Low-frequency Averaging

$$\langle x(t)\rangle\big|_{T_s} \equiv \frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} x(\tau)\,d\tau \longrightarrow \text{Removes switching ripple on } x(t)$$

for an inductor

$$\langle i_L(t)\rangle\big|_{T_s} = \frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} i_L(\tau)\,d\tau$$

what is:

$$\frac{d\langle i_L(t)\rangle\big|_{T_s}}{dt} = \frac{d}{dt}\left[\frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} i_L(\tau)\,d\tau\right] = \frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} \frac{d}{d\tau} i_L(\tau)\,d\tau = \frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} \frac{v_L(\tau)}{L}\,d\tau$$

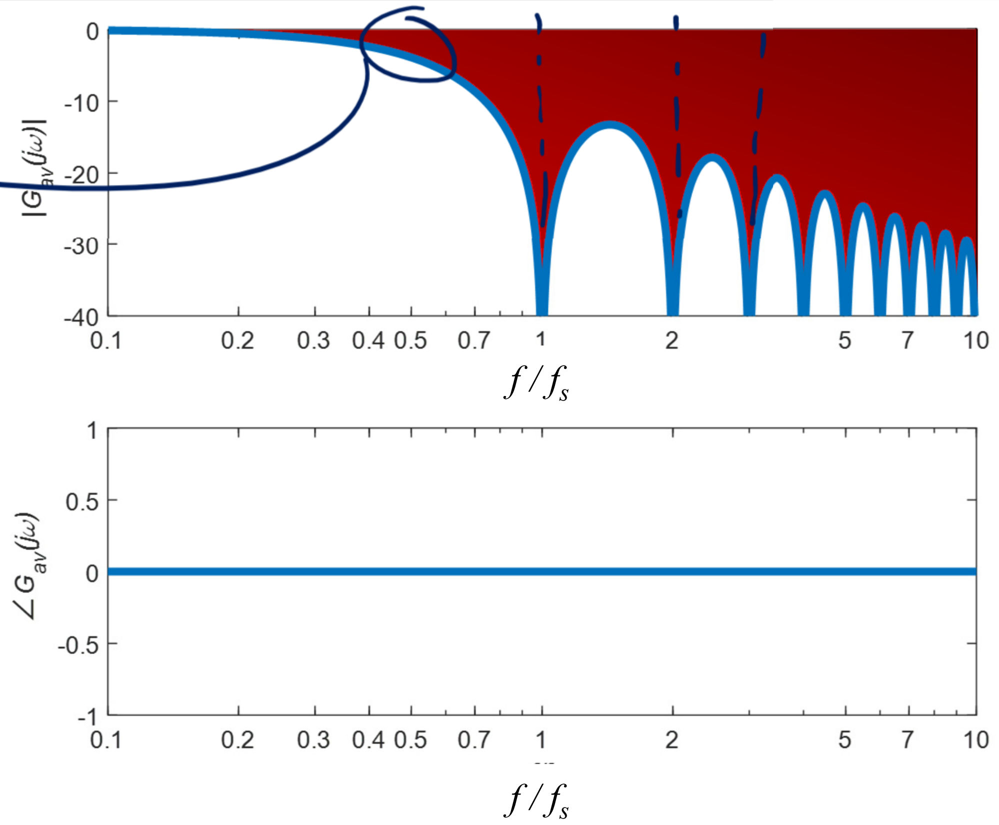
$$\boxed{L\frac{d\langle i_L(t)\rangle\big|_{T_s}}{dt} = \langle v_L(t)\rangle\big|_{T_s}}$$

& same for capacitor

as we go on. $\langle x(t)\rangle\big|_{T_s} \equiv \langle x\rangle$ (short-hand notation)

Averaging Operator

Average Models innacurate above $\sim f_s/10$

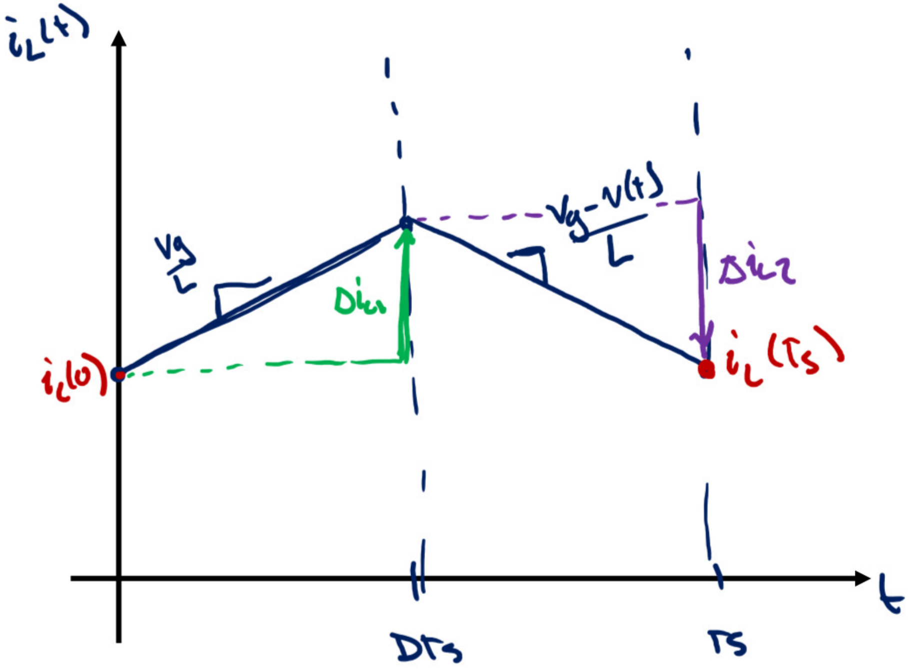


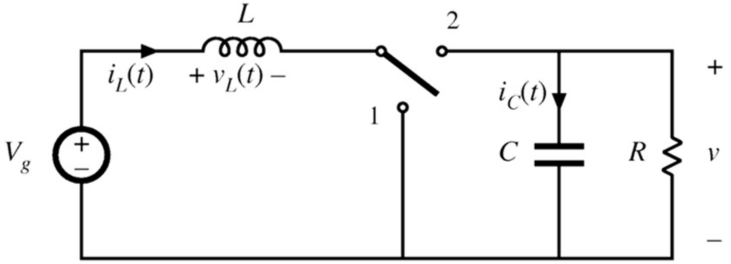
$$\mathcal{F}\left\{\frac{1}{T_s}\int_{t-\frac{T_s}{2}}^{t+\frac{T_s}{2}} x(\tau)\,d\tau\right\} = \frac{e^{j\omega\frac{T_s}{2}}X(j\omega) - e^{-j\omega\frac{T_s}{2}}X(j\omega)}{j\omega T_s}$$

$$= \mathrm{sinc}\left(\omega\frac{T_s}{2}\right)\cdot X(j\omega)$$

$$= G_{av}(j\omega)\cdot X(j\omega)$$

Averaging in Steady-State





$$i_L(T_s) = i_L(0) + \underbrace{\frac{1}{L}\int_0^{DT_s} V_g\,dt}_{\Delta i_{L1}} + \frac{1}{L}\int_{DT_s}^{T_s} V_g - v(t)\,dt$$

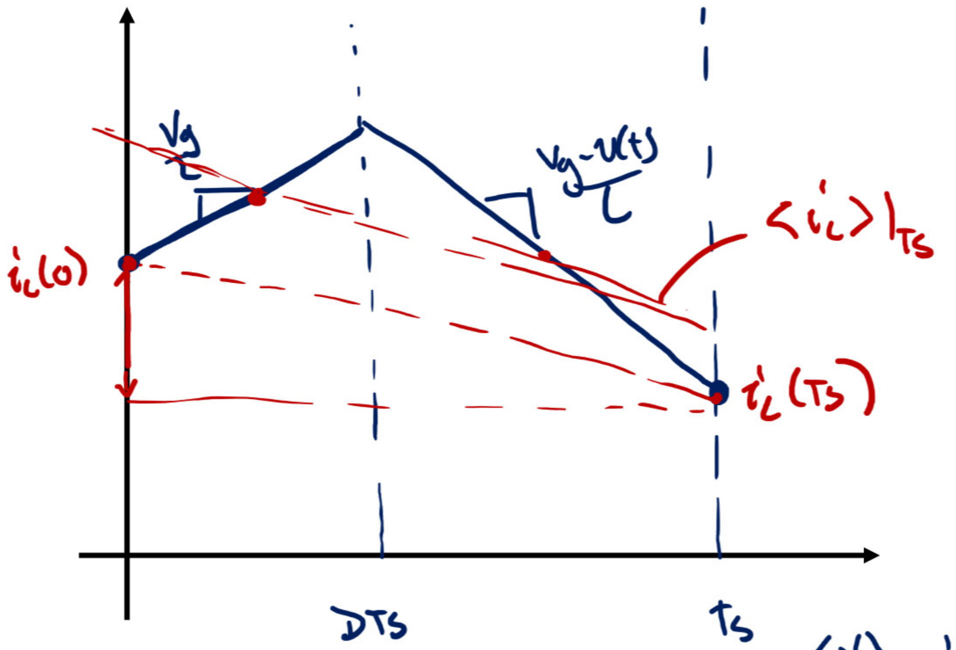
small Ripple Approx $v(t) \to V$

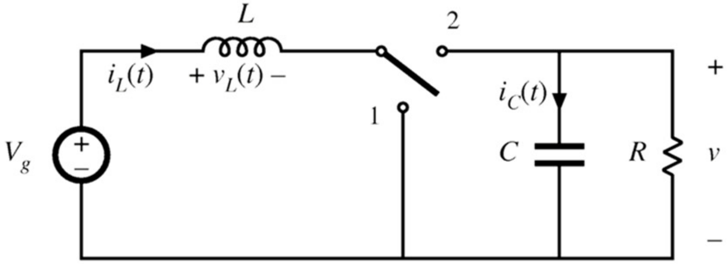
$$i_L(T_s) = i_L(0) + \frac{V_g}{L}DT_s + \frac{V_g}{L}D'T_s - \frac{V}{L}D'T_s$$ 1

2 $$L\left[\frac{\cancelto{0}{i_L(T_s) - i_L(0)}}{T_s}\right] = V_g - D'V$$

$$L\left.\frac{\cancelto{0}{\Delta i_L}}{\Delta t}\right|_{T_s} = V_g - D'V \longrightarrow 0 = V_g - D'V, \quad M = \frac{1}{D'}$$

Averaging in Transient Operation





$$i_L(T_s) = i_L(0) + \Delta i_{L1} + \Delta i_{L2}$$

Rather than SRA replacing signals w/ DC, we'll replace them with their average value (slowly time-varying)

$$i_L(T_s) = i_L(0) + \frac{\langle V_g \rangle}{L} T_s - \frac{\langle v \rangle}{L} d'(t) T_s$$

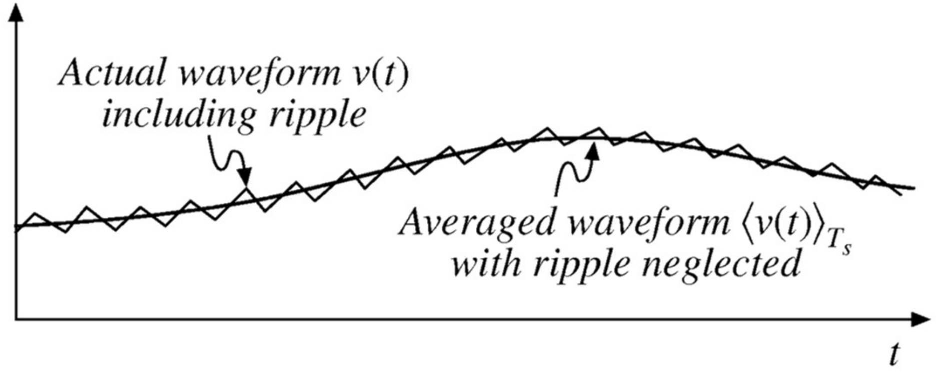
$$L \frac{[i_L(T_s) - i_L(0)]}{T_s} = \langle V_g \rangle - \langle v \rangle d'(t)$$

$$\boxed{L \frac{d\langle i_L(t) \rangle}{dt} = \langle v_L(t) \rangle = \langle V_g(t) \rangle - \langle v(t) \rangle d'(t)}$$

Averaging: Discussion

Replace signals with slowly-varying averages

— All switching ripple removed from signal



Limitations:

(1) Must have small switching ripple

(2) Valid only for $f \ll f_s$ (common $f < f_s/10$)

(3) Generally, circuit is still nonlinear after averaging

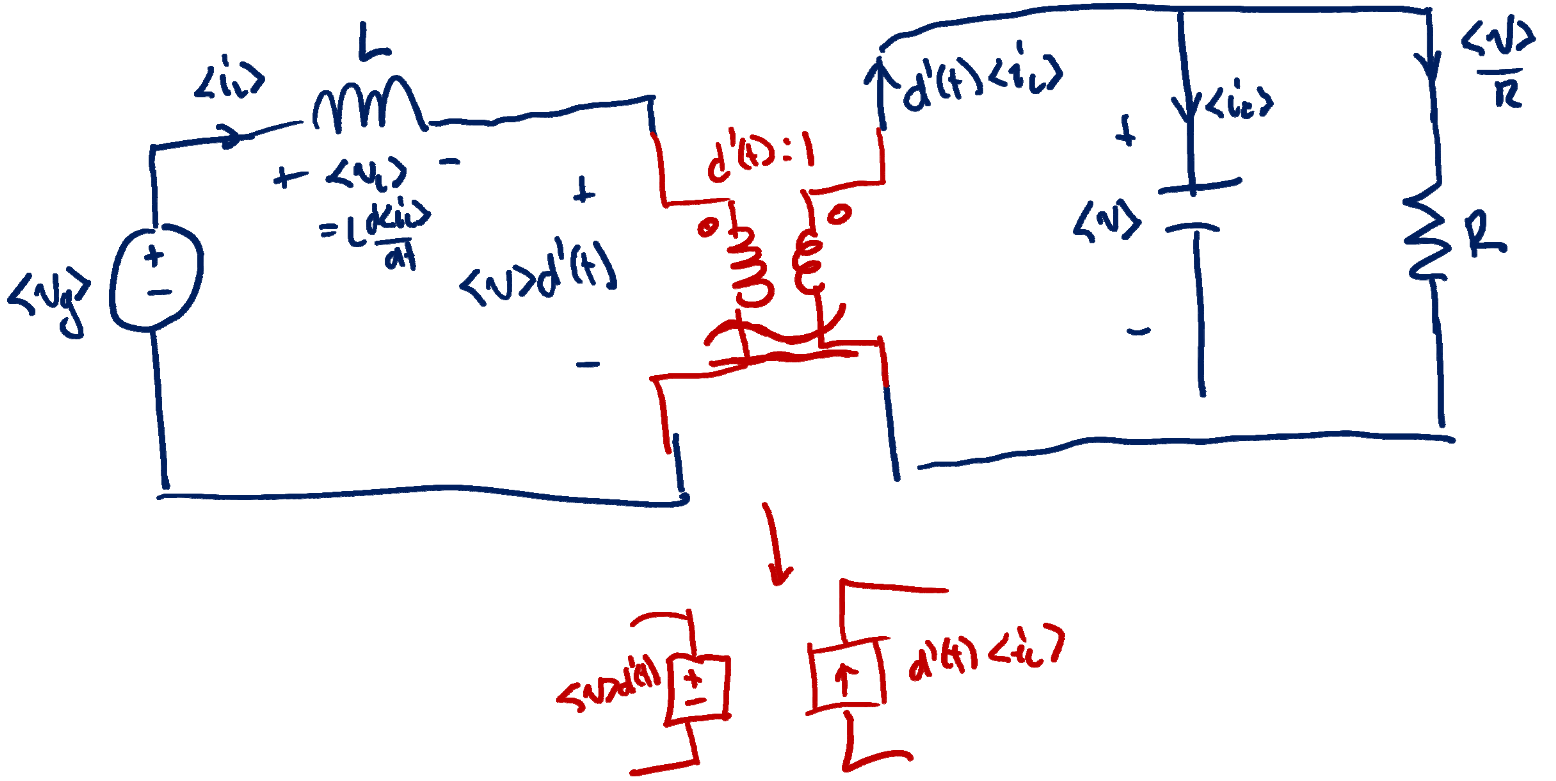
Averaged (Nonlinear) Model

Boost Converter

$$\langle v_L \rangle = L \frac{d\langle i_L \rangle}{dt} = \langle v_g \rangle - \langle v \rangle d'(t)$$

$$\langle i_C \rangle = C \frac{d\langle v \rangle}{dt} = d'(t)\langle i_L \rangle - \frac{\langle v \rangle}{R}$$

} Nonlinear, averaged model (is Time-invariant)

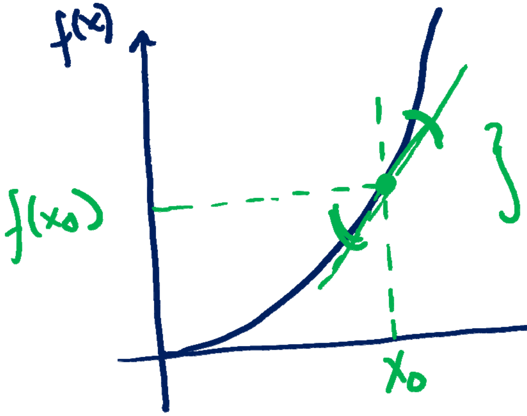


Nonlinear, time-invariant averaged model useful for simulation

SPICE can linearize this (Time-invariant) circuit

(i.e. ac simulations work)

Small Signal Modeling: Linearization



Nearby

$$f(x) \approx f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x - x_0)$$