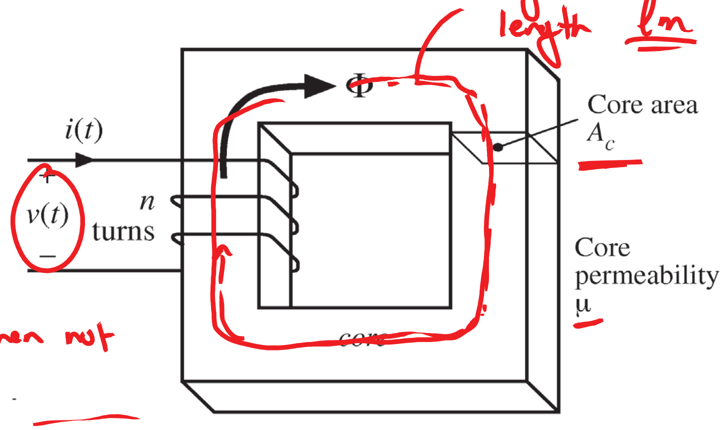
Inductor Example

Simplifying assumptions:

1. $\mu \gg \mu_0$ & flux stays entirely within the core
2. H & B fields are uniform throughout the core
3. $B = \mu H$ in the core (when not saturated)



Faraday:

$$V_{turn} = A_c \frac{dB(t)}{dt}$$

$$\boxed{v(t) = n A_c \frac{dB(t)}{dt}}$$

Ampere:

$$H l_m = I_{enclosed}$$

$$\mathcal{F}_w = \boxed{H l_m = n i(t)}$$

Core Material:

$$\boxed{B = \mu H} \quad (\text{if } \max|B| < B_{sat})$$

$$v(t) = n A_c \frac{d}{dt}\left(\mu \frac{n i(t)}{l_m}\right) = \underbrace{\frac{n^2 A_c \mu}{l_m}}_{L} \frac{di(t)}{dt}$$

Magnetic Circuits

$V \leftrightarrow \mathcal{F}$

Based on conservative fields ($\vec{E}$ / $\vec{H}$)

Potential independent of path

↓

KVL

$\Phi \leftrightarrow I$

Always flows in closed loop

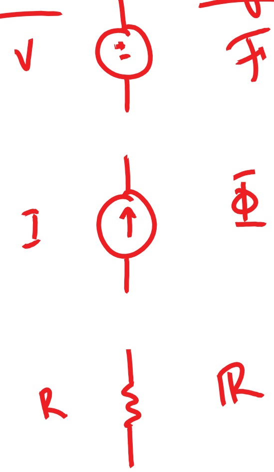
no single point src/sink

↓

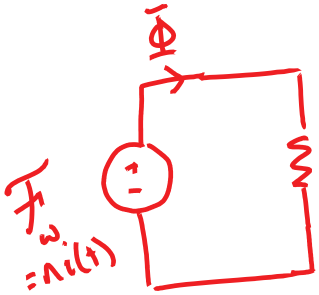
KCL

$R \leftrightarrow \mathcal{R}$

Electric ⟷ Magnetic



Inductor Magnetic Circuit Model



$\mathcal{R}_c = \frac{1}{\mu} \frac{l_m}{A_c}$

$$\Phi = \frac{\mathcal{F}_w}{\mathcal{R}_c} = \frac{n\, i(t)}{\frac{l_m}{\mu A_c}}$$

$$\Phi = \frac{n \mu A_c}{l_m} i(t)$$

Faraday: $v(t) = n \frac{d\Phi}{dt}$

$$v(t) = n \frac{d}{dt}\left(\frac{n \mu A_c}{l_m} i(t) \right)$$

$$\boxed{v(t) = \underbrace{\frac{n^2 \mu A_c}{l_m}}_{L} \frac{di}{dt}}$$ ✓

Saturation Limits

(Gapless Inductor)

Process: first assume inductor is not saturated, analyze using $B = \mu H$ → afterwards check if $|B(t)| > B_{sat}$

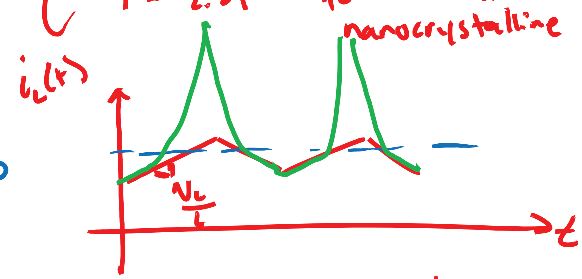
typical B_{sat} { 0.2 - 0.8T for ferrite; 1 - 2.2T for laminated iron, nanocrystalline }

previous example:

$L = \frac{\mu n^2 A_c}{l_m}$

saturation $\mu \downarrow \sim 1000$

so $L \downarrow \sim 1{,}000$



$v(t) = n A_e \frac{dB}{dt}$ → $\boxed{B(t) = \frac{1}{n A_e} \int_0^t v(t)\, dt}$

$v(t) = L \frac{di}{dt} = n A_c \frac{dB}{dt}$

$LI = n A_c B$

$I = \frac{n A_c B}{L}$

@ B_{sat}

$\boxed{I_{sat} = \frac{l_m}{\mu n} B_{sat}}$

for ungapped inductor

to $\uparrow I_{sat}$

(1) $\uparrow B_{sat}$

(2) $\uparrow l_m$

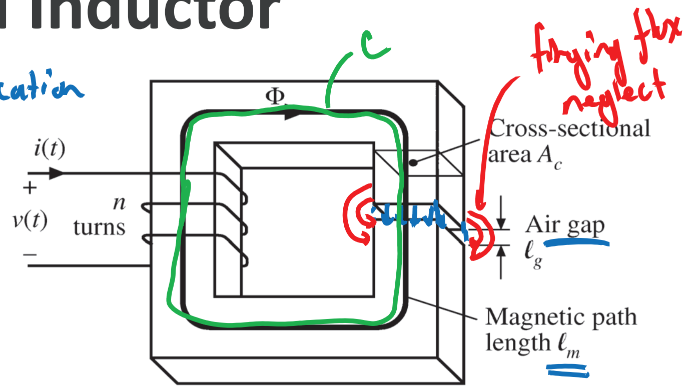
(3) $\downarrow \mu$

(4) $\downarrow n$

} also $\downarrow L$

Example: Gapped Inductor

Additional assumption/simplification
(4) No fringing flux



Faraday:

$$v(t) = n\frac{d\Phi}{dt} = nA_c\frac{dB}{dt}$$

Ampere:

$$\oint_C H\cdot dl = I_{enclosed} = n\,i(t) = H_c(l_m - l_g) + H_g l_g$$

Material characteristics:

in core $B = \mu H_c$ (unsaturated), in air $B = \mu_0 H_g$

$$n\,i(t) = \frac{B}{\mu}(l_m - l_g) + \frac{B}{\mu_0}l_g$$

$l_m \gg l_g$

$$B(t) = \frac{n\,i(t)}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}}$$

$$v(t) = nA_c\frac{d}{dt}\left(\frac{n\,i(t)}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}}\right)$$

$$v(t) = \frac{n^2 A_c}{\frac{l_m}{\mu} + \frac{l_g}{\mu_0}}\frac{di}{dt}$$

L